

$A(1,1)*x(1)+A(1,2)*x(2) = b(1)$   
 $A(2,1)*x(1)+A(2,2)*x(2) = b(2)$   
 where  $A(i,j)$  a square matrices,  $x(i)$ ,  $b(i)$  are vectors

**How to get Schur complement:**

- 1)  $A(1,1)=L*U$
  - 2)  $A(1,2)=(L*U)^{-1}*A(1,2)$
  - 3)  $b(1)=(L*U)^{-1}*b(1)$
  - 4)  $A(2,2)=A(2,2)-A(2,1)*A(1,2)$
  - 5)  $b(2)=b(2)-A(2,1)*b(1)$
- then, the Schur complement is stored at  $A(2,2)$  and  $b(2)$

1)  
 $A(1,1)=L*U$

LAPACK DGETRF for double precision  
 ZGETRF for complex

SUBROUTINE ZGETRF( M, N, A, LDA, IPIV, INFO )

INTEGER

INFO, LDA, M, N

INTEGER

IPIV( \* )

DOUBLE

PRECISION A( LDA, \* )

M (input) INTEGER The number of rows of the matrix A.  $M \geq 0$ .

N (input) INTEGER The number of columns of the matrix A.  $N \geq 0$ .

A (input/output) DOUBLE PRECISION array, dimension (LDA,N) On entry, the M-by-N matrix to be factored.

On exit, the factors L and U from the factorization  $A = P*L*U$ ; the unit diagonal elements of L are not stored.

LDA (input) INTEGER The leading dimension of the array A.  $LDA \geq \max(1,M)$ .

IPIV (output) INTEGER array, dimension (min(M,N)) The pivot indices; for  $1 \leq i \leq \min(M,N)$ , row i of the matrix was interchanged with row **IPIV**(i).

INFO (output) INTEGER = 0: successful exit

< 0: if INFO = -i, the i-th argument had an illegal value

> 0: if INFO = i, U(i,i) is exactly zero. The factorization has been completed, but the factor U is exactly singular, and division by zero will occur if it is used to solve a system of equations.

2)

$A(1,2)=(L*U)^{-1}*A(1,2)$

in other words (where  $A=A(1,2)$ )

$R = U^{-1}*L^{-1}*A$

$L^{-1}*A = B$

$U^{-1}*B=R$

Algorithm:

a) Given A and L, solve  $A=L*B$

b) Given B and U, solve  $B=L*R$  for R

DTRSM - ZDTRM( SIDE, UPLO, TRANS, DIAG, M, N, ALPHA, A, LDA, B, LDB )

$B<=L^{-1}*A$

**A is of size mxn**

**L is of size mxm**

SIDE = 'L'

UPLO = 'L' lower triangular matrix

TRANS = 'N' no transpose of L

DIAG = indicates if the diagonal of L is to be taken to equal the identity matrix (DIAG = "Unit" ) or the values in the matrix (DIAG = "Non unit" ).

M=m, N=n

ALPHA=1

A <- L

B <- A

The leading dimensions of the matrices are given in LDA(for L) and LDB (for A).

**$R \leftarrow U^{-1} * B$**

**B is of size nxn**

**U is of size nxn**

SIDE = 'L'

UPLO = 'U' upper triangular matrix

TRANS = 'N' no transpose of U

DIAG = indicates if the diagonal of  $U$  is to be taken to equal the identity matrix (DIAG = "Unit" ) or the values in the matrix (DIAG = "Non unit" ).

M=n, N=n

ALPHA=1

A <- U

B <- B

The leading dimensions of the matrices are given in LDA(for U) and LDB (for B).

**3)**

**$b(1) = (L * U)^{-1} * b(1)$**

in other words (where  $b = b(1)$ )

$d = U^{-1} * L^{-1} * b$

$L^{-1} * b = e$

$b = L * e$

$d = U^{-1} * e$

$e = U * d$

Algorithm:

a) Given  $b$  and  $L$ , solve  $b = L * e$  for  $e$

a) Given  $e$  and  $U$ , solve  $e = U * d$  for  $d$

DTRSM - ZDTRM( SIDE, UPLO, TRANS, DIAG, M, N, ALPHA, A, LDA, B, LDB )

**$L^{-1} * b = e$**

**b is of size mx1**

**L is of size mxm**

SIDE = 'L'

UPLO = 'L' lower triangular matrix

TRANS = 'N' no transpose of L

DIAG = indicates if the diagonal of  $L$  is to be taken to equal the identity matrix (DIAG = "Unit" ) or the values in the matrix (DIAG = "Non unit" ).

M=m, N=1

ALPHA=1

A <- L

B <- b

The leading dimensions of the matrices are given in LDA(for L) and LDB (for b).

**$d \leftarrow U^{-1} * e$**

**e is of size mx1**

**U is of size mxm**

SIDE = 'L'

UPLO = 'U' upper triangular matrix

TRANS = 'N' no transpose of U

DIAG = indicates if the diagonal of  $U$  is to be taken to equal the identity matrix (DIAG = "Unit" ) or the values in the matrix (DIAG = "Non unit" ).

M=m, N=1

ALPHA=1

A <- U

B <- e

The leading dimensions of the matrices are given in LDA(for U) and LDB (for e).

#### 4) $A(2,2)=A(2,2)-A(2,1)*A(1,2)$

DGEMM/ZGEMM (double / complex)

ZGEMM (transa, transb, l, n, m, alpha, a, lda, b, ldb, beta, c, ldc)

$C = \alpha A * B + \beta C$

here alpha = -1, beta = 1, A(2,1)=mult, B=A(1,2), C=A(2,2)

transa = 'N' **A** is used in the computation.

transb = 'N', **B** is used in the computation.

l is the number of rows in matrix **C**.

n is the number of columns in matrix **C**.

m is the number of columns in matrix **A**.

alpha is the scalar alpha.

a is the matrix **A**, where: **A** has l rows and m columns.

If transa equal to 'N', its size must be lda by (at least) m.

lda is the leading dimension of the array specified for a.

b is the matrix **B**, where: **B** has m rows and n columns.

ldb is the leading dimension of the array specified for b.

beta is the scalar beta.

c is the l by n matrix **C**.

ldc is the leading dimension of the array specified for c.

On Return c is the l by n matrix **C**, containing the results of the computation.

#### 5) $b(2)=b(2)-A(2,1)*b(1)$

ZGEMV

( TRANS, M, N, ALPHA, A, LDA, X, INCX, BETA, Y, INCY )

TRANS = 'N' for  $y := \alpha A * x + \beta y$ ,

M = number of rows in A

N = number of columns in A

ALPHA=-1

A(2,1)

LDA On entry, LDA specifies the first dimension of A as declared in the calling (sub) progra

X(1)

INCX=1

BETA=1

Y(1)

INCY=1