Wstęp do oddziaływań hadronów

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Wykład 12

Parity operator

- The parity operator performs spatial inversion through the origin:

$$\psi'(\vec{x},t) = \hat{P}\psi(\vec{x},t) = \psi(-\vec{x},t)$$

- To preserve the normalisation of the wave-function $\hat{P}$ must be unitary:

$$\langle\psi|\psi\rangle = \langle\psi'|\psi'\rangle = \langle\psi|\hat{P}^{\dagger}\hat{P}|\psi\rangle$$

- Since in addition $\hat{P}\hat{P} = I$ this implies that $\hat{P}$ is hermitian (what means it corresponds to an observable quantity) with eigenvalues $P = \pm 1$:

$$\hat{P}\hat{P}\psi(\vec{x},t) = \hat{P}\psi(-\vec{x},t) = \psi(\vec{x},t)$$

- It was shown previously that the parity operator for Dirac particles is $\hat{P} = \gamma^0$, and spin-half particles have opposite parity to the corresponding anti-particles (convention: particles $+1$, anti-particles -1):

$$P(e^-) = P(\mu^-) = P(\tau^-) = P(\nu) = P(q) = +1$$

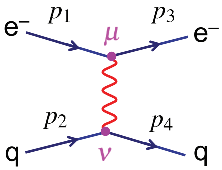
$$P(e^+) = P(\mu^+) = P(\tau^+) = P(\bar{\nu}) = P(\bar{q}) = -1$$

- From QFT it can be shown that vector bosons responsible for electromagnetic, strong and weak forces all have negative intrinsic parity:

$$P(\gamma) = p(g) = P(W^{\pm}) = P(Z) = -1$$

Parity conservation in QED

- The matrix element for the process $e^- q \to e^- q$ is:



$$M = \frac{Q_q e^2}{q^2} j_e \cdot j_q$$

with $\quad j_e^\mu = \overline{u}(p_3)\gamma^\mu u(p_1) \quad$ and $\quad j_e^\nu = \overline{u}(p_4)\gamma^\nu u(p_2)$

- Transformation of Dirac spinors and adjoint spinors under parity operator:

$$u \xrightarrow{\hat{P}} \hat{P}u = \gamma^0 u \quad \Rightarrow \quad \overline{u} = u^\dagger\gamma^0 \xrightarrow{\hat{P}} (\hat{P}u)^\dagger\gamma^0 = u^\dagger\gamma^{0\,\dagger}\gamma^0 = u^\dagger\gamma^0\gamma^0 = \overline{u}\gamma^0$$

- Transformation of the four-vector current under parity operator:

$$0: \qquad j^0 \xrightarrow{\hat{P}} \overline{u}\gamma^0\gamma^0\gamma^0 u = \overline{u}\gamma^0 u = j^0$$

$$k = 1,2,3: \qquad j^k \xrightarrow{\hat{P}} \overline{u}\gamma^0\gamma^k\gamma^0 u = -\overline{u}\gamma^k\gamma^0\gamma^0 u = -\overline{u}\gamma^k u = -j^k$$

- Consequently the four-vector current scalar product remains unchainged under parity transformation, i.e. the QED matrix elements are parity invariant, what means that parity is conserved in QED.
- The QCD vertex is similar, what means that parity is conserved in QCD.

Parity violation in β-decay

Example: Let us consider two decays (J^P values are shown in brackets):

$$\rho^0(1^-) \to \pi^+(0^-) + \pi^-(0^-) \quad \text{and} \quad \eta(0^-) \to \pi^+(06-) + \pi^-(0^-)$$

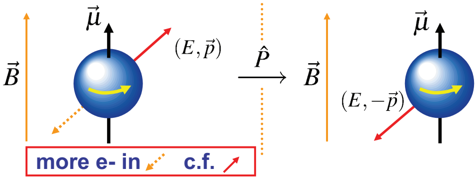
Conservation of parity in the two strong decays can be expressed as:

$$P(\rho^0) = P(\pi^+) \cdot P(\pi^-) \cdot (-1)^{\ell=1} \quad \Rightarrow \quad -1 = (-1)(-1)(-1) \quad \text{allowed}$$
$$P(\eta) = P(\pi^+) \cdot P(\pi^-) \cdot (-1)^{\ell=0} \quad \Rightarrow \quad -1 \neq (-1)(-1)(+1) \quad \text{not allowed}$$

- Parity violation in a weak β-decay of polarized cobalt-60 nuclei (Wu, 1957):

$$^{60}\text{Co} \to ^{60}\text{Ni}^\star + e^- + \overline{\nu}_e$$

- The ^{60}Co nuclei, which posses permanent magnetic moment $\vec{\mu}$ were aligned in strong magnetic field $\vec{B}$ and a β-decay electrons were detected at different polar angles with respect to this axis.
- Both, $\vec{\mu}$ and $\vec{B}$, are axial vectors.
- If parity were conserved we expect equal rates for electrons in directions along and opposite to the nuclear spin.
- Observe more e^- emitted in the hemisphere opposite to direction of $\vec{B}$.



- Conclusion: parity is violated in weak interactions (different form of vertex).

Scalars, pseudoscalars, vectors and axial vectors

- The parity properties of different rank quantities:

	Rank	Parity	Example	Bilinear form	Boson spin
Scalar	0	+	temperature, mass	$\overline{\psi}\phi$	0
Pseudoscalar	0	−	helicity, $h \propto \vec{S}\cdot\vec{p}$	$\overline{\psi}\gamma^5\phi$	0
Vector	1	−	momentum	$\overline{\psi}\gamma^\mu\phi$	1
Axial vector	1	+	angular momentum	$\overline{\psi}\gamma^\mu\gamma^5\phi$	1

- The requirement of the Lorentz invariance of the interaction matrix element severely restricts the possible form of bilinear combinations of two spinors $\overline{u}(p')\Gamma u(p)$, where Γ is a 4×4 matrix formed from products of Dirac γ-matrices.
- Apart of the four listed above bilinear covariants, one more (tensor which corresponds to spin-2 boson) is possible.
- The most general Lorentz invariant form for the interaction between a fermion and a boson is a linear combination of the bilinear covariants.

$V - A$ structure of the weak interaction

- The most general Lorentz invariant form for the interaction between a fermion and a spin-1 (vector) boson is a linear combination of the vector and axial vector currents (g_V and g_A are vector and axial vector coupling constants):

$$j^\mu \propto \overline{u}(p')(g_V \gamma^\mu + g_A \gamma^\mu \gamma^5) u(p) = g_V j_V^\mu + g_A j_A^\mu$$

- Transformation of the four-vector current under parity operator:

$$0: \qquad j_A^0 \xrightarrow{\hat{P}} \overline{u}\gamma^0\gamma^0\gamma^5\gamma^0 u = -\overline{u}\gamma^0\gamma^5 u = -j_A^0$$

$$k = 1,2,3: \qquad j_A^k \xrightarrow{\hat{P}} \overline{u}\gamma^0\gamma^k\gamma^5\gamma^0 u = (-)(-)\overline{u}\gamma^k\gamma^0\gamma^0\gamma^5 u = \overline{u}\gamma^k\gamma^5 u = j_A^k$$

- Consequently the scalar product of axial vector four-vector currents remains unchanged under parity transformation.
- However, the scalar product of a vector and axial vector currents changes sign under parity transformation:

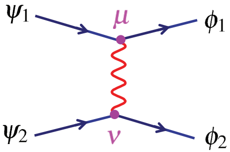
$$j_{V1} \cdot j_{A2} \xrightarrow{\hat{P}} j_1^0(-j_2^0) - (-j_1^k) j_2^k = -j_{V1} \cdot j_{A2}$$

- Hence the combination of vector and axial vector currents provides a mechanism to explain the observed parity violation in the weak interaction.

$V - A$ structure of the weak interaction

- Consider general charged-current weak interaction process $\psi_1\psi_2 \to \phi_1\phi_2$:

$$\begin{aligned} j_1 &= \overline{\phi_1}(g_V\gamma^\mu + g_A\gamma^\mu\gamma^5)\psi_1 = g_V j_1^V + g_A j_1^A \\ j_2 &= \overline{\phi_2}(g_V\gamma^\mu + g_A\gamma^\mu\gamma^5)\psi_2 = g_V j_2^V + g_A j_2^A \\ M_{fi} &\propto j_1 \cdot j_2 = g_V^2 j_1^V \cdot j_2^V + g_A^2 j_1^A \cdot j_2^A + \\ &\qquad + g_V g_A (j_1^V \cdot j_2^A + j_1^A \cdot j_2^V) \end{aligned}$$



- Consider now the parity transformation of the scalar product of the currents:

$$j_1 \cdot j_2 \xrightarrow{\hat{P}} g_V^2 j_1^V \cdot j_2^V + g_A^2 j_1^A \cdot j_2^A - g_V g_A (j_1^V \cdot j_2^A + j_1^A \cdot j_2^V)$$

- Relative strength of the parity violating part is given by $\dfrac{g_V g_A}{g_V^2 + g_A^2}$
- If either g_V or g_A is zero then parity is conserved.
- Maximum violation occurs when $|g_A| = |g_V|$ and corresponds to pure $V - A$ or $V + A$ interaction.
- From experiment it is known that the weak interactions due to $W^\pm$ bosons exchange, are of the $V - A$ type with vertex factor: $\dfrac{-ig_W}{\sqrt{2}}\dfrac{1}{2}\gamma^\mu(1-\gamma^5)$
- The four-vector current is given by: $j^\mu = \dfrac{g_W}{\sqrt{2}}\overline{u}(p')\dfrac{1}{2}\gamma^\mu(1-\gamma^5)u(p)$

Chiral structure of the weak interaction

- Any spinor can be decomposed into left- and right-handed chiral states:

$$u = P_R u + P_L u = \frac{1}{2}(1+\gamma^5)u + \frac{1}{2}(1-\gamma^5)u = a_R u_R + a_L u_L$$

- In QED only chiral spinors RR and LL give non-zero valus in vector current:

$$\overline{\psi}\gamma^\mu\phi = \overline{\psi}_R\gamma^\mu\phi_R + \overline{\psi}_R\gamma^\mu\phi_L + \overline{\psi}_L\gamma^\mu\phi_R + \overline{\psi}_L\gamma^\mu\phi_L$$

e.g. $$\overline{\psi}_R\gamma^\mu\phi_L = \frac{1}{2}\psi^\dagger(1+\gamma^5)\gamma^0\gamma^\mu\frac{1}{2}(1-\gamma^5)\phi = \frac{1}{4}\overline{\psi}\gamma^\mu(1+\gamma^5)(1-\gamma^5)\phi = 0$$

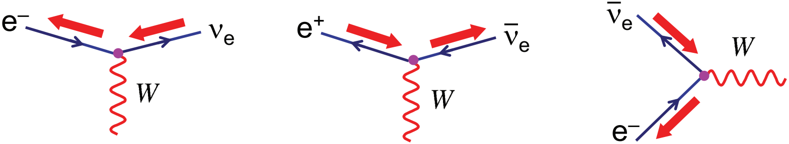
- For the weak interaction, the $V-A$ vertex factor includes the lef-handed chiral projection operator, therefore:

$$j^\mu_{RR} = \frac{g_W}{\sqrt{2}}\overline{u}_R(p')\gamma^\mu\frac{1}{2}(1-\gamma^5)u_R(p) = \frac{g_W}{\sqrt{2}}\overline{u}_R(p')\gamma^\mu P_L u_R(p) = 0$$

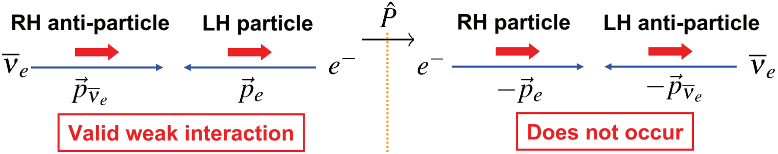
- For antiparticle spinors P_L projects out RH chiral states: $\frac{1}{2}(1-\gamma^5)v = v_R$
- Only the left-handed chiral components of particle spinors and right-handed chiral components of antiparticle spinors participate in charged current weak interactions.

Helicity structure of the weak interaction

- In the ultra-relativistic limit ($E \gg m$) chiral and helicity states are the same.
- The $V - A$ term in the weak interaction vertex projects out LH helicity particle states and RH helicity antiparticle states, e.g. the only possible electron-neutrino interactions are:



- The helicity structure of the weak interactions is the origin of parity violation.
- e.g. weak interaction of a high energy LH e^- and RH $\bar{\nu}_e$ is allowed. However in parity mirror, the vector quantities are reversed, $\vec{p} \to -\vec{p}$, but the axial vector spins of the particles remain unchanged, giving a RH particle and LH antiparticle. Hence the parity operation transforms an allowed weak process into one that is not alolowed.



Helicity in pion decay