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A determination of the total roll separating force during rolling of bimetal rods in the grooves

The proper design of the rolling process of the bimetal rods in the grooves requires taking into account many constraints affecting the process. During the technology design the rolling separating force is a parameter among many others, which should be determined.

In this paper a theoretical formula to determine the roll separating force during the bimetallic rods rolling in grooves, is proposed. It is obtained by analysis of the computer modeling results of stress distributions in the deformation zone as well as the results of experimental rolling. The rolling process in the passes round-horizontal oval is studied. Rolling force obtained from the proposed formula is compared to modeling by the SortRoll program that uses the full 3D plasticity solution of the finite element method (FEM).

In the theoretical and experimental research, bimetallic specimen with the different ratio of the soft clad layers to hard core as well as the different flow stress ratio are used.

The proper design of the process of the rolling of the bimetal rods in grooves requires that several limitations affecting the process should be considered. When technology is being developed, among other things, energy and force parameters, should be taken into account. Literature reports numerous solutions intended for determining these parameters during the rolling of monometallic materials [1-3]. Each of those solutions has some simplifications which, more or less, affect the accuracy of determined values. In the literature, there are no theoretical solutions or analysis of the force parameters during the rolling of the doublelayer rods in stretching passes. In the 1960s, studies were conducted only with the aim of the determination of the force parameters for the rolling of multi-layered sheets on the smooth roll face [4, 5]. Relationships that were derived by those studies cannot be, however, used for the analytical determination of the overall roll separating force during the rolling of bimetallic rods.

A subject of the paper is a development of analytical model of rolling force during rolling of bimetallic rods. Results of calculation by analytical model are compared with results of FEM model and experimental data.

Therefore, the present study has derived theoretical formulae that enable the calculation of the overall roll separating force during the rolling of the bimetallic rods in the stretching passes. The rolling force obtained by using the proposed relationship was compared with the results of computation performed by the SortRoll software which use the full 3D plasticity solution by finite element method (FEM). The results were compared with the measurements of the overall roll separating force recorded during the rolling.

An analytical determination of the overall roll separating force during the rolling of the bimetal rods in the passes

A theoretical relationship for determination of the overall roll separating force during the rolling of the bimetal rods is proposed in this study. This has been obtained by analysis of the results of computer simulations of stress distributions in the roll gap and experimental testing of the mode of bimetal rod flow during the rolling in the oval-vertical ovaloval-round pass design.

The overall roll separating force in the rolling of the bimetal rods can be calculated by the following formula:

$$P_{bim} = p_{av_{bim}} \cdot S_{dbim} \tag{1}$$

An area of the horizontal projection of the contact metal of the roll with surface, S_{dbim} , was determined as for the case of rolling monometallic rods; however, there are no methods for determination of the average pressure $p_{av_{bim}}$ and its respective components in literature for rolling bimetal rods.

Based on the analysis of the mode of flow of respective bimetal rod layers and the results of works [6,7] it has been found that the increase of the elongation of the bimetal rod core, comparing with the elongation of the steel rod "core", occurs due to additional stresses produced by an unequal deformation of the soft (copper) layer and the steel core. **Fig. 1a** illustrates the effect of stresses during the rolling of a round monometallic rod composed of an apparent steel outer "layer", F_{st} , and an apparent steel "core", F_{st} . Fig. 1b shows a schematic of the action of additional tensile stresses σ_{dod} onto the steel core of a bimetal round rod with a cross-section composed of the copper layer F_{Cu} and the steel core F_{st} during the rolling in the oval pass.

As a result of the action of the additional stress (being a tensile stress for the core), the resistance to the plastic flow of the core in the rolling direction decreases. This leads to a decrease of strip spread, and thus to an increase of elongation compared to monometallic rod.

Under identical friction conditions during the rolling of monometallic rods, $\lambda_{Cu} = \lambda_{st} = \lambda_0$ and $n_{Cu} = n_{st} = n_0$, thus:

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Rolling



Fig. 1: a schematic diagram of stresses occurring during the rolling round rod in the oval pass: a) action of tensile stresses onto the ,,core" and the ,,layer", respectively, in monometallic (steel) rod; b) actions of additional tensile stresses onto the steel core and the copper layer in bimetal rod.

$$p_{av_{bim}} = \frac{n_0 \left(\sigma_{pst} \cdot F_{st} + \sigma_{pCu} \cdot F_{Cu}\right)}{F_{bim}} \cdot \frac{\ln \lambda_0}{\ln \lambda_{bim}}$$
(2)

A coefficient of bimetal stress state in the roll gap, n_0 , occurring in formula (2), can be determined from the same relationships as the coefficient, n, for monometallic rods, since it accounts for the effect of the same parameters. In this study, for the determination of the coefficient, n_0 , the empirical relationships proposed by Brovman [1] and Chekmarev [2] were used.

A $\ln\lambda_0/\ln\lambda_{bim}$ factor occurring in formula (2), which defines the increase of elongation during the rolling of the bimetal rods compared to the rolling of copper or steel rods, was determined from the results of the tests of the rolling of steel and bimetal rods, respectively, and plastometric tests. The calculation results are presented in **Fig. 2**.

After the approximation of the experimental data shown in Fig. 2, the equation of a function has been obtained, which describes the effect of the copper layer share in the strip cross-section and the yield stress values of the metals of the respective bimetal rod layers on the increase of elongation of the bimetal rod compared to the monometallic rod. The factor $ln\lambda_0/ln\lambda_{bim}$ is described by the following equation:

$$\frac{\ln \lambda_0}{\ln \lambda_{bim}} = f(U_{Cu}) + \frac{\left[1 - f(U_{Cu})\right] \cdot \left(\frac{\sigma_{pCu}}{\sigma_{pst}} - 0.2\right)}{0.8}$$
(3)

$$f(U_{Cu}) = 1 - 2.83 \cdot U_{Cu} + 8.11 \cdot U_{Cu}^2 - -8.22 \cdot U_{Cu}^3 + 2.93 \cdot U_{Cu}^4$$
(4)

where U_{Cu} – copper layer share in the bimetal strip crosssection; σ_{pCu} – yield stress for the copper layer; σ_{pst} – yield stress for the steel core.

Ultimately, the formula for the calculation of the overall



Fig. 2: relationship of elongation ratio in the rolling of bimetal rods to the rolling of steel rods as a function of copper layer share in the strip cross-section and the yield stress ratio of the components in the bimetal rod

STEEL GRIPS 2 (2004) Suppl. Metal Forming 2004

roll separating force P_{bim} during the rolling of the bimetal rods takes on the following form:

$$P_{bim} = \begin{cases} \frac{n_0 \left(\sigma_{pst} \cdot F_{st} + \sigma_{pCu} \cdot F_{Cu}\right)}{F_{bim}} \\ \left[f\left(U_{Cu}\right) + \frac{\left[1 - f\left(U_{Cu}\right)\right] \cdot \left(\frac{\sigma_{pCu}}{\sigma_{pst}} - 0.2\right)}{0.8} \\ \right] \\ \end{cases} \\ S_{dbim} \end{cases}$$
(5)

For the calculation of energy and force parameters occurring during hot rolling, it is necessary to know the yield stress in specific thermo-mechanical conditions. Moreover the accuracy of computations using a computer program is dependent on the properties of materials used for examinations. Experiments undertaken were aimed to establish the effect of deformation parameters on the value of yield stress for the St3S and the 55 steel.

The experimental examination of the properties of the St3S and 55 steel were carried out at the Institute of the Modeling and Automation of Plastic Working Processes of the Czestochowa University of Technology using a dilatometer–plastometer, type DIL 805A/D. Plastometric tests were performed using deformation rates of 1.0 s^{-1} , 5.5s^{-1} and 10 s^{-1} and the temperatures of plastometric tests were, respectively: 700°C, 800°C, 900°C and 1000°C. The results of plastometric tests of M1-E copper were taken from the literature [8, 9]. For M1-E copper plastometric tests were performed using deformation rates of 0.1 s^{-1} and 2.5 s^{-1} and the temperatures of 0.1 s^{-1} and 2.5 s^{-1} and the temperature [8, 9]. For M1-E copper plastometric tests were performed using deformation rates of 0.1 s^{-1} and 2.5 s^{-1} and the temperatures of plastometric tests were, respectively: 700° C, 800° C and 900° C.

In order to obtain a mathematical relationship making the value of yield stress, s_p , dependent on deformation parameters, ($\boldsymbol{\varepsilon}$, $\dot{\boldsymbol{\varepsilon}}$, t), the results of the performed tests were approximated with a functional relationship described by Equation (6). The flow stress s_p dependence on strain intensity $\boldsymbol{\varepsilon}$, strain rate $\dot{\boldsymbol{\varepsilon}}$ and temperature t for the St3S and 55 steel and the M1-E copper is approximated by Henzel-Spittel formula expressed as [10]:

$$\sigma_p = A_1 \cdot \varepsilon^{A_2} \cdot \dot{\varepsilon}^{A_3} \cdot \exp^{\left(-t \cdot A_4\right)} \tag{6}$$

The coefficients A_1 , A_2 , A_3 , A_4 of the St3S steel, the 55 steel and the M1-E copper are given in **Table 1**.

The determination of the overall roll separating force by the FEM during the rolling of the bimetal rods in passes

The numerical model for the three-dimensional metal flow of bimetallic rods during the rolling in the grooves of any arbitrary shape has been developed based on principles published in the works [11, 12], by employing the finite-element method. The solution was sought for from the Markov functional χ [13]:

$$\chi = \int_{V} \sigma_i \xi_i dV + \int_{V} \sigma_0 \xi_0 dV - \int_{S} \sigma_\tau v_\tau dS$$
(7)

where $\sigma_i(\xi_i, \varepsilon_i, t)$ – dependence of yield stress σ_i on strain rate intensity ξ_i , strain intensity ε_i and temperature t; V – deformed metal volume; ξ_0 – relative volume change rate; σ_0 – mean stress; v_{τ} – velocity of slip of metal over the tool; S – metal - tool contact surface area; σ_{τ} – stress of the friction at the metal-tool interface.

The method of taking boundary conditions into account is described in detail in the article [11]. The method uses penalty function for solving contact problems, namely for the prevention of material penetration into the tool and for accounting for the effect of friction forces.

The friction law was written considering the direction of metal slip over to the roll:

$$\sigma_{\tau} = m\sigma_i \quad sign(v_{\tau}) = m\sigma_i \frac{v_{\tau}}{|v_{\tau}|}$$
(8)

where m - friction factor.

To facilitate searching for the direction of action of the friction stresses, the following transformations were used:

$$\int_{S} \sigma_{\tau} v_{\tau} dS = \int_{S} m \sigma_{i} v_{\tau} \frac{v_{\tau}}{|v_{\tau}|} dS =$$

$$= \int_{S} \frac{m \sigma_{i}}{|v_{\tau}|} v_{\tau}^{2} dS = \int_{S} K_{\tau} v_{\tau}^{2} dS$$
(9)

where

$$K_{\tau}^{(p)} = \frac{m\sigma_{i}^{(p-1)}}{\left|v_{\tau}^{(p-1)}\right|}$$
(10)

where p – iteration number over the boundary conditions.

Taking the nonlinearity of the work-hardening curve into consideration, required the use of the iteration procedures shown below:

$$\int_{V} \sigma_i \xi_i dV = \int_{V} \mu \xi_i^2 dV \tag{11}$$

where

ł

$$u^{(p)} = \frac{\sigma_i^{(p-1)}}{\xi_i^{(p-1)}}$$
(12)

After carrying them out, functional (7) could be written in a form easy to be solved by the FEM:

 Table 1: parameters of function (6) for the St3S steel, the 55 steel

 and the M1-E copper

material	A 1	A 2	A 3	A 4	mean
					square error
St3S	1347.399	0.2197	0.0495	-0.0021	0.237
55	4710.612	0.174	0.0789	-0.0034	0.244
M1-E	543.329	0.1705	0.0744	-0.0026	0.133

$$\chi = \int_{V} \mu \xi_i^2 dV + \int_{V} \sigma_0 \xi_0 dV - \int_{S} K_\tau v_\tau^2 dS$$
(13)

Calculations of respective components of the stress tensor were performed using the relationship below:

$$\sigma_{ij} = \delta_{ij}\sigma_0 + \frac{2\sigma_p}{3\xi_i}\xi_{ij} \tag{14}$$

Whereas, stresses normal to the area of the metal and roll contact surface were determined from the relationship:

$$\sigma_n = a_x \sigma_x + a_y \sigma_y + a_z \sigma_z \tag{15}$$

The overall roll separating force per roll was determined by numerical integration over the area of each element contact with the roll:

$$P = \int_{S} \sigma_n dS = \sum_{e=1}^{N_e} \int_{S_e} \sigma_{ne} dS_e$$
(16)

Prismatic finite elements with the triangular basis are used for the FEM solution. Velocity distribution in each element is approximated by 15 nodes while mean stresses approximated by 6 nodes.

A finite element mesh is generated automatically by following sequence:

- reading of cross-section contours for the groove and the billet (i.e. core + cladding layer), prepared in a CAD program (such as AutoCAD);
- generating a two-dimensional grid of triangular elements;
- creating a three-dimensional grid from the obtained two-dimensional grid. While doing this, the principle of elongation of individual triangular elements in the rolling direction, and for the rolls, the principle of rotation of flat elements around the axes of corresponding rolls are applied;
- arranging the initial position of the feedstock and the rolls.

Algebraic equations, which are determined by discretization of the equations (7-13), are solved by a frontal method.

Rolling of steel and bimetallic copper – steel rods

Steel and bimetallic rods with an outer diameter of about 22mm and a copper layer share of 15, 30, 45, 50 and 55% Cu after explosive welding, were rolled on a D 320 threestand two-high shape mill [14, 15]. The stock material was round St3S and 55 steel covered with an M1-E copper layer. Heating of the bimetallic stock of initial length of 250 mm was carried out in a chamber furnace. Rods heated up to a temperature of 960°C. Rolling speed was approx. 0.45 m/s. During the rolling, the value of the overall roll separating force was measured and recorded using strain gauges [16].

As a result of rolling in 6 passes, rods of a diameter of about 14.0 mm were obtained. Six elongation grooves were used in the horizontal oval – vertical oval – horizontal oval – round passes design. During the rolling, the copper layer thickness decreased by 1 to 4% compared with rods after explosive welding.

The number of rolling passes was chosen depending on the thickness of the copper layer in the bimetal stock. **Fig. 3** shows the shape and dimensions of ovals used in the investigation. If the Cu layer share was 15 to 30%, then rolling was carried out in 4 rolling passes (with horizontal oval 1a as pass no. 1); if, on the other hand, the Cu layer share was 45 to 55%, then rolling was carried out in 6 rolling passes (with horizontal oval 1b as pass no. 1). The study was limited to the first rolling pass only.

The view of template examples is shown in **Fig. 4**. In this figure, a deformation of the copper layer can be seen.

Results and discussion

A theoretical analysis of the process of rolling of steel rods and copper–steel bimetal rods in the first pass (where the deformation is maximal) has been performed in the study.



Horizontal oval 1a

Fig. 3: shape and dimensions of grooves used during the rolling process



Fig. 4: view of lateral samples taken from the billet after rolling in first pass: a) the 55 steel rod rolling in the oval 1a; b) the 55 steel rod rolling in the oval 1b; c) the copper-55 steel bimetallic rod rolling in the oval 1a (15% Cu); d) the copper-55 steel bimetallic rod rolling in the oval 1a (30% Cu); e) the copper-St3S steel bimetallic rod rolling in the oval 1b (45% Cu); f) the copper-St3S steel bimetallic rod rolling in the oval 1b (50% Cu); g) the copper-St3S steel bimetallic rod rolling in the oval 1b (55% Cu)

The initial stock temperature for monometallic and bimetallic rods was taken based on measurements using an IG8 pyrometer manufactured by IMPAC.

The following input data were taken for simulation:

- stock temperature of 960°C for steel rods and Cu– steel bimetal rods;
- tool (upper and lower roll) temperature of 60°C;
- ambient temperature of 20°C;
- friction factor, m = 0.85;
- coefficient of heat exchange between the material and the tool, α = 3000 [W/Kmm²];
- coefficient of heat exchange between the material and the air, $\alpha_{pow} = 100 [W/Kmm^2]$.

The comparison of billet shape and dimensions, obtained from computer simulations and measured is shown in **Fig. 5**.

The data shown in Fig. 5 indicates that the shape of the bimetallic billet after simulation is consistent, to a considerable degree, with the shape of templates taken after each pass. The greatest differences were observed for the billet with an initial copper layer share of 50 and 55% (Figs. 5f and 5g). For other cases, good agreement between the billet width obtained from simulation and the measured values was achieved. In no case, was any difference between the measured billet height and that obtained from simulation noted. Also, the distribution of copper layer thickness was consistent with that measured on the templates taken from the strip.

Results of the calculated overall roll separating force

Knowing the values of yield stresses for respective bimetal components and for monometallic steel rods, the stress state coefficient n_0 was determined using the empirical relationships given by Brovman and Chekmarev. Then, the overall roll separating force in the rolling of bimetal rods was calculated using relationship (5) and using the SortRoll computer program based on FEM. The obtained results of theoretical calculations were compared to the results obtained from experimental tests (**Tables 2 and 3**). The summarized calculation results and the values recorded during measurements are shown in Fig. 6 for all roll pass design arrangements and different stock materials.

The analysis of the data given in Tables 2 and 3 and shown in Fig. 6 indicates that, of the two formulae used for determining the stress state coefficient n, the smallest deviations were obtained using Chekmarev's relationship [2], whereas the use of Brovman's relationship [1] for determining the stress state coefficient n produced larger devia-



tions. At the same time, the overall roll separating force calculated using n Brovman's relationship for the determination of the stress state coefficient n was, for most rolling passes, lower than the recorded force. On the other hand, the best results were obtained by using the computer program utilizing the FEM for the three-dimensional solution of the plasticity problem. During numerical modelling of

Rolling

the rolling of both steel rods and bimetal rods with a different shape of the soft cladding layer the error did not exceed 10%, and in most cases it was lower than 5%.

The accuracy of calculations of the overall roll separating force made by using formula (5) depends also on the copper layer thickness and the yield stress values of the metals of respective bimetal rod layers. As the formulae used for the determi-

nation of these relationships have been obtained from the approximation of the experimental test results, they may either underestimate or overestimate the obtained results. The use of the FEM for the determination of the overall roll separating force is not associated with such shortcomings. Therefore, the differences between calculation and measurement results were the smallest when relationship (5) was used.

В

Experiment

FEM simulation











Table 2: comparison of rolling force values calculated and recorded during the rolling steel rods according to the round-horizontal oval scheme

Rod	Band dimensions,			Roll separating force / mean square error					
	mm				[kN] [%]				
	Before the After the			Pexp.	Ptheoret.	Ptheoret.	Ptheoret.		
	rolli	ing		ing	kN	Chekm	Brovman	FEM	
	nu	DU	nı	DI					
steel 55 (oval 1a)	22	22	12.7	31.6	235.6	245.6/4.2	204/-13.4	242.3/2.8	
steel St3S (oval 1a)	22	22	12.7	31.5	226.7	315.8/39.3	224.4/-1.0	212.4/-6.3	
steel 55 (oval 1b)	22	22	12.5	30.8	205.7	240.2/16.8	190.2/-7.5	209.2/2.0	
steel St3S (oval 1b)	22	22	12.6	30.7	185.2	178.5/-3.6	188.7/1.9	184.3/-0.5	

Conclusions

The investigation carried enables the following observation to be made and conclusions to be drawn:

The process of the rolling of the bimetal rods in the passes is characterized by a greater deformation nonuniformity compared to the rolling of monometallic rods, therefore the accurate determination of energy and force parameters requires the use of more complex em-

> pirical relationships that will account for the interaction of the bimetal layers and resistances to the plastic flow of respective components.

The method of calculation of the overall roll separating force in the rolling of bimetal rods, presented in this study, yields results that are in good agreement with the values measured during the experimental tests. In most cases, deviations from the measurements did not exceed 10÷15%. Therefore, it can be recommended for practical use when developing new bimetal rod rolling technologies.

Fig. 5: comparison of experimental and theoretical shapes and dimensions of the billet after rolling in first pass: a) the 55 steel rod rolling in the oval 1a; b) the 55 steel rod rolling in the oval 1b; c) the copper-55 steel bimetallic rod rolling in the oval 1a (15% Cu); d) the copper-55 steel bimetallic rod rolling in the oval 1a (30% Cu); e) the copper-St3S steel bimetallic rod rolling in the oval 1b (45% Cu); f) the copper-St3S steel bimetallic rod rolling in the oval 1b (50% Cu); g) the copper-St3S steel bimetallic rod rolling in the oval 1b (55% Cu)

Table 3: comparison of rolling force values calculated and recorded during the rolling bimetallic rods according to the round-horizontal oval scheme

Rod	Band dimensions, mm				Roll separating force / mean square error [kN] [%]			
	Before the roll- ing		After the rolling		Pexp. kN	Ptheoret. Chekm	Ptheoret. Brovman	Ptheoret. FEM
	h0	b0	h1	b1				
core - steel St3S, 15% Cu; (oval 1a)	21.7	21.7	12.4	25.9	180.9	225.9/24.9	168.5/-6.9	173.4/-4.1
core - steel 55, 15% Cu; (oval 1a)	21.7	21.7	12.5	25.7	198.9	216.5/8.8	159.7/-19.7	196.0/-1.5
core - steel 55, 30% Cu; (oval 1a)	21.7	21.7	12.5	30.7	165.7	170.1/2.7	125.8/-24.1	171.4/3.4
core - steel 55, 45% Cu; (oval 1b)	21.7	21.7	14.0	23.6	87.8	118.6/35.1	87.5/-0.3	82.9/-5.6
core - steel 55, 50% Cu; (oval 1b)	22.7	22.7	13.8	26.8	113	140/23.9	103.6/-8.3	125/10.6
core - steel St3S, 55% Cu; (oval 1b)	21.0	21.0	13.6	24.5	72.4	106.8/47.5	79.6/9.9	77.7/7.3



Fig. 6: roll separating force of metal on rolls during the rolling process steel and bimetallic rods according to the round – horizontal oval pass

• The use of the FEM for the computation of the overall roll separating force made it possible to reduce the deviations below 10%. Moreover, it is possible to accurately determine the strip shape and dimensions after individual passes, which, in the case of bimetal rod rolling, is a particularly difficult and complex problem.

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Rolling