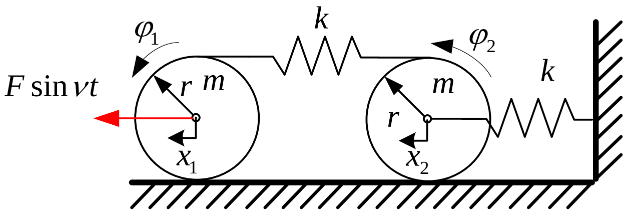
Drgania wymuszone:

Sporządź charakterystyki amplitudowo częstotliwościowe dla układu na poniższym rysunku:



Dane:

$k = 1000 N/m$

$m = 2kg$

$r = 0,2m$

$F = 20N$

Równania więzów:

$$\begin{cases} \dot{x}_1 = \dot{\varphi}_1 r \\ \dot{x}_2 = \dot{\varphi}_2 r \end{cases}$$

$$q = \{\varphi_1, \varphi_1\}$$

$$E = \frac{mr^2}{4}\dot{\varphi}_1^2 + \frac{mr^2}{4}\dot{\varphi}_2^2 + \frac{m\dot{x}_1^2}{2} + \frac{m\dot{x}_2^2}{2}$$

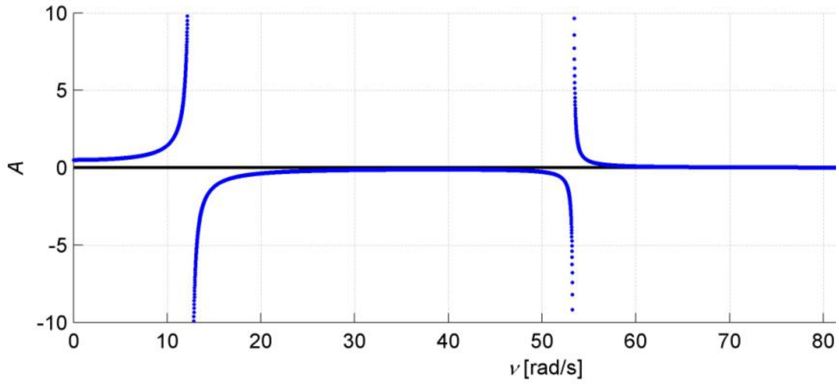
$$E = \frac{3}{4}mr^2\left(\dot{\varphi}_1^2 + \dot{\varphi}_2^2\right)$$

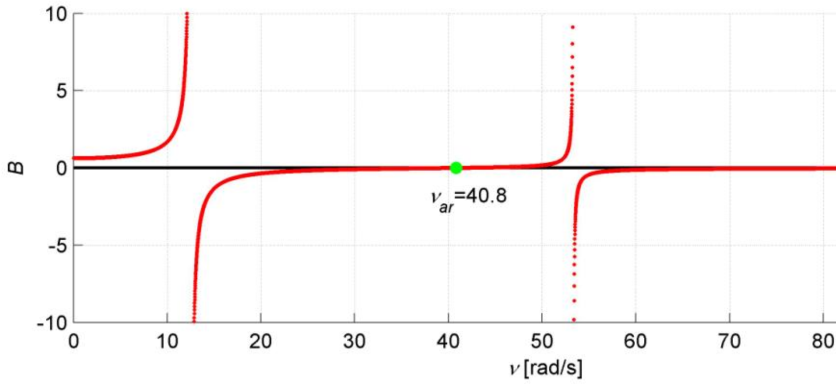
$$U = \frac{1}{2}kx_2^2 + \frac{1}{2}k\left(x_1 + \varphi_1 r - x_2 - \varphi_2 r\right)^2$$

$$U = \frac{1}{2}kr^2\varphi_2^2 + 2kr^2\left(\varphi_1 + \varphi_2\right)^2$$

$$\delta W = F\sin\nu t \cdot \delta x_1 = Fr\sin\nu t \cdot \delta\varphi_1$$

$$Q_{\varphi 1} = Fr\sin\nu t \qquad Q_{\varphi_2} = 0$$





Pochodne:

$$\frac{d}{dt}\left(\frac{\partial E}{\partial \dot{\varphi}_1}\right)=\frac{3}{2}mr^2\ddot{\varphi}_1$$

$$\frac{\partial U}{\partial \varphi_1}=4kr^2\left(\varphi_1-\varphi_2\right)$$

$$\frac{3}{2}mr^2\ddot{\varphi}_1+4kr^2\left(\varphi_1-\varphi_2\right)=Fr\sin\nu t$$

$$\frac{d}{dt}\left(\frac{\partial E}{\partial \dot{\varphi}_2}\right)=\frac{3}{2}mr^2\ddot{\varphi}_2$$

$$\frac{\partial U}{\partial \varphi_1}=kr^2\varphi_2-4kr^2\left(\varphi_1 -\right.$$

$$\frac{3}{2}mr^2\ddot{\varphi}_2+5kr^2\varphi_2-4kr^2$$

$$\begin{cases}\ddot{\varphi}_1+\frac{8}{3}\frac{k}{m}\left(\varphi_1-\varphi_2\right)=\frac{2F}{3mr}\sin\nu t\\ \ddot{\varphi}_2+\frac{10}{3}\frac{k}{m}\varphi_2-\frac{8}{3}\frac{k}{m}\varphi_1=0\end{cases}$$

$$\begin{cases}\ddot{\varphi}_1+\frac{8}{3}\frac{k}{m}\varphi_1-\frac{8}{3}\frac{k}{m}\varphi_2=f_0\sin\nu t\\ \ddot{\varphi}_2+\frac{10}{3}\frac{k}{m}\varphi_2-\frac{8}{3}\frac{k}{m}\varphi_1=0\end{cases}$$

gdzie: $f_0=\frac{2F}{3mr}=33,34$

Poszukiwane rozwiązania w postaci (całka szczególna):

$$\varphi_1=A\sin\nu t \qquad \ddot{\varphi}_1=-A\omega^2\sin\nu t$$

$$\varphi_2=B\sin\nu t \qquad \ddot{\varphi}_2=-B\omega^2\sin\nu t$$

$$\begin{cases}-\nu^2A+\frac{8}{3}\frac{k}{m}A-\frac{8}{3}\frac{k}{m}B=f_0\\ -\nu^2B+\frac{10}{3}\frac{k}{m}B-\frac{8}{3}\frac{k}{m}A=0\end{cases}$$

$$\begin{cases}A\left(-\nu^2+\frac{8}{3}\frac{k}{m}\right)-B\frac{8}{3}\frac{k}{m}=f_0\\ -A\frac{8}{3}\frac{k}{m}-B\left(-\nu^2+\frac{10}{3}\frac{k}{m}\right)=0\end{cases} \qquad \begin{cases}A\left(-\nu^2+1333,33\right)-B\cdot 1333,33=f_0\\ -A\cdot 1333,33-B\left(-\nu^2+1666,67\right)=0\end{cases}$$

$$W=\begin{vmatrix}-\nu^2+1333,33 & -1333,33\\ -1333,33 & -\nu^2+1666,67\end{vmatrix}=\left(-\nu^2+1333,33\right)\left(-\nu^2+1666,67\right)-1333,33^2$$

$$W=\nu^4-3000\nu^2+444452.2222$$

Częstości drgań własnych obliczamy przyrównując wyznacznik główny do 0.

$W=0 \;\rightarrow\; \omega^4-3000\omega^2+444452.2222=0$ - równanie charakterystyczne

$$\Delta=9000000-8888884,4444=7222191,11$$

$$\sqrt{\Delta}=2687,41$$

$$\omega_1^2=\frac{3000-2687,41}{2}=156,29 \qquad \omega_1=12,5\frac{rad}{s} \qquad f_1=2Hz$$

$$\omega_2^2=\frac{3000+2687,41}{2}=2843,71 \qquad \omega_2=53,33\frac{rad}{s} \qquad f_2=8,5Hz$$

$$W_1 = \begin{vmatrix} 166,67 & -1333,33 \\ 0 & -\nu^2 + 1666,67 \end{vmatrix} = 33,34\left(-\nu^2 + 1666,67\right)$$

$$W_2 = \begin{vmatrix} -\nu^2 + 1333,33 & 166,67 \\ -1333,33 & 0 \end{vmatrix} = 1333,33 \cdot 33,34 = 44445,22$$

Amplitudy w funkcji częstości wymuszenia przyjmują postać:

$$A(\nu) = \frac{W_1}{W} = \frac{33,34\left(-\nu^2 + 1666,67\right)}{\nu^4 - 3000\nu^2 + 444452.22}$$

$$A(0) = 0,125 rad$$

$$Antyrezonans:$$

$$-\nu_{ar}^{\ 2} + 1666,67 = 0$$

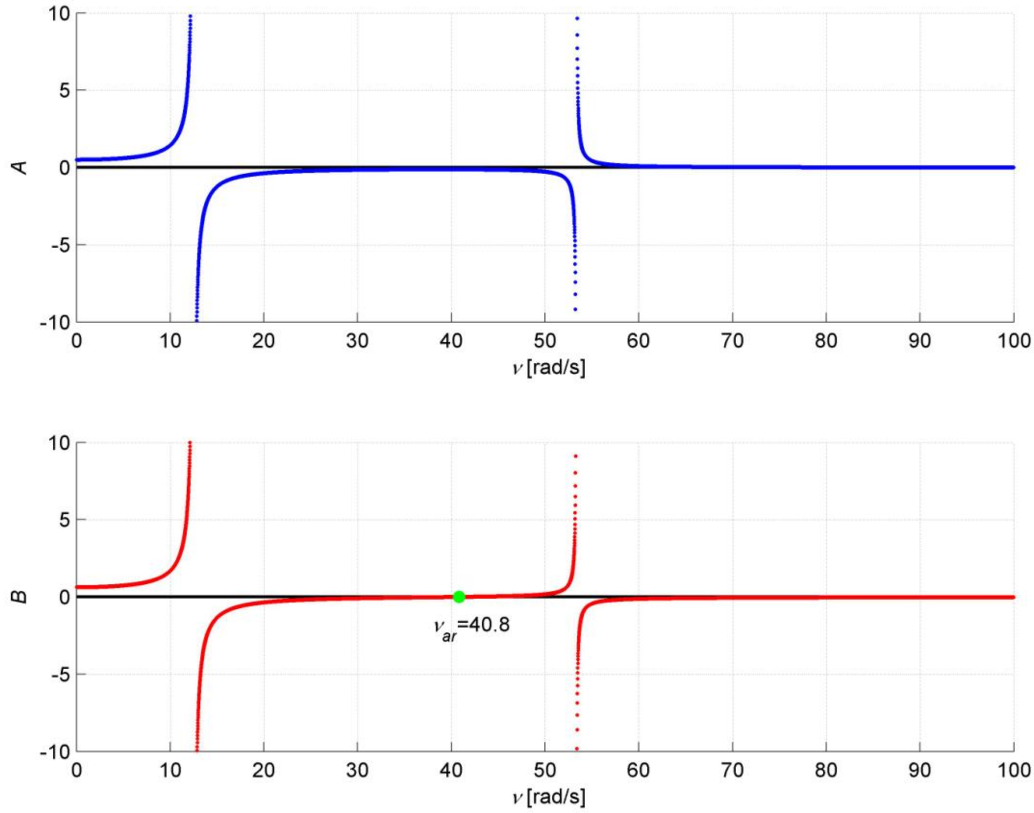
$$\nu_{ar} = 40,82 \frac{rad}{s}$$

$$B(\nu) = \frac{W_2}{W} = \frac{44445,22}{\nu^4 - 3000\nu^2 + 444452.22}$$

$$B(0) = 0,5 rad$$

$$Antyrezonans:$$

$$nie\ \ występuje!!!$$



Rys. 1 Charakterystyka amplitudowo częstotliwościowa