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Assessment of thermal diffusion in the natural biological environment

Abstract: In this article, the application of the Ångström method in estimating the thermal diffusion parameter in an example of natural biological environment is described. The present study is based on measurements performed in forest soils. The main advantage of the suggested procedure is that it does not need the collection of samples of the soil for a later study in the laboratory. Collecting such data is typically very laborious and requires the preservation of the natural content and structure of all elements of the soil profile in the samples, such as organic components, soil parts, or rocks. The harmonic time runs necessary to implement the Ångström method were computed with the short-time Fourier transform. The obtained results, compared with reference values taken from literature, confirm the accuracy of the proposed methodology.

Keywords: Ångström method; natural environment; thermal diffusivity.

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Introduction

The natural heat exchange is a very important energy transport process that is crucial in the functioning of numerous biological systems, and the possibility of modeling might provide a useful tool in studying the structure of such systems and their internal processes [1]. Forest soils can provide the data to verify the possibility of the implementation of the harmonic frequency-based Ångström method [2] to assess the values of heat transport model parameters. Furthermore, researchers have focused on the selected example of the biological system because of its importance in human life, economy, and environmental studies [3–6].

There are three ways the heat energy moves between two bodies of different temperatures [7]: (a) convection, (b) radiation, and (c) conductivity. The convective process is based on the mobility of gases or fluids, which are warmer than the surrounding environment. During radiation, the heat energy is first transformed into electromagnetic radiation, which then travels between bodies, even in vacuum. Conductivity is possible only between bodies of different temperatures that are in immediate contact. Because of the heat energy transport process, the temperature values of the bodies tend to equilibrate.

A very important parameter that describes the heat transport properties of the systems is thermal diffusivity. With the value of thermal diffusivity known, some further profound study of such systems with respect to their thermal behavior is available [8]. In cases when either the contents of the investigated system is unknown or it is impossible to extract the sample without considerably affecting the system, the thermal diffusivity has to be estimated from immediate temperature measurements performed in situ. When it is possible to collect the time runs of the temperature values, the Ångström method [2] can be applied.

The aim of the present study is to investigate the possibility of the implementation of the above-mentioned method based on data collected in forest soils and to assess the resulting efficiency when compared with typical values provided in reference sources. Forest soils typically have a very complicated structure, which results from a variety of possible elements, organic, mineral, and soil skeleton. The rock component in highland soils may exceed even 30%.

Heat transfer in soils

While considering the heat exchange problems in soil, the macroscopic phenomenological approach typically has to be implemented. It is based on the assumption that the elements of the studied structure are virtually continuous and uniformly distributed. Such a macroscopic approach is used in most of engineering studies. The topics related to heat transfer in soil are usually considered with the additional conditions that the soil porosity must be constant and the soil skeleton cannot be deformed. Such hypothetical assumptions are generally accepted in agriculture and hydrogeology [5]. Forest soils are characterized by layers with considerable organic components, which have a noticeable impact on their physical properties. The description of heat phenomena in soil is somewhat simplified by the fact that neither heat capacity nor thermal conductivity are sensitive to temperature. Nevertheless, the changes in soil humidity affect the mentioned parameters considerably, which makes the analysis more sophisticated. In addition, the thermal field in the soil profile depends on pulsating heating conditions on the soil surface, where the heat is provided by solar radiation during the day and emitted from the soil to the atmosphere during the night.

In the mathematical modeling of the heat transport, typically, a one-dimensional heat flow, along the x-axis, in the soil volume unit, ∂x, during time unit, ∂t, is considered. The equation for heat conductivity was proposed in 1822 by Fourier.

$$\frac{\partial T}{\partial t}=D\frac{\partial^2 T}{\partial x^2} \qquad (1)$$

where T denotes the time-dependent temperature; coefficient D, which is the thermal diffusivity or temperature equilibration coefficient, is a result of the product of heat conductivity λ and heat capacity C. Therefore, thermal diffusivity depends strongly on soil humidity because both λ and C depend on the amount of water in the soil. Yet D is independent of changes in parameter T. The realistic values of D are within the range from 4×10^{-7} to 8×10^{-7} m²/s, typically about 5×10^{-7} m²/s [5].

The heat capacity C is the body parameter describing the amount of heat energy it can accumulate when heated, with simultaneous increase in body temperature by one unit. In the case of soil consisting of all three phases, solid, liquid, and gas, the heat capacity is computed as a weighted sum of the heat capacity of each of the elements, for example, minerals, water, air. The solid phase is typically divided into two elements: organic and mineral. In practical calculations, the following equation is usually applied [5]:

$$C=1.89\,X_m+2.56\,X_o+4.19\,X_w \text{ (J/cm}^3\text{/K)} \qquad (2)$$

where symbols X_i denote the fraction of a given soil component in the soil total volume, with m, o, and w representing the mineral parts, organic parts, and water, respectively.

The soil thermal conductivity coefficient λ is measured as the amount of heat that is transported in 1 s through the soil slice of with a surface of 1 m² and depth of 1 m, assuming that the temperature difference between opposite surfaces of the slice is 1 K. The soil components do not affect the heat conductivity in the same manner as they do in the case of heat capacity because the heat-conductive elements may be arranged in parallel or in series. The overall value of the λ coefficient results from the mutual location of the highly conductive solid parts that are connected by less conductive liquid or gas phase. For a very dry soil, in which the contact between soil granules is limited only to the actual touching areas, λ values lower than 4.19 W/m/K are typically assumed. Increasing humidity rapidly increases the λ value, and for wet soils, it ranges from 4 to 21.0 W/m/K. The values of the C, λ, and D parameters for selected soil elements are presented in Table 1.

Measurement of the heat diffusivity

The immediate measurement of thermal diffusivity D is possible only when heat transfer is unsteady. For cases with steady heat transfer, the temperature distribution does not depend on parameter D. Nevertheless, for relatively simple shapes and uncomplicated border conditions, the analytical solution of the differential equation for unsteady heat transport is available. Assuming a physical heat transfer model for the semi-infinite body and periodic temperature changes on the system border (the border condition of the first kind) according to the following formula:

$$T(0,t)=T_0\cos(2\pi ft) \qquad (3)$$

where f denotes the frequency (in hertz); thus, the solution of Eq. (1) takes the form

$$T(x,t)=T_0\exp\left\{-x\sqrt{\frac{2\pi f}{2D}}\right\}\cos\left(2\pi ft-x\sqrt{\frac{2\pi f}{2D}}\right) \qquad (4)$$

Table 1 Values of heat capacity C, conductivity λ, and diffusivity D for selected soil ingredients [5].

Component	Heat capacity C, J/m³/K	Thermal conductivity λ, W/m/K	Thermal diffusivity D, m²/s
Quartz (solid rock)	2.0×10^{6}	8.8	4.2×10^{-6}
Argillaceous soil	2.0×10^{6}	2.9	1.14×10^{-6}
Organic substance	2.5×10^{6}	0.25	0.1×10^{-6}
Water	4.19×10^{6}	0.57	0.036×10^{-6}
Ice (T=0°C)	2.0×10^{6}	2.18	1.09×10^{-6}
Humid air	0.0013×10^{6}	–	–

According to the Ångström method, coefficient D can be calculated from the amplitude changes in two points, A_1 and A_2, located at mutual distance Δx and from the angle shift $\Delta\varphi$ between the periodic signals measured at these points. The thermal diffusivity coefficient may be computed from the following formula:

$$D=\frac{1}{2}\frac{2\pi f(\Delta x)^2}{\Delta\varphi\ln\left(\frac{A_1}{A_2}\right)} \tag{5}$$

A schematic graph of the measured soil profile along with the time runs of the temperature values is depicted in Figure 1.

Computations and obtained results

The data set used in the described computations was collected during a widely planned and realized gradient hydrologic research conducted by the Department of Forest Engineering, University of Agriculture, Krakow, in the summer of 2004. A subset of data that represents the measurements performed on the soil under a 41-year-old maturing spruce tree stand for four consecutive sunny days, without rainfall, has been selected. The measured temperature values were collected from two shallow layers at a depth of 0 cm (Tg_1) and 5 cm (Tg_2). Such layers consisted mainly of organic parts, with 99% of the organic contents within the 0- to 2.4-cm layer and 79% within the 2.4- to 5-cm layer.

In the time runs of the measured temperature presented in Figure 2A, the characteristic daily periodicity may be observed. However, the collected time runs do not fulfil the assumption about harmonic constraint at the surface [3, 7, 9, 10]. Such a requirement, which is necessary in applying the Ångström method, may be satisfied with use of the Fourier-based frequency representation of measured signals. To implement such an approach, the runs were divided into 24-h sections, containing 240 values measured at every 6 min, and then the short-time Fourier transform [11] was applied.

$$XT[m]=\sum_{n=0}^{N-1}T[n]\exp\left\{-j2\pi\frac{n}{N}m\right\} \tag{6}$$

where $T[n]$ is the discrete time series of the measured temperature values, N is the number of samples in the series, n, m are indices, such that n, m=0, ..., N-1. The Fourier transform data XT allow the measured series to be approximated with the use of the linear combination of the harmonic basis series:

$$T[n]=\frac{|XT[0]|}{N}+\frac{2}{N}\sum_{m=1}^{\frac{N-1}{2}}|XT[m]|\cos(2\pi f_m n+\measuredangle XT[m]) \tag{7}$$

where | | is the absolute value of the complex number, $\measuredangle$ is the phase angle of the complex number. The frequency values f_m for which the frequency domain coefficients are computed,

$$f_m=\frac{m}{Nt_s}, \tag{8}$$

where t_s=6 min is the distance in time between consecutive samples.

The value $A_k=\frac{2}{N}|XT[K]|$ computed directly from the frequency domain coefficients describes the amplitude of k-th harmonic component, with period $1/f_k$ and which may be immediately used in Eq. (5). The same applies to the computed phase angle difference $\Delta\varphi=\measuredangle XT[k+1]-\measuredangle XT[m]$. For further studies, only the daily component was selected (k=1), which represents the basic cycle resulting from soil heating due to solar radiation. Figure 2 presents the results from the described computations time runs of the harmonic signals A_1 and A_2 (B) and the phase difference time run, expressed as delay in hours (C). During the whole studied interval, the temperature changes within a 24-h period were about 2°C higher in layer Tg_1 than in layer Tg_2, and the phase difference between both layers turned out to be about 1 h. Figure 2D shows the computed values of the thermal diffusivity coefficient D. The obtained values stay within range from 3×10^{-7} to 5×10^{-7} m²/s, which is in very good

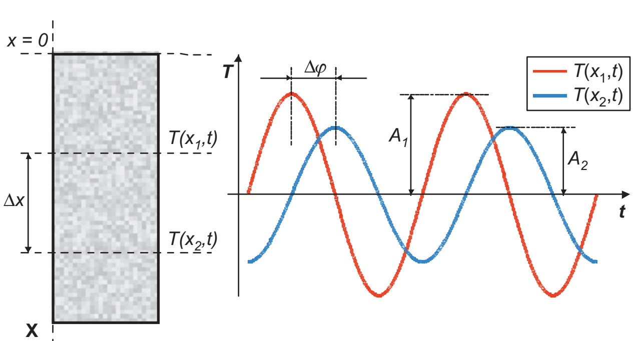

Figure 1 Schematic drawing of the soil profile and temperature signals according to Eq. (4).

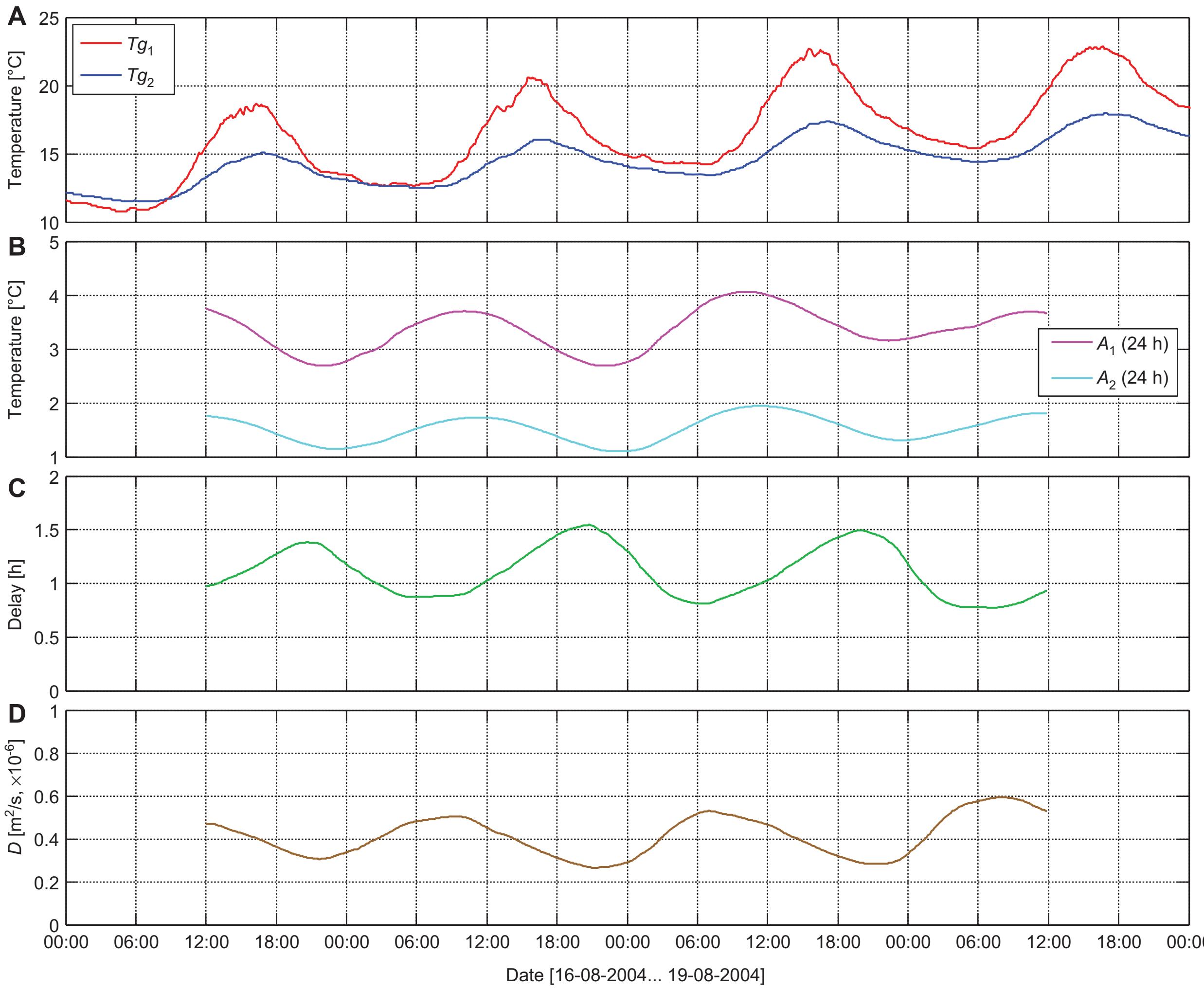

Figure 2 The time runs of measured data and computer parameters (from the top): (A) measured temperature values, (B) approximation temperature runs, (C) phase difference between the two considered layers in the form of time delay, (D) estimated thermal diffusivity coefficient *D*. The shorter runs for the computed values result from the block-based computations typical for the short-time Fourier transform.

agreement with reference results (Table 1). The considered soil contained 88% organic ingredients, whereas for the purely organic soil, $D=1\times10^{-7}$ m²/s [5].

Concluding comments

The presented results positively verify the hypothesis that the Ångström method may be efficiently applied to assess the thermal diffusivity coefficient in unsteady heat transfer conditions. Such an approach could be particularly useful in circumstances where the extraction of the investigated samples from their natural environment is troublesome or costly or may affect the obtained results considerably. The laboratory conditions when treating the separated sample assume the isolation of the sample from the surrounding environment, which differs noticeably from real conditions. The soil in the forest ground is a very good example; however, the suggested methodology could be easily applied in many other areas of research, especially when considering biological tissues [1].

The obtained results, presented in Figure 2 as time runs of A_1, A_2, $\Delta\varphi$, and D, indicate that some secondary heat radiation from the inside regions of the earth should be also included in further studies, either as an additional source of radiation or as the subject of an investigation.

Harmonic-based approximation and parameterization in a manner similar to that described above with the Fourier transform may be developed with use of related trigonometric transforms [12, 13], also involving wavelet-like decomposition. A considerable improvement of the efficiency of the method while applied to more sophisticated tasks should be expected with the support of artificial intelligence technology [14], which makes it possible to compensate for some missing values or incomplete simplified description adapted to realistic conditions when compared with profound theoretical model.

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