

# Explainable Artificial Intelligence. Model Discovery with Constraint Programming

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- 1 Introduction
- 2 Towards Exact Model-Based Reasoning
- 3 Abductive Model-Based Diagnosis
- 4 Structure discovery with Constraints
- 5 Concluding Remarks

# Presentation Outline

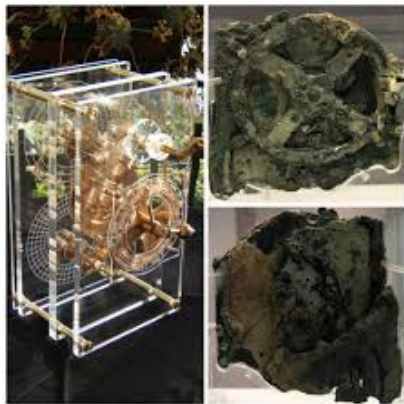
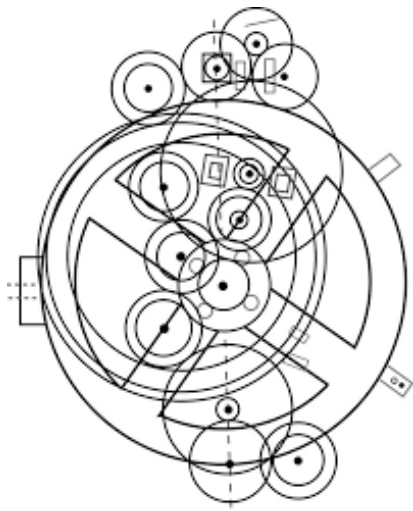
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# An Eternal Question: How Does it Work?



**Figure:** The Antikythera mechanism; recovered on May 17, 1901. The instrument has been variously dated to about 87 BC, or between 150 and 100 BC, or in 205 BC  
[https://en.wikipedia.org/wiki/Antikythera\\_mechanism](https://en.wikipedia.org/wiki/Antikythera_mechanism)

# How Does it Work? Model-Based Reasoning



Components + Connections + Causality = Operation

# XAI: Explainable Artificial Intelligence – WHY?

## Why do we care about **Explainability in Artificial Intelligence**?

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- readable and interpretable models,
- reduction of models (knowledge is more concise than data),
- well-defined domain of application,
- predictable behaviour,
- **Reliable Artificial Intelligence**,
- safe AI solutions,
- easy adaptation and modification,
- **Trustable Artificial Intelligence**.

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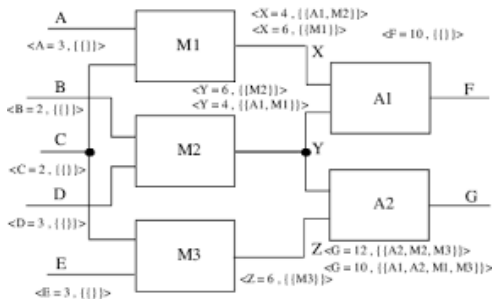
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# Model-Based Reasoning – A Perfect XAI Paradigm?

## Discovering Causal Structure: Motivation:

- majority of ML models cover *shallow* knowledge only,
- most of them are on decision/classification type; no *functional* output,
- often: fuzzy/rough/probabilistic output,
- no investigation of the *guts* — what is inside?
- starting point: **explanation of behaviour**; **diagnostic reasoning**.

- variables, values, signals,
- components, links,
- internal structure,
- input – **internal state** – output,
- operation, constraints,
- functionality.





# Level of Details in Structure Discovery

## Values of variables:

- binary 0/1; true/false, ternary -/0/+, qualitative, integer numbers.

## Connections:

- existence, direction, breaks, shortcuts, complex.

## Components:

- parametric identification, selection one-of, function discovery.

## Overall Structure:

- causality, structure - causal graph, logical and functional dependencies.

# Towards Exact Model-Based Reasoning

## CSP statement

- $X = \{X_1, X_2, \dots, X_k\}$  — variables,  $D = \{D_1, D_2, \dots, D_k\}$  — their domains,
- $C = \{(S_i, R_i) : i = 1, 2, \dots, n\}$  — constraints;  $S_i$  — scope;  $R_i$  — relation.

## CSP solution

A solution to CSP:  $(X, D, C)$  — any assignment of values to variables of  $X$ :

$$\{X_1 = d_1, X_2 = d_2, \dots, X_k = d_k\},$$

where  $d_i \in D_i$ , and for any constraint in  $(S_i, R_i) \in C$ ,  $R_i$  is satisfied.

## A CSP Example

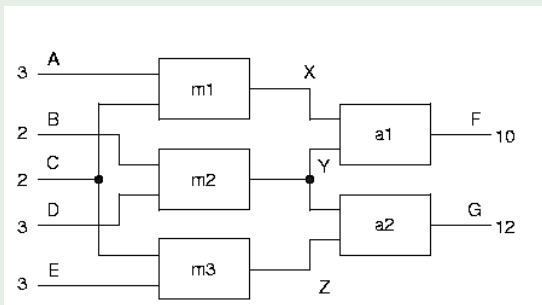
$$\begin{array}{r} \text{S E N D} \\ + \text{M O R E} \\ \hline \text{M O N E Y} \end{array}$$

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# Abductive Consistency-based Diagnosis

## The multiplier-adder system to be diagnosed

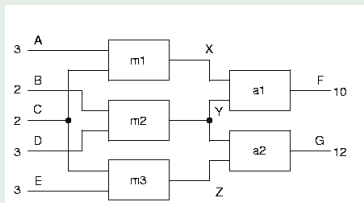


## Consistency-Based Diagnosis

- MISBEHAVIOR:  $F=10$  (should be 12); note that  $G=12$  is O.K.
- ABDUCTION — CONFLICTS:  $\{a1, m1, m2\}, \{a1, m1, a2, m3\}$ ,
- REPAIR — DIAGNOSES:  $\{a1\}, \{m1\}, \{m2, m3\}, \{a2, m2\}$ .

# An example problem continued: case of $\{m1\}$

## Model for diagnosis: $m1$



% K1/M1 = multiplier error

$A * C * K1 \neq X * M1,$

$B * D \neq Y,$

$C * E \neq Z,$

$X + Y \neq F,$

$Y + Z \neq G,$

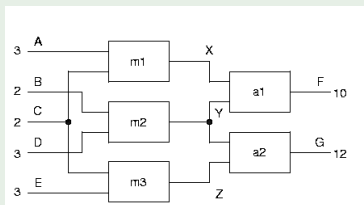
$K1 \neq > 0, M1 \neq > 0.$

**Solution:**

$X=4, Y=6, Z=6, K1=2, M1=3$

# An example problem continued: case of {a1}

## Model for diagnosis: a1



% A1 = addition error

$A * C \neq X,$

$B * D \neq Y,$

$C * E \neq Z,$

$X + Y - A1 \neq F,$

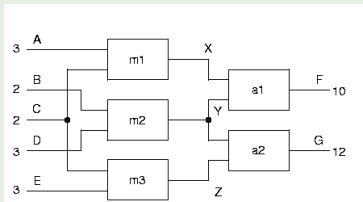
$Y + Z \neq G.$

Solution:

$X=6, Y=6, Z=6, A1=2$

# An example problem continued: case of $\{a2, m2\}$

## Model for diagnosis: $\{a2, m2\}$



```
% A2 = addition error  
% K2/M2 = multiplier error
```

```
A * C #= X,  
B * D * K2 #= Y * M2,  
C * E #= Z,  
X + Y #= F,  
Y + Z + A2 #= G,  
K2 #> 0, M2 #> 0.
```

Solution:

X=6, Y=4, Z=6, K2=2, M2=3, A2=2

# Constraints for parametric identification problems

A generic procedure:

- 1 take a minimal diagnosis for detailed examination;
- 2 for any component define the variable(s) to capture its misbehavior,
- 3 define the domains of the variables
- 4 and the constraints imposed on these variables for the analyzed case,
- 5 and constraints modeling the possible faulty work of all the components
- 6 run the constraint solver and analyze the results.

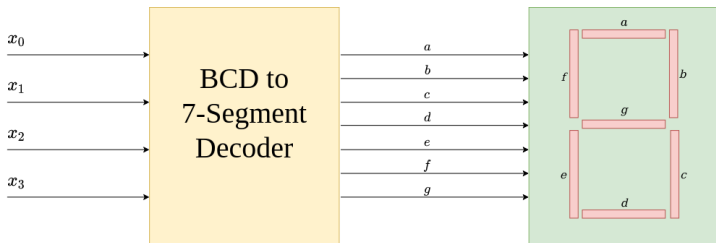
Results: detailed numerical explanations – numerical models of faulty behavior – for all the diagnoses.



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# Example: The 7-Segment Display



**Figure:** An example scheme presenting the input and output of the controller and connection to the 7-segment display

The inputs of the controller are the binary signals  $x_3, x_2, x_1, x_0$ . The digit  $D \in [0, 1, \dots, 9]$  to be displayed is calculated as:

$$D = x_3 * 2^3 + x_2 * 2^2 + x_1 * 2 + x_0$$

# Input-Output Data

## Inputs and Outputs Specification

INP = [ 0,0,0,0	OUT = [ 1,1,1,1,1,1,0
0,0,0,1	0,1,1,0,0,0,0
0,0,1,0	1,1,0,1,1,0,1
0,0,1,1	1,1,1,1,0,0,1
0,1,0,0	0,1,1,0,0,1,1
0,1,0,1	1,0,1,1,0,1,1
0,1,1,0	1,0,1,1,1,1,1
0,1,1,1	1,1,1,0,0,0,0
1,0,0,0	1,1,1,1,1,1,1
1,0,0,1 ];	1,1,1,1,0,1,1 ];

The input table INP has 4 columns representing the values of  $x_3, x_2, x_1, x_0$ , respectively, and 10 rows, each of them representing a single digit of 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. The output table OUT has 7 columns, each of them corresponding an appropriate display segment a, b, c, d, e, f, g, and exactly 10 rows, again each of them representing a single digit of 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 .

# Assumptions for Model Discovery and CP

- we shall look for a **structure** in **Disjunctive Normal Form** (DNF),
- there will be two levels of components, the first one with AND gates (binary multiplication) and the second one with OR-gates (binary addition); the negations of input signals are assumed to be available,
- the **number of the AND-gates** should be **minimal**,
- the **number of the OR-gates is 7**; this is the consequence of the fact that there are 7 independent outputs  $a, b, c, d, e, f, g$ ,
- the **function of the controller** must reconstruct the specified input-output behavior **in an exact way**.

The search areas open for the Constraint Programming:

- search for **connections**, from the input signals  $x_3, x_2, x_1, x_0$  and their negations to the inputs of the AND-gates; note that the case of the lack of the connection (if the signal is not used) must also be represented,
- the **number of connections should be minimal**; this is so to keep the transparency and uniqueness of the solution,
- the **number of the AND-gates should be as small as possible**,
- for repeated AND-gates only one physical realization is necessary.

# Encoding Search for Connections

The key element here is the array  $W$  encoding the existence (or not) of the input connections for each of the AND-gate. We applied the following encoding:

- $W[i, j] = 1$  codes a connection of (positive occurrence) of  $x_j$  to an input of the  $i$ -th AND-gate,
- $W[i, j] = -1$  codes a connection of negative occurrence (logical negation) of  $x_j$  to an input of the  $i$ -th AND-gate,
- $W[i, j] = 0$  codes a lack of connection of  $x_j$  as an input of the  $i$ -th AND-gate; in fact, the value of  $x_j$  is replaced with a constant 1 (a logical trick for assuring the correct operation of the AND-gate).

The corresponding function is a function taking as the input the appropriate  $x_j$  value and depending on the value of  $W[i, j]$  returns the exact or negated value of  $x_j$ , or just 1 in case of lack of the connection.

## Encoding connections in MiniZinc

```
function var int: foo(var -1..1: w, var 0..1: x) =  
    if w==1 then x  
    elseif w==-1  
    then abs(x-1)  
    else 1 endif;
```

# Encoding AND and OR gates and the Goal

## Encoding the AND-gates for signals $x[3], x[2], x[1], x[0]$

```
function var int: and4(array[1..4] of var 0..1:x,  
                      array[1..4] of var -1..1:w) =  
    product(i in 1..4)(foo(w[i],x[i]));
```

## Encoding the OR-gate output

```
function var int: sumM(array[1..M] of var 0..1: m) =  
    if sum(j in 1..M)(m[j]) > 0 then 1 else 0 endif;
```

## The number of input connection to be minimized; $M =$ number of inputs:

```
Y = sum(i in 1..M, j in 1..4)(abs(W[i,j]));  
solve minimize Y;
```

## For every input the appropriate output must be rendered:

```
N = 10; % N= number of input cases to be covered  
constraint forall(i in 1..N) (OUT[i,S] =  
    sumM(j in 1..M) (and4(INP[i,..],W[j,..])));
```

# Identification results for segment d

An example of identification results for segment d function.

Function d:

$M = 5$  (number of AND-gates discovered),

$Y = 10$  (minimal number of connections).

## The weights of 5 AND gates

$W = [0, 1, -1, 1,$   
 $0, -1, 1, 0,$   
 $1, 0, 0, 0,$   
 $0, 0, 1, -1,$   
 $0, -1, 0, -1]$

$Y = 10$

The function is defined as:

$$d = x_2 \bar{x}_1 x_0 + \bar{x}_2 x_1 + x_3 + x_1 \bar{x}_0 + \bar{x}_2 \bar{x}_0.$$

# Summary of the Results

NoTS = the Number of Total Solutions (admissible solutions that are not necessary optimal ones),

TT = the total time of search in case we search for all solutions.

**Table:** Selected numerical results for the 7-segment display controller reconstruction

Segment	M (No. of AND-gates)	Y (No. of connections)	NoTS	TT [s]
a	4	6	4608	3.53
b	3	5	96	0.07
c	3	5	504	0.11
d	5	10	80640	83.56
e	2	4	12	0.00
f	4	7	8064	1.93
g	4	7	3840	2.31

The presented examples shows that Constraint Programming can be helpful for solving the problem of structure discovery in case of finite-values discrete systems and in presence of auxiliary, partial structural and functional knowledge.



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# Concluding Remarks

## Conclusions:

- Exploring mutual interplay of **Model-Based Reasoning**, and **Constraint Programming** seems inspiring, especially in modeling **Structure Discovery**,
- Both Boolean and exact numerical models can be investigated,
- Structural knowledge can be discovered with Constraint Programming,
- Signals + Connections + Functions = System Model.

## Further Issues and Work Directions:

- development of a parameterized set of block functions – potential components of larger systems,
- exploring imprecise cases with `solve minimize` of fitness evaluation,
- exploring methods for Causal Graphs discovery.
- typical ML-repository data is insufficient for causal/structural investigation,
- possible data extensions: causal, logical, functional and temporal aspects/knowledge,
- knowledge extensions: components, connections, causality, constraints,...

Thank you for your attention!  
Questions? Remarks? Constructive critics?

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