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The rod and hole paradox re-examined

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Abstract

In the rod and hole paradox as described by Rindler (1961 *Am. J. Phys.* **29** 365–6) ('length contraction paradox'), a rigid rod moves at high speed over a table towards a hole of the same size. A bystander expects the rod to fall into the hole, but a co-moving observer expects it to pass unhindered over the hole. According to the accepted solution as first described in that paper, the entire rod must fall somewhat into the hole and therefore cannot remain rigid when the hole moves underneath it. We present an improved approach that is based on retardation due to speed of stress propagation, and in which proper stiffness is not affected. After showing how to solve the similar but simpler car and hole paradox, we find with the same approach that the rod as depicted in Rindler's paper will not fall into the hole as was claimed and that it may pass practically unhindered over it.

Introduction

In accordance with the principle of relativity, the Lorentz transformations [1] describe differences in observation depending on motional speed. If the length of an object at rest is L_0 , then when it moves relative to a coordinate system with a speed v it is expected to appear contracted from L_0 to L as follows:

$$L = L_0/\gamma \quad (1)$$

where $\gamma = 1/\sqrt{1 - v^2/c^2}$ and c is the speed of light in vacuum.

In a co-moving coordinate system the length L' will still appear to be equal to L_0 .

The often counter-intuitive consequences led over the years to a number of paradoxes. One of these is the rod and hole paradox, which was first published in 1961 [2]. The conclusions of that paper have been regularly copied and the paradox reappeared in a modern physics book [3] in the form of a snowboarder who encounters a hole that is slightly shorter than the board. According to that book, based on [2], at sufficient speed the snowboard will fall into the hole. Before we take a closer look at that paradox, we will first consider the following variant that was presented as a real contradiction [4].

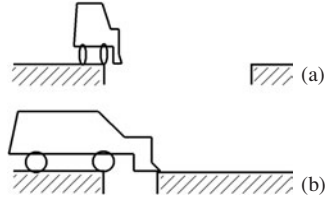


Figure 1. Situation as seen from stationary frame (a) and from moving frame (b).

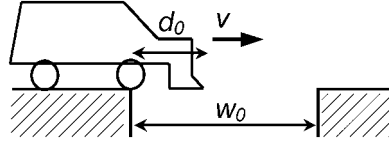


Figure 2. Schematic side view with proper lengths (when in rest or Newtonian).

1. Car and hole paradox

A car is in inertial motion on a road, going at a speed near to that of light and there is a hole in the road in front of it. For this discussion we will assume that the car is constructed in such a way that it has a spoiler that almost touches the road (see figure 1).

According to a bystander on the road, the car is contracted, so that it looks certain that the car will fall a little and collide with the outer edge. But according to the driver of the car, the spoiler may already be beyond the contracted hole when the wheels reach it. Clearly, as depicted in figure 1(b), the front of the car cannot collide with the hole. As this looks like a contradiction, we should do a careful calculation.

We will refer to two inertial coordinate systems: the ‘stationary frame’ which is attached to the road, and the ‘moving frame’ which is attached to the car while it is in inertial motion.

Let d_0 be the proper distance from the front of the spoiler to the wheel axis, and w_0 the proper hole width (figure 2). With ‘proper’ we mean as measured in a reference frame in which the object is in rest.

As soon as the front wheels reach the left edge of the hole, they lose their support and a stress propagates through the car at a proper propagation speed u_0 . To keep it simple we do not account for a suspension system and assume exactly horizontal stress propagation.

1.1. Seen from stationary frame (road)

With the stress propagation in the same direction as the motion of the car, the apparent speed of stress propagation u is assumed to be [5]

$$u = \frac{u_0 + v}{1 + u_0 v / c^2}. \quad (2)$$

It takes a time Δt before the front of the spoiler’s motion is affected, where

$$\Delta t = \frac{d}{u - v} \quad \text{with} \quad d = d_0 / \gamma. \quad (3)$$

During that time the car has moved a distance $\Delta x = v \Delta t$ so that the spoiler tip should pass the hole if

$$v \Delta t > (w_0 - d). \quad (4)$$

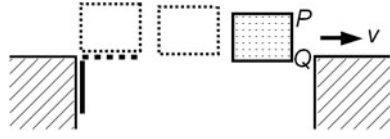


Figure 3. Situation as seen from the stationary frame according to [2]. A stationary observer releases a trap door in the hole at high speed when the rod is just completely above the hole. The entire rod is thought to fall downwards into the hole. In the drawing $\gamma = 4$; in the text $\gamma = 10$.

As $d = d_0/\gamma$, and by substituting Δt from equation (3)

$$w_0 < \left(\frac{u}{u-v} \right) \frac{d_0}{\gamma}. \quad (5)$$

Substituting u from equation (2), we obtain

$$w_0 < \gamma(1 + v/u_0)d_0. \quad (6)$$

We conclude that for a hole shorter than w_0 , the spoiler tip should not collide with the wall.

1.2. Seen from moving frame (car)

In the moving frame, the width w' of the hole appears to be reduced to w_0/γ , and the effect of the hole's edge on the wheels will reach the spoiler after a time $\Delta t' = d_0/u_0$.

The driver of the car thus concludes that the spoiler tip will pass the hole if

$$d_0/u_0 > (w_0/\gamma - d_0)/v \quad (7)$$

or

$$w_0 < \gamma(1 + v/u_0)d_0. \quad (8)$$

As was to be expected, this is in perfect agreement with the calculation using the stationary frame. There is no contradiction.

Note that we assumed that proper stiffness is unaffected by motional speed: the spoiler will not bend down due to its speed relative to the hole.

2. Rod and hole paradox

A 'rigid' rod of 25 cm length moves at high speed over a table with a 25 cm wide hole that has a trap door in it. When the rod is just completely above the hole, a stationary observer quickly releases the door. That observer ('A') expects the rod to fall somewhat and be stopped by the far edge of the hole, but a co-moving observer ('B') expects it to pass unhindered over the hole. The gravitational field can be replaced by a magnetic field if the rod is made of iron. The paper [2] stated that 'There is no doubt that A's description of events is correct. The rod simply cannot remain rigid in B's internal frame.' The author apparently assumed that points P and Q are immediately in free fall (see figure 3) and that consequently in its proper frame (in its own rest frame) the rod loses its rigidity and bends into the hole. But that would be in disagreement with the known laws of physics: a rod's proper material properties such as stiffness do not change due to its speed relative to another system.

The rod will move in accordance with solid plate mechanics. Since infinite rigidity does not exist, the rod is slightly compressed under its own weight and at each point where the support vanishes there will be an immediate downward motion of the bottom surface. If the downward displacement is more than the necessary clearance for sliding, the moving observer

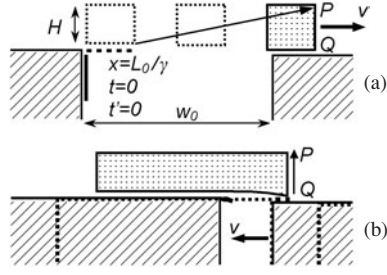


Figure 4. Motion of the rod relative to the table—allowing for decompression—during the stress propagation inside the rod (thin arrows) as observed from the stationary frame (a) and from the moving frame (b). Clearance and decompression are greatly exaggerated.

will have to agree with the stationary observer that the rod is going to touch the far edge of the hole. However, this paradox has as prerequisite that a rigid rod can pass over a small hole. For a further discussion of the paradox we will therefore assume that, if required, the far side of the hole is adjusted to allow for downward motion of point Q as expected by the moving observer (figure 4(b)).

Full ‘falling’ and ‘bending’ of the rod in the sense of the paradox can be distinguished from decompression by regarding the downward motion of point P as illustrated in figure 3. Before the entire rod falls as interpreted from the stationary frame, or before it fully bends as seen from the moving frame, the stress change must propagate from the bottom to the top front corner (figure 4, from Q to P)—this retardation effect was overlooked in [2].

We will calculate the time and distance that are required for this stress propagation.

As the trap door rests in the stationary frame, point Q is affected by the release at a distance of $1/\gamma$ of the hole width to the right of the left wall. The proper width of the hole is again w_0 , the rod’s length is L_0 and its (invariant) height is H . Its proper speed of stress propagation is u_0 .

2.1. Seen from stationary frame (table)

The stress propagates from Q to P in a time Δt .

Using the relativistic equation for time dilation

$$\Delta t = \gamma \Delta t' \quad (x' = \text{constant}) \quad (9)$$

we obtain

$$\Delta t = \gamma H/u_0. \quad (10)$$

During that time, the rod has advanced by $\Delta x = v \Delta t$.

The gap is made up of the difference between hole width and contracted rod length. The rod will reach the other side of the hole before point P is affected if this gap is smaller than the advance $v \Delta t$, thus if

$$(w_0 - L_0/\gamma) < v\gamma H/u_0. \quad (11)$$

Taking as in [2] $w_0 = L_0$, the rod will reach the other side before it fully falls if

$$\gamma \frac{H}{L_0} \frac{v}{u_0} + \frac{1}{\gamma} > 1. \quad (12)$$

2.2. Seen from moving frame (rod)

The stress arrives at P after a time $\Delta t' = H/u_0$.

During that time the right edge of the hole has moved a distance $\Delta x' = v \Delta t'$ to the left.

The starting position of point Q is for all observers at a distance of $1/\gamma$ of the hole width to the right of the left wall (figure 4), so that the gap is equal to the contracted gap of the stationary frame. The rod will reach the other side of the hole before point P is affected if this gap is smaller than the advance $v \Delta t'$, thus if

$$(w_0 - L_0/\gamma)/\gamma < vH/u_0. \quad (13)$$

With $w_0 = L_0$, the rod will reach the other side before it fully bends if

$$\gamma \frac{H}{L_0} \frac{v}{u_0} + \frac{1}{\gamma} > 1, \quad (14)$$

which is again in perfect agreement with the equation obtained for the rest frame (equation (12)).

For iron, $u_0 \approx 6000 \text{ m s}^{-1}$ (speed of sound). Taking the numerical values of [2], i.e. $L_0 = 25 \text{ cm}$ and $\gamma = 4$, only a thin foil of thickness $H < 1 \text{ }\mu\text{m}$ will fully fall while above the hole, and for $\gamma = 10$ this should occur with $H < 0.5 \text{ }\mu\text{m}$. In the moving frame this will appear as bending at the place of the hole. An iron rod as depicted in [2] (figure 3, about 5 cm thick) will advance kilometres beyond the hole before the top surface starts to move down.

With a force equal to that of terrestrial gravitation, the decompression displacement of the iron rod above the hole will be much smaller than the diameter of an atom, as estimated with Hooke's law. The free-fall displacement will even be negligible; consequently the rod may pass practically unhindered over the hole. We did not examine dynamical aspects in detail.

3. Conclusions

The accepted solution of the rod and hole paradox leads to a rejection of the fundamental physics concept of cause and effect: it ignores the fact that motional speed does not affect proper stiffness. We presented an improved approach in which the rod does not become less rigid in its own frame and that is based on retardation due to the speed of stress propagation.

The rod and hole paradox illustrates that infinite rigidity is impossible and that bending in one frame may appear as falling in another frame. But a rod as depicted will only experience a very incomplete decompression above the hole; apart from that, the rod will not fall into the hole. If the displacement due to decompression of the rod is less than the clearance for sliding or small compared to atomic size, the rod will pass practically unhindered over the hole and will not be stopped by it.

We also gave the solution for the similar car and hole paradox. The difference in observed speed of stress propagation compensates exactly for the difference in observed length.

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