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RELATIONSHIP BETWEEN MAXIMUM VALUE OF ANNUAL TREE STAND HEIGHT INCREMENT AND AGE OF ITS OCCURRENCE

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Abstract: The paper presents the analytical formula describing the relationship between the maximum value of an annual tree stand height increment and the age at which it takes place. The estimation of considered parameters was based on Czarnowski's theory of tree stand dynamics. The empirical data from 19 height growth benchmarks representing all site quality indexes of two species – beech and spruce – from experimental thinning plots were taken into consideration.
Keywords: tree-stand growth model; maximum value of annual stand growth increment; age of annual stand growth increment occurrence; tree stand management

Introduction

The tree stand dynamics are among the most important factors developing the components of water exchange balance between the atmosphere, tree stand, and the soil in various forests. There is a linear dependence between transpiration and the amount of biomass production; interception is directly proportional to the surface area of plants. In turn, the site quality index has a basic significance for the biometric features of the tree stand, which condition the energy of solar radiation reaching the litter level and the attenuation of wind velocity within the tree stand [6]. The site quality index is usually expressed by a Roman numeral (absolutely useless in modeling) or by the average height of a 100-year-old tree stand for the given species. There have also been some attempts to describe the site quality index in relation to the amount of biomass of over-ground parts of plants [1].

The objective of the paper is to find an analytical formula, combining the maximum value of an annual height increment (ΔH_m) and the age of its occurrence (A_m). The relationship of these biometrical features of tree stand is observed in the site surveys [1] [1] [3]:

- values of ΔH_m are dependent on tree species: the heliophilous species have bigger values of ΔH_m at an earlier age A_m than the shade-tolerant species,
- the higher the site quality index, the bigger the ΔH_m increments for a given species,
- the higher the site quality index, the earlier the age A_m of the maximum value of an annual tree stand height increment,
- values of ΔH_m and A_m may be influenced by silvicultural practices; for instance, thinning and also by violently affecting biotic and abiotic environments,
- the age A_m may be delayed when the analyzed tree stand was growing under forest canopy; this delay may have no influence on the value of ΔH_m.

The maximum value of an annual tree stand height increment and the age at which it takes place can be easily interpreted from the biological point of view. The culmination point of a height increment ΔH_m may be taken to mean a manifestation of stand growth ability and the A_m – age of the ΔH_m occurrence – may be understood as tree stand's response to growth conditions. The value of an annual height increment is high in early age on high-quality forest sites. If the site quality index is low, the growth activity of stand decreases and the culmination point occurs later.

There is a lack of formulae describing the relationship between ΔH_m and A_m which could be applied in forest practice. Such equations are essential for forest practice to anticipate and to describe effects of silvicultural practices [7] [8]. The goal of this paper is to express by a formula the field-noticeable interdependence between the maximum value of an annual height increment and the age of its occurrence.

Tree stand growth equations

Growth equations describe the change of height with age, which follows a sigmoid curve. In the beginning the curve is concave up, while it becomes convex in later life. But one should remember that all mathematical models express just the idealized shape of the growth curve. Although many equations have been proposed to describe stand growth [5] [11] [13], most of them can be used only to supply an empirical fit. In turn, all parameters in such equations are not regarded as having any absolute significance for the theory of growth [5]. So the usefulness of any growth model should be verified not only by the values of goodness of fit, but above all the direct biological interpretation of parameters is essential [9].

Czarnowski's equation is one of a very few 'growth functions' directly based on biological premises. Referring to Czarnowski's theory [2], the height increment of an even-aged tree stand may be computed by the following formula:

$$\Delta H = \Delta H_m \left(1 - \left| \frac{A - A_m}{A + A_m} \right| \right) \qquad (1)$$

where: ΔH – an annual height increment [m] in consecutive growing years A, ΔH_m – the maximum annual height increment [m], which takes place at the age of A_m [years].

The formula (1) may be easily interpreted from a biological point of view, because both parameters in the equation are related to field-measurable biometric features of tree stands, especially of cone-shaped coniferous stands. The height of an even-aged tree stand may be calculated after the integration over the curve (1). It is essential to take into consideration that the function (1) has the singular point at the age of A_m, so the integration must be divided to two intervals delimitated by the value of A_m:

$$H(A) = \begin{cases} 2 \cdot \Delta H_m \cdot A_m \cdot \left[\frac{A}{A_m} - \ln\left(1 + \frac{A}{A_m}\right) \right] & \text{for } A \le A_m \\ 2 \cdot \Delta H_m \cdot A_m \cdot \left[\ln\left(1 + \frac{A}{A_m}\right) + 1 - 2 \cdot \ln(2) \right] & \text{for } A \ge A_m \end{cases} \qquad (2)$$

The Figure 1 presents the changeability of tree stand height and growth increment respectively, based on formulae (1) and (2). Although in a natural environment old trees are usually destroyed by different kinds of disasters such as a hurricane, a stroke of lightning or by a fire, some of them reveal the continuous increment. The annual increment of such trees is slight, but they manage to rich impressive dimensions. Moreover the tree height is often reduced by strong winds forcing trees to regenerate continuously. Such continuous ability to regenerate points out that trees are able to grow in height and in width until death [2].

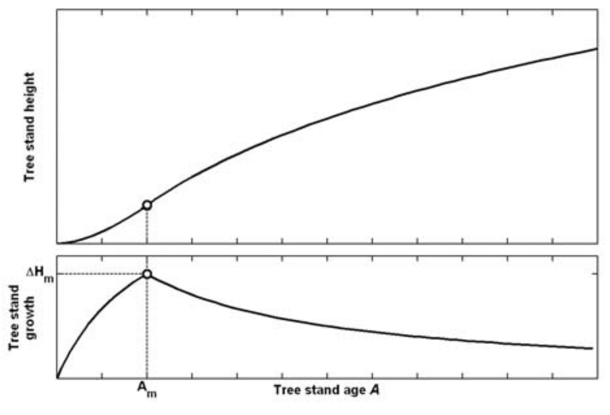

Fig. 1. Curve of tree stand height (above) and theoretical curve of tree stand height increment (below)

Relationship between ΔH_m and A_m

Two species of tree stands were analysed: a deciduous one (B type thinning beech) and a coniferous one (B type thinning spruce) – see Figure 2. All calculations were carried out on the basis of data on tree heights taken from the growth tables [12]. This means that data were referred to tangible tree stands with specific thinning techniques with similar intensity. Apart from the yield tables, the growth tables contain indisputable results of tree stand height measurements in experimental plots in the field. Such measurement results fulfil all requirements of empirical data. B type thinning describes the intensity of forest thinning that is the removal of growing trees carried out due to different criteria, for instance the tree height. It should be taken into consideration that the tree stand heights are in fact non-controversial in comparison to other features included in the yield tables. Moreover the average height of a tree stand is nearly independent of thinning intensity, but there is strong relation between the height and the size index [8]. One should remember that silvicultural practices have a direct influence on height growth benchmarks – if the lowest trees are thinned during the entire life of a tree stand, the average height of the tree stand is increased.

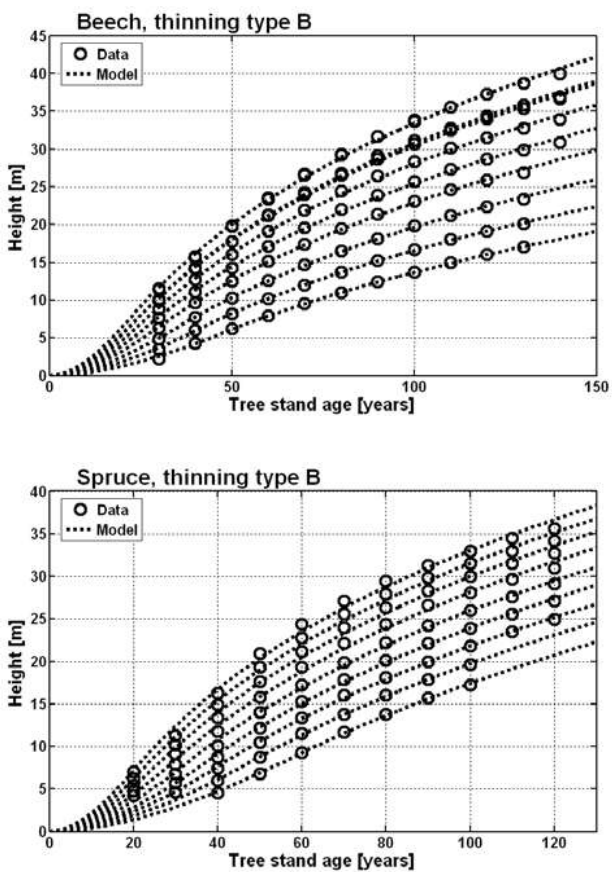

Fig. 2. Approximations of tree stand height by formula (2)

In the beginning, the pairs of parameters (A_m and ΔH_m) were estimated using the formula (2) for all stands growing benchmarks, separately for each species. The detailed results of approximation and the goodness of fit are presented in Table 1 and in Figure 2. The goodness of fit values for measured and estimated data prove that the tree stand dynamics are thoroughly expressed by Czarnowski's formula (2): (a) the variability of tree stand height is explained in almost 100% (parameter $100R^2$), (b) average estimation errors are usually lower than 0.5m, only in few cases for spruce stand errors are bigger, but still lower than 0.7m. The values of errors are bigger for the highest mature tree stand growing on the best sites, where accurate measurements of tree height are difficult.

The field-observed relationship between the maximum value of annual growth increment and the age of its occurrence is confirmed by the estimated pairs of parameters A_m and ΔH_m (Table 2) – the better the site index, the earlier the culmination point occurs. This relationship can be analytically expressed by the formula:

$$\Delta H_m = \frac{\alpha}{A_m - \beta} + \gamma \qquad (3)$$

where: ΔH_m – the maximum annual height increment [m], which takes place at the age of A_m [years], α, β, γ– coefficients to be calculated during the identification of the model equation. The results of identification of the formula (3) are presented in Table 2 and in Figure 3. The values of goodness of fit confirm that the form of equation (3) was correctly matched and, according to values of correlation coefficient R or $100R^2$, the variability of ΔH_m is explained in practically 100%. The standard deviation and the average error of estimation are so low that they can be simply omitted.

Conclusion

The empirical data from 19 height growth benchmarks representing all site quality indexes of two species: beech and spruce from experimental thinning plots were analysed in the paper. The field-noticeable relationship between the maximum value of an annual stand height increment and the age of its occurrence can be analytically expressed by the formula (3) with very high accuracy. The essential subject for the future research is to investigate the proper meaning of parameters α, β and γ from the biological point of view. On the other hand, the influence of the smoothing function (2) on parameters ΔH_m and A_m values should be also taken into consideration. Based on biological assumptions relationship between ΔH_m and A_m may allow to work out a formula to calculate a site quality class or annual biomass productivity from field measurements of tree stand height and age. Due to the type of considered data, the worked out function may be only applied to single-species and even-aged tree stands. It is also essential to investigate whether the presented relationship (3) may be used more generally and be applied to the uneven-aged tree stands [4].

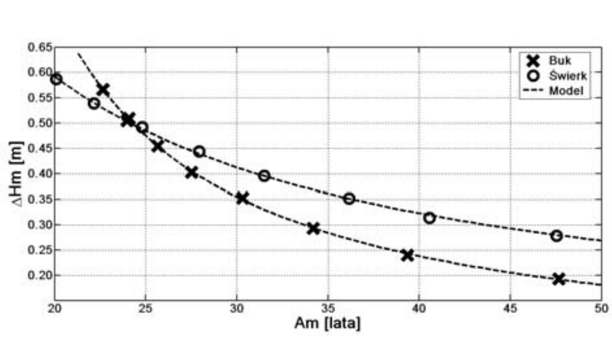

Figure 3. Relationship between ΔH_m and A_m

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