AGH University of Science and Technology in Cracow

Department of Electronics

Laboratory Manual
Physics_1

Title:

Damped oscillations in RLC circuits

Experiment No.

8

2010 r.

1. Goal

To observe damped oscillations in the RLC circuit and measure the amplitude, period, angular frequency, damping constant and log decrement of damped oscillatory signals. To find the critical resistance for which the critical damping occurs.

2. What to learn?

Transfer of energy in LC circuit. The electrical-mechanical analogy. Differential equation describing damped simple harmonic motion in the RLC circuit.. Solution of this equation. Angular frequency of the damped oscillator. Damping constant. Angular frequency of the undamped oscillator. Forced oscillations and resonance. Kirchhoff's rules. Log decrement of damped oscillatory signals. Critical damping. How does the oscilloscope work?

3. Background

A circuit consisting of an inductor, a capacitor, and a resistor connected in series (Fig. 1) is called an RLC circuit. The resistance of the resistor R represents all of the resistance in the circuit. .With the resistance R present, the total electromagnetic energy U of the circuit (the sum of the electrical energy and magnetic energy) decreases with time because some portion of this energy is transferred to thermal energy in the resistance. Because of this loss of energy, the oscillations of charge, current and potential difference continuously decrease in amplitude, and the oscillations are said to be damped.

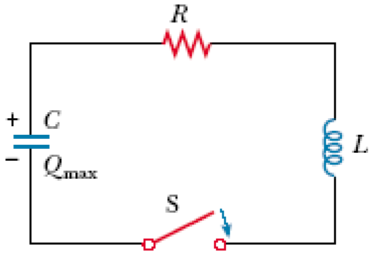

Fig.1. A series RLC circuit

Let's write the equation for the total electromagnetic energy U in the circuit at any instant. As we know the resistance doesn't store electromagnetic energy, so we have:

$$U = U_B + U_E = \frac{Li^2}{2} + \frac{q^2}{2C} \qquad *$$

This total energy decreases because energy is transferred to thermal energy. The rate of this transfer is:

$$\frac{dU}{dt} = -i^2 R \qquad **$$

where the minus sign indicates that U decreases.

By differentiating * with respect to time and then substituting the results in ** we obtain:

$$\frac{dU}{dt} = Li\frac{di}{dt} + \frac{q}{C}\frac{dq}{dt} = -i^2 R$$

$$i = \frac{dq}{dt} \text{ and } \frac{di}{dt} = \frac{d^2q}{dt^2}$$

$$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{1}{C}q = 0 \qquad ***$$

$$\frac{d^2q}{dt^2} + 2\frac{R}{2L}\frac{dq}{dt} + \frac{1}{LC}q = 0$$

$$\frac{R}{2L} = \beta, \qquad \frac{1}{LC} = \omega_0^2$$

$$\frac{d^2q}{dt^2} + 2\beta\frac{dq}{dt} + \omega_0^2 q = 0$$

The solution to this differential equation for damped oscillations in an RLC circuit is:

$$q = Qe^{-\frac{Rt}{2L}}\cos(\omega t + \varphi) \qquad ****$$

where:

$\omega = \sqrt{\omega_0{}^2 - \left(\frac{R}{2L}\right)^2}$ is the angular frequency of the damped oscillations,

$\omega_0 = \frac{1}{\sqrt{LC}}$ is the angular frequency of the undamped oscillations.

$$\omega = \sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2}$$

$\beta = \frac{R}{2L}$ is a circuit damping constant

The equation **** describes a sinusoidal oscillation with an exponentially decaying amplitude $Qe^{-\frac{Rt}{2L}}$.

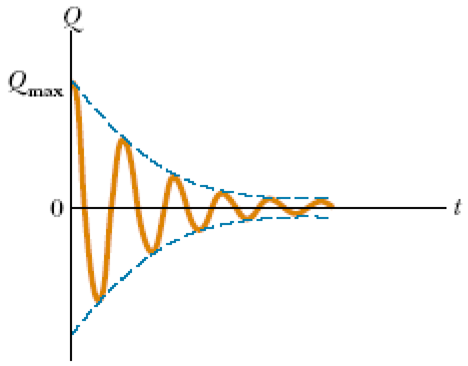

Fig. 2 Charge versus time for the damped RLC circuit.

The RLC circuit is analogous to the damped harmonic oscillator illustrated in Fig. 3 and described by the equation:

$$m\frac{d^2x}{dt^2}+b\frac{dx}{dt}+kx=0$$

Q corresponds to the position x of the block at any instant, L to the mass m of the block, R to the damping coefficient b, and C to $1/k$, where k is the force constant of the spring.

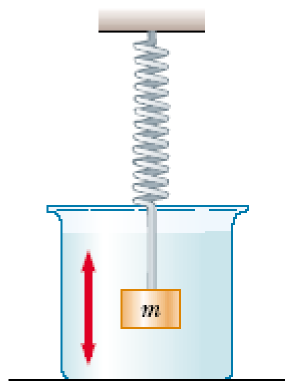

Fig. 3. A mechanical analogous to the RLC circuit – a block–spring system moving in a viscous medium.

The log decrement is determined from the ratio of the voltages of successive maxima of the damped oscillatory signal. This measurement is taken from the oscilloscope display, as shown in the Fig. 4. The figure shows the damped signal for two different values of resistance for fixed value of capacitance. The arrows indicate the first and second maxima.

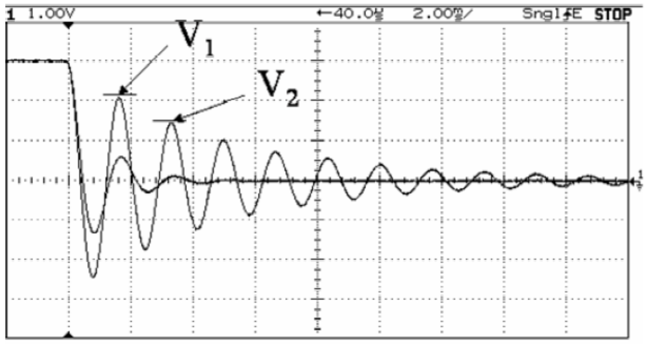

Fig. 4. Typical oscilloscope trace showing log decrement measurement

For damped oscillations, the log decrement is equal to:

$$\delta = \ln\left(\frac{V_1}{V_2}\right) = \frac{R}{2L}T$$

Critical damping in an RLC circuit is achieved when:

$$\frac{1}{LC} - \left(\frac{R}{2L}\right)^2 = 0\,,$$

from which we obtain:

$$R_{critical} = 2\sqrt{\frac{L}{C}}$$

We vary the resistance and search for a signal that has no undershoot and has a maximum decay rate. Typical oscilloscope traces for the overdamped, underdamped and critical damping are shown in Fig. 5.

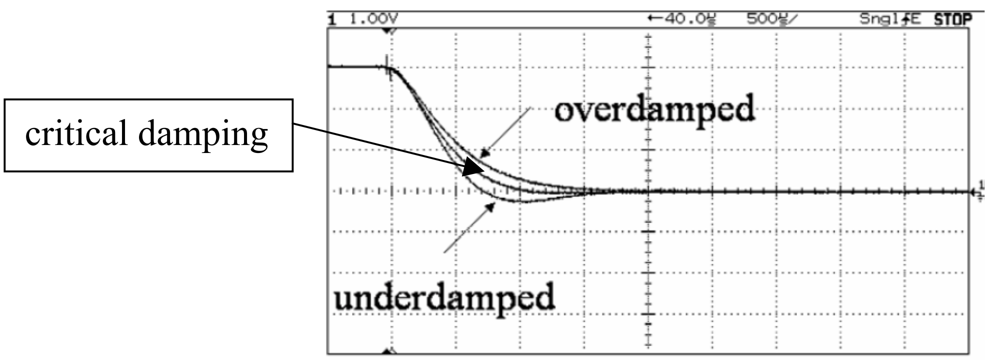

Fig. 5. Typical oscilloscope traces for critical damping

4. Equipment

RLC circuit consists of a capacitor C combined with its charging system, a decade resistor and a decade coil – all connected in series . The oscilloscope monitors the potential difference $u_C(t)$ across the capacitor as a function of time.

5. Measurements

1. Set the RLC circuit as shown in Fig. 6 . Fig. 7 shows the decade coil, the decade resistor and the capacitor with its charging system.

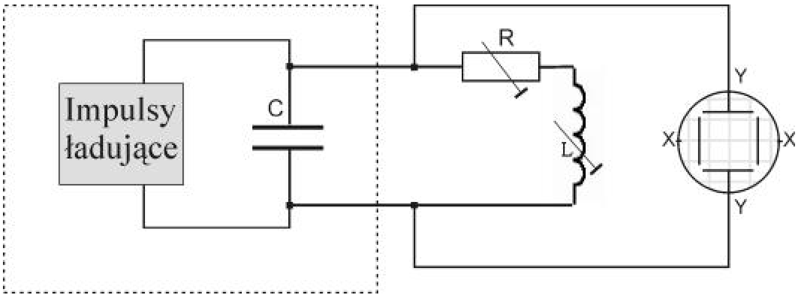

Fig. 6. The measurement system

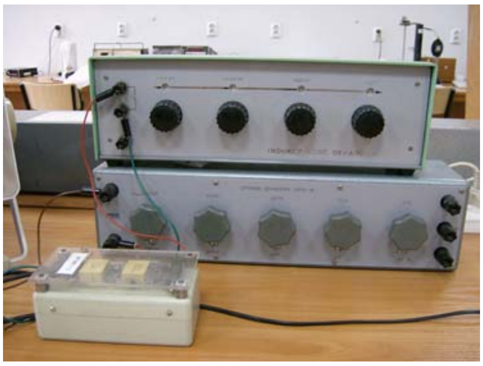

Fig. 7. The decade coil, the decade resistor and the capacitor with its charging system.

2. Set the inductance at the decade coil to the certain value L_1 . Set the resistance at the decade resistor to $R_0=0$. Observe the damping oscillations on the oscilloscope selecting the proper time base setting and vertical gain.
3. Measure on the oscilloscope display the period of oscillations T (Fig. 8). To increase accuracy, measure periods of a few successive oscillations and calculate the average value which will be further used to get the experimental angular frequency ω_{exp} of oscillations: $\omega_{\exp} = \dfrac{2\pi}{T}$.
4. Measure on the oscilloscope display the amplitude of maxima U_2, U_4, (Fig. 8)

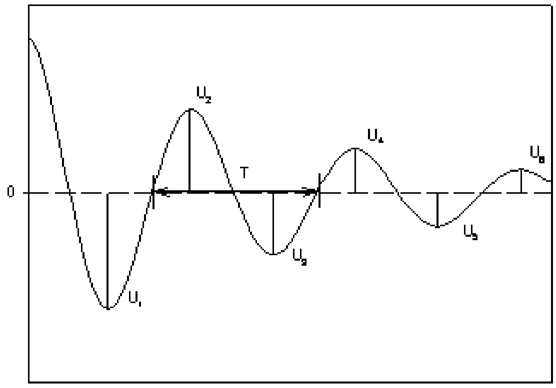

Fig. 8.. Damping oscillations

5. Keep the inductance L_1 at the decade coil unchanged and set the another resistance R_1 at the decade resistor. Observe the damping oscillations for the settings (L_1, R_1) and repeat the measurements 3-4. Set the resistance R_2 at the decade resistor and repeat the observations. Investigate how the increase in resistance R affects the oscillation parameters.
6. Increase R until the critical aperiodic oscillations occur and write down the $R_{critical}$ in the measurement table.
7. Change the inductance to L_2 at the decade coil and for resistances $R_0=0$, R_1, R_2 repeat all measurements according to 2-6. Investigate how the increase in inductance L affects the oscillation parameters.
8. Write down the results in Table 1.

6. Data Handling

1. Calculate the log decrements $\delta = \ln\left(\frac{U_i}{U_{i+2}}\right)$ and the experimental damping coefficient β_{exp} of oscillations: $\beta_{\exp} = \frac{\ln \frac{U_i}{U_{i+2}}}{T}$ for all combinations (L_1, R_0), (L_1, R_1), (L_1, R_2), (L_2, R_0), (L_2, R_1), (L_2, R_2)

2. Calculate the parasitic resistance of a coil R_p taking advantage from the fact that at the beginning there is always set $R=0$ at the resistance decade. the total resistance R_{total} in the RLC circuit consists of the resistance of a decade R and the parasitic resistance of a coil R_p: $R_{total} = R + R_p$.

$$\beta_{\exp} = \frac{R_{total}}{2L}, \quad R_{total} = \beta_{\exp} \cdot 2L$$

When $R = 0 \Rightarrow R_{total} = R_p \Rightarrow R_p = \beta_{\exp} \cdot 2L$

3. Calculate experimental angular frequency of oscillations $\omega_R = \dfrac{2\pi}{T}$.

4. For the resonance conditions in RLC circuit, calculate capacitance C for each inductance L using the equations:

$$\omega_R = \sqrt{\omega_0^{\ 2} - \left(\frac{R_{total}}{2L}\right)^2}, \ \omega_0 = \sqrt{\frac{1}{LC}}$$

4.Calculate the uncertainty of the capacitance ΔC

5.For the case of critical damping, compare the experimental value of critical resistance R_C with its theoretical value calculated on the basis of the equation:

$$R_{Ctheory} = 2\sqrt{\frac{L}{C}}$$

5. Write the conclusions.

Literature:

1. Halliday, Resnick “Fundamentals of Physics - 8th edition”, John Wiley 2007,
2. Zięba “Pracownia Fizyczna Wydziału Fizyki I Techniki Jądrowej AGH”, Uczelniane Wydawnictwo Naukowo-Dydaktyczne 1999.

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Table 1

Inductance L[H]	Resistance R[Ω]	Period of oscillations T [ms]	Amplitude of oscillations U_i, U_{i+2} [V]	Critical Resistance experimental $R_{critical-exp}$[Ω]	Damping coefficient experimental β_{exp}	Total resistance R_{total} [Ω]	Parasitic resistance R_{par} [Ω]	Angular frequency of oscillations experimental ω_{exp} [rad/s]	Capacitance C[F]	Uncertainty ΔC	Critical resistance theory $R_{critical}$ [Ω]
L_1	R=0		U_2								
			U_4								
	R_1		U_2								
			U_4								
	R_2		U_2								
			U_4								
L_2	R=0		U_2								
			U_4								
	R_1		U_2								
			U_4								
	R_2		U_2								
			U_4								