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Methodology based on volumetric biomass density to estimate solar radiation transmission through tree stands

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ABSTRACT

Solar radiation is a key factor which affects the hydrological balance of water exchange between a tree stand, the atmosphere and the ground. This paper presents a methodology of determining solar radiation transmission through tree stands based on vertical distribution of volumetric biomass density. Volumetric biomass density expresses the ratio of the biomass of all plants above the ground to the volume occupied by the whole tree stand (including the empty spaces between plants). It can be calculated based on empirical equations which describe the vertical biomass arrangement of every particular tree in a stand. Furthermore, the calculation scheme may be applied to any arbitrarily chosen layer in a given tree stand. Measurements of solar insolation were recorded at 4 levels for two mainly fir stands (*Abies alba Mill.*): a 135-year-old stand that is managed as a selection forest system, and a one-storied 115-year-old stand. The developed model, which is based on the Beer-Lambert law, estimates the solar radiation transmission through a tree stand with very high precision ($R^2 > 0.99$). The model parameters largely depend on tree height, tree diameter measured at breast height (130 cm), and the height of the top surface of the investigated layer above ground level. The model and parameterization are proposed mainly for fir stands and depend solely on easily measurable biometric features; thus, it should be readily adaptable to stands composed of other tree species by using appropriate coefficients that differentiate these stands from fir stands. A thorough understanding of the factors determining solar radiation transmission through tree stands may considerably expand the knowledge of the water exchange balance within forest complexes, snow melting rate, as well as the estimation of site productivity and forest succession.

1. Introduction

The amount of solar radiation energy reaching the ground surface from above the treetops is one of the main factors influencing the growth conditions of the forest community (Beaudet and Messier, 1998). It also regulates site productivity (Vose and Allen, 1988), biomass and the spatial arrangement of foliage (Chen et al., 1997), forest succession (Kobe et al., 1995; Piedallu and Gégout, 2008) and snow melting rate (Hardy et al., 2001; Ellis and Pomeroy, 2007). Furthermore, solar radiation is a fundamental component of water exchange between the atmosphere, stands, and the soil. In addition, the dynamics of the soil water reserve is one of the main habitat factors. Therefore, developed models ought to link the variability of water balance components with the biometric features of the studied stand and all parameters used in such equations should have physical or ecological interpretation (Suliński, 1993; Suliński and Owsiak 2009; Suliński and Starzak, 2009; Sypka and Starzak, 2013; Sypka et al., 2016a). Such equations are useful for comparing the water balance of forest ecosystems and tracking changes occurring in them in relation to the dynamics of the stand, silvicultural practices, biotic and abiotic factors, and events that generally cause changes in the biometric features of a tree stand (Suliński and Starzak, 2019).

The subject of solar radiation attenuation inside forest communities and its relationship to the biometric features of stands has been analysed for a long time (Rauner, 1972). The Beer-Lambert law, according to which absorbance is a function of bulk density and medium thickness, may be the most adaptable formula. Beer's law and its modifications, which take for example *LAI*, site index and basal area into account, are often used (Hale, 2001; Sonohat et. al., 2004). There are also models based on hemispherical photography (Hardy et al., 2004). Models of

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varying complexity have been developed according to different specifications (Ligot et al., 2014). Generally, variations in the conditions of solar radiation transmission through a stand are expressed by its biometric features, while the stand is treated as one layer (Reid et al., 2014). Although assuming that a canopy is horizontally homogeneous might be not suitable for complex, unevenly aged or mixed stands, this supposition is commonly used with good results (Forrester et al., 2021). Some more advanced models take the three-dimensional geometry of the forest canopy into consideration (Li et al., 1995). Forest-cover heterogeneity is characterized by simple geometric shapes for tree trunks and crowns (Courbaud et al., 2003). Such models usually depend upon various parameters that are problematical to measure in the field: for example, foliage area volume density, crown geometry, and the shape of gaps in the canopy. Furthermore, the *LAI* and *LAD* indexes are widely used to characterize the energy transmission through a stand, but such parameters do not directly describe the amount of biomass in a tree trunk zone and the vegetation that occupies the free space between trunks. Thus, without appropriate numerical relationships between these indexes and biomass, they may only be equivalent to the biomass of woody tree parts, i.e. trunks and branches. This issue seems to be important because, as mentioned above, the Beer-Lambert law is based on the bulk density of a medium (mass of the medium divided by its volume). In this context, it is necessary to construct a formula that defines a relationship between the solar radiation transmittance, of any selected layer within a tree stand and its characteristics, which are calculated on the basis of standard biometric data that is taken from forest management plans or can be effortlessly measured on the ground.

This paper presents transmission of solar radiation through tree stands and shows how to calculate a stand's volumetric biomass density and its vertical distribution within a stand. The aim of the study was to develop an appropriate formula that describes values of solar radiation transmittance, in a tree stand by means of biometric stand characteristics. An additional objective is the development of a formula that can calculate the volume and weight of individual trees whilst taking into account the vertical distribution of biomass in multi-storied and mixed-species stands. This formula will make it possible to use models which express universal relationships when extrapolating results from single measurements to large areas. Models which describe general relationships may be important in forest practice for predicting natural changes in stands due to stand development and the consequences of management measures (mainly weeding and thinning).

2. Methods and measurements

The transmittance of different media, i.e. the ratio of the light energy transmitted through a body to that falling on it, is generally described by Beer's law: $I_{out}/I_{in} = e^{-\mu \cdot x}$. This equation relates physical properties, i.e. material density, μ, to the optical path length through a sample, x. Therefore, application of Beer's law within a forest community requires correct computation of the volumetric biomass density of a tree stand and the development of numerical algorithms that enable its calculation for any arbitrarily chosen layer in a given tree stand. Additionally, in hydro-meteorological research, all measurements can be taken at only a few carefully chosen points, meaning the obtained results have to be extrapolated over a large spatial scale. For that reason, the investigated sites should be typical and have properties representative of all study regions.

2.1. Vertical distribution of tree stand volumetric biomass density

Considering ecological assumptions concerning how the empty spaces in a tree stand are filled with biomass (Suchecki, 1953; Czarnowski, 1989; Suliński, 1997), a tree stand's volumetric density can be defined as the ratio of the total biomass of all above-ground plant parts to the volume occupied by the tree stand using the following formula:

$$\overline{VSD} = \frac{M_t}{V_t} = \frac{\sum_i (\rho_i \cdot V_i + M_{li}) + M_g}{A \cdot \overline{H}} \tag{1}$$

where: $\overline{VSD}$ denotes the tree stand's mean volumetric biomass density [kg·m^{-3}]; M_t defines the total biomass of the tree stand [kg]; V_t is the volume occupied by the tree stand; ρ_i describes the average density of fresh wood for a given tree [kg·m^{-3}]; V_i represents the volume of a particular tree (a trunk and all branches) in a tree stand [m^3]; M_{li} expresses the biomass of leaves or needles of a given tree [kg]; M_g stands for the total biomass of the groundcover; A is the area covered by a tree stand [m^2] (usually $A = 10^4$ to standardize tree stand volume for 1 ha); $\overline{H}$ represents the mean height of the dominant stand [m]. In the presented research, the biomass of the groundcover was omitted ($M_g = 0$). Eq. (1) was constructed under the assumption that the total biomass of a tree stand can be calculated as the sum of the volumes of the trunks and branches of particular trees, multiplied by the appropriate average density of fresh wood, as shown in Table 1 (Dz.U., 2012), and the biomass of leaves or needles of trees, i.e. fresh foliage. In spite of the fact that each tree species has a specific weight, and the weight of individual parts of trees of one species also varies, the notion of a stand's volumetric biomass density should be understood as a generalization, i.e. the average value of a specific layer within a space occupied by a stand. Such a definition may be applied to calculate the volumetric biomass density for any chosen layer of a tree stand. Hence, each layer within a stand may be recognized as an endless slab of a certain thickness. Consequently, it is possible to assume, in a simplified way, that the distribution of biomass density is uniform within each thin horizontal layer.

Vertical distribution of a particular tree's volume of trunk and branches can be calculated based on its solid of revolution. After a series of numerical experiments using different bodies, a custom-made 3D figure based on rotating a 2$^{\text{nd}}$-order polynomial curve around the horizontal axis was created. The parabola which defines the relationship between tree radius, r, and tree height, h, reaches its global minimum at tree height $\left(H, \frac{D_H}{2}\right)$; at ground level, the radius of a tree trunk base is related to the value of $\frac{D_0}{2}$, i.e. $\left(0, \frac{D_0}{2}\right)$:

$$r(h) = \frac{D_0 - D_H}{2 \cdot H^2}(h - H)^2 + \frac{D_H}{2} \tag{2}$$

where D_0 is the diameter of the tree trunk base at ground level [m], and D_H indicates the diameter at tree height H. The shape of such a solid is shown in Fig. 1. Therefore, based on disc integration (Anton, 1984; Ayres and Mendelson, 2008), the total volume of such a 3D figure may be calculated by the following equation:

$$V = \pi \int_0^H r^2(h)dh = \frac{\pi}{60} \cdot H \cdot \left(3{D_0}^2 + 4D_0D_H + 8{D_H}^2\right) \tag{3}$$

where D_0 defines the diameter of the tree trunk base at ground level [m], and D_H [m] denotes the diameter at tree height H [m]. The diameter of the tree trunk base at ground level, D_0, and the diameter at tree height, D_H, can be computed based on data published in volumetric tables

Table 1
Average density of fresh wood, ρ [kg·m^{-3}] (Dz.U., 2012).

Species	ρ	Species	ρ	Species	ρ	Species	ρ
Douglas fir	660	Alder	770	Elm	820	Poplar	670
Fir	780	Ash	800	Goat willow	740	Robinia	840
Larch	830	Aspen	710	Hornbeam	960	Sycamore	910
Scots pine	740	Beech	980	Linden	670	White poplar	710
Spruce	750	Birch	810	Oak	950	Willow	770

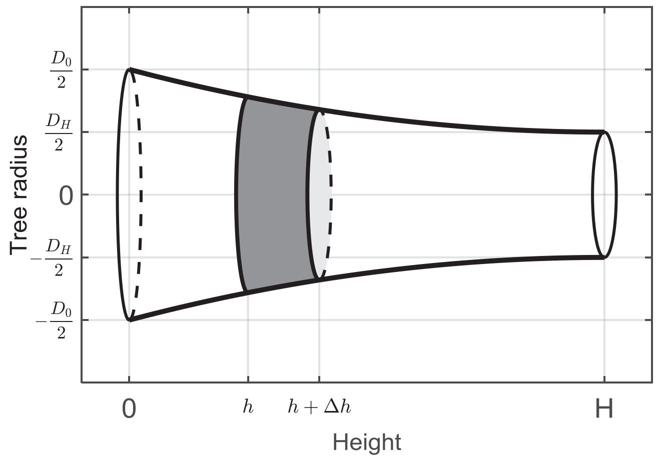

Fig. 1. The profile of the solid used to calculate tree volume (a trunk and all branches). A funnel-like figure was obtained by rotating a plane curve, i.e. a 2nd-order polynomial, around a horizontal axis. The parabola which describes the relationship between tree radius, *r*, and tree height, *h*, was defined under the assumption that it reaches its global minimum at tree height $\left(H, \frac{D_H}{2}\right)$; at ground level, the radius of a tree trunk base is related to the value of $\frac{D_0}{2}$, i.e. $\left(0, \frac{D_0}{2}\right)$. Such a solid of revolution makes it possible to calculate the total volume of this funnel-like solid and the volume of an arbitrarily chosen slice at height h and thickness Δh.

(Czuraj et al. 1966; Czuraj 1991) using the following empirical formulae, which take into account only the fundamental biometric features of a tree, i.e. tree height and diameter at breast height :

$$D_0 = DBH + 1.796\cdot(H+1)^{-1.15}\cdot\left(\frac{DBH}{100} + 0.01\right)^{1.03} \tag{4}$$

$$D_H = \alpha\cdot D_0{}^{\beta}\cdot(1 - \gamma\cdot 0.01\cdot H) \tag{5}$$

where: D_0 represents the diameter of the tree trunk base at ground level [m]; D_H [m] denotes the diameter at tree height H [m]; *DBH* is the diameter at breast height [cm]; α, β, γ are species-dependent coefficients which should be calculated in the process of formula identification, the results of which are presented in Table 2. Hence, the volume of an arbitrarily chosen slice at a height of h and a thickness of Δh (the grey area shown in Fig. 1) can be calculated as follows:

$$\Delta V(h) = \pi\int_{h}^{h+\Delta h} r^2(h)dh = \pi\frac{(D_o - D_H)^2}{20\cdot H^4}\cdot\left[(h+\Delta h - H)^5 - (h-H)^5\right] + \pi\frac{(D_o - D_H)\cdot D_H}{6\cdot H^2}\cdot\left[(h+\Delta h - H)^3 - (h-H)^3\right] + \pi\frac{D_H^2}{4}\cdot\Delta h \tag{6}$$

where D_0 defines the diameter of the tree trunk base at ground level [m], and D_H [m] denotes the diameter at tree height H [m].

Eq. (3) explains the volume of a particular tree with extremely high precision of almost 100% for all the main forest-forming tree species in moderate climates. Secondly, very low standard deviation values and average errors of estimation in the goodness-of-fit statistics (Table 2) confirmed the proper choice of the forms of Eqs. (3), (4) and (5).

The total biomass of leaves or needles may be calculated on the basis of the allometry theory using the following empirical equation, which was introduced by Suliński (1993, 2019):

$$M_l = \delta\cdot(100\cdot(D_0 - DBH))^{\kappa}\cdot V^{0.66} \tag{7}$$

Table 2
Calculated coefficients and goodness-of-fit statistics for model (3).

Species	α	β	γ	K	$100\cdot R^2$	σ	ν
Douglas fir	0.505	0.960	0.301	66	99.9	0.008	1.0
Fir	0.883	1.020	1.130	1891	99.9	0.082	1.4
Larch	0.128	0.287	−3.230	1691	99.9	0.054	1.6
Scots pine	0.628	0.971	0.963	1226	99.9	0.051	1.9
Spruce	0.475	0.700	0.340	1754	99.9	0.083	2.4
Alder	0.538	1.000	0.271	353	99.9	0.008	1.3
Ash	0.494	0.977	−0.210	1056	99.9	0.024	1.4
Aspen	0.458	0.945	0.0257	1242	99.9	0.024	1.4
Beech	0.640	1.090	0.0207	1421	99.9	0.013	0.5
Birch	0.348	0.869	−0.525	591	99.9	0.014	2.0
Elm	0.476	0.887	0.264	713	99.9	0.022	2.2
Goat willow	0.376	0.804	−0.0032	287	99.9	0.008	3.5
Hornbeam	0.606	1.150	−0.227	872	99.9	0.026	2.4
Linden	0.432	0.950	0.0205	1045	99.9	0.020	1.7
Oak	0.745	1.090	0.580	1985	99.9	0.072	1.6
Poplar	0.427	0.999	0.136	1371	99.9	0.012	0.5
Robinia	0.366	0.752	0.202	460	99.9	0.011	1.1
Sycamore	0.588	0.978	0.156	665	99.9	0.016	1.4
White poplar	0.449	0.899	0.148	1082	99.9	0.054	2.6
Willow	0.302	0.871	−1.110	670	99.9	0.019	2.3

K – number of cases, R – correlation coefficient, σ – standard deviation of estimation [m^3], ν – average error of estimation [%]

where: M_l denotes the total biomass of leaves [kg]; D_0 represents the diameter of the tree trunk base at ground level, Eq. (4) [m]; *DBH* is the diameter at breast height [cm]; δ, κ are species-dependent coefficients which should be estimated in the process of formula identification. Eq. (7) was verified using data from yield tables (Burger, 1945, 1947a, 1947b, 1950, 1951, 1953; Karmanova et al., 1987; Usolcev, 1988; Osuch, 1994). The results of the formula identification are presented in Table 3. The total biomass of leaves of a particular tree can be expressed by Eq. (7) with very high precision of over 91% (with the exception of birch, i.e. 88.5%). Due to the lack of data for all species in Table 2, the missing coefficients, which are denoted in italics in Table 3, were filled with values for similar species (Instrukcja, 2012).

Thereafter, the computed total biomass of leaves (or needles) should be added to the biomass of a tree in the thickness of its crown layer. At research plots in dense tree stands, it was impossible to measure the crown span for particular trees. Hence, it was simply assumed that the biomass of leaves has a vertical spatial distribution based on a quadric shape which depends on the species: a cone shape was assumed for coniferous stands and an oval shape for deciduous stands. The biomass of leaves in an arbitrarily chosen slice (within a crown layer, similarly to the slice shown in Fig. 1) may be calculated as the ratio of the volume of that slice to the whole volume of the crown body multiplied by the total biomass of the leaves or needles. Hence, the appropriate formulae have the following forms:

Coniferous stands :

$$m_l(h) = M_l\cdot\frac{1}{CT^3}\cdot\left\{(h - H + \Delta h)^3 - (h - H)^3\right\} \tag{8}$$

Deciduous stands :

$$m_l(h) = M_l\cdot\left\{\frac{2}{CT^3}\left(h - H + \frac{CT}{2}\right)^3 - \frac{2}{CT^3}\left(h - H + \frac{CT}{2} + \Delta h\right)^3 + \frac{3}{2}\frac{\Delta h}{CT}\right\} \tag{9}$$

Table 3
Calculated coefficients and goodness-of-fit statistics for model (7).

Species	δ	κ	K	$100{\cdot}R^2$	σ	ν
Douglas fir	34.4	1.49	21	98.4	3.47	15.1
Fir	40.4	0.788	88	97.2	15.9	25.2
Larch	10.3	0.732	99	93.0	3.54	22.6
Scots pine	18.6	0.551	210	91.2	3.05	31.6
Spruce	46.5	0.179	245	95.7	9.61	27.3
Aspen	20.9	0.267	73	92.0	1.81	24.7
Beech	18.6	0.519	91	99.2	1.61	15.7
Birch	36.2	0.282	101	88.5	4.36	30.3
Hornbeam	21.0	0.285	61	99.1	0.475	8.7
Linden	18.3	1.81	34	94.5	0.914	18.1
Oak	19.3	0.446	53	98.6	2.47	17.9
Alder (Birch) *	*36.2*	*0.282*				
Ash (Birch) *	*36.2*	*0.282*				
Elm (Oak) *	*19.3*	*0.446*				
Goat willow (Birch) *	*36.2*	*0.282*				
Poplar (Aspen) *	*20.9*	*0.267*				
Robinia (Birch) *	*36.2*	*0.282*				
Sycamore (Beech) *	*18.6*	*0.519*				
White poplar (Birch) *	*36.2*	*0.282*				
Willow (Aspen) *	*20.9*	*0.267*				

K – number of cases, R – correlation coefficient, σ – standard deviation of estimation [kg], ν – average error of estimation [%].
[*] due to the lack of data, coefficients were filled with values for similar species denoted in parenthesis.

where: m_l is the biomass of leaves in a slice [kg]; M_l denotes the total biomass of leaves [kg]; H defines tree height [m]; CT represents the thickness of the tree crown [m]; h is the height over the ground within the crown layer, i.e. $H - CT \leq h \leq H$ [m]; Δh is the thickness of a slice [m]. Thus, the volumetric biomass density in a given slice can by computed by the ratio between the sum of biomasses estimated for all particular trees to the volume of a slice:

$$VSD_{\Delta h}(h) = \frac{\sum_i \Delta V_i(h) \cdot \rho_i + m_{li}(h)}{A \cdot \Delta h} \tag{10}$$

where: $VSD_{\Delta h}$ denotes a tree stand's total volumetric biomass density in a slice of thickness Δh [kg·m^{-3}]; ΔV_i represents the volume of a slice of a particular tree (a trunk and branches) in a tree stand [m^3], described by Eq. (6); ρ_i is the average density of fresh wood for a given tree [kg·m^{-3}] (Table 1); m_{li} expresses the biomass of leaves or needles of a given tree within a slice [kg], described by Eqs. (8) or (9); A is the area occupied by a tree stand [m^2]. Fig. 2 shows the vertical biomass distribution for a trunk and branches (left), leaves/needles (centre), and a whole fir tree (right). The presented profiles were calculated based on the aforementioned equations under the assumption that $\Delta h = 0.05$ m.

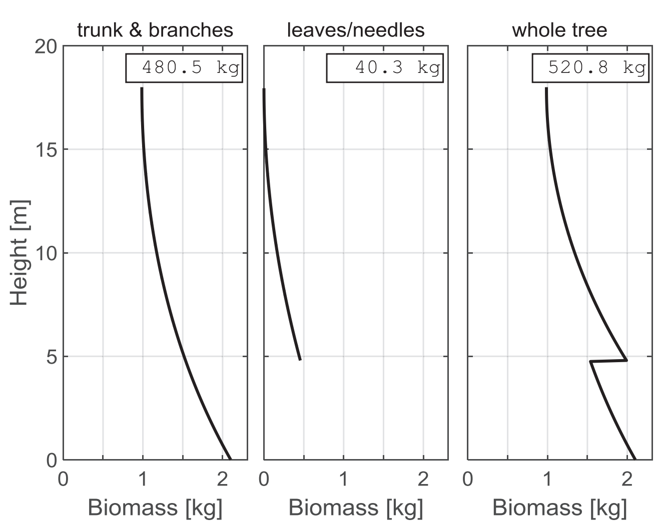

Fig. 2. A vertical biomass distribution of a particular fir tree ($DBH = 24.7$ cm, $H = 18$ m and $CT = 13.2$ m) calculated for a slice thickness of $\Delta h = 0.05$ m. Distribution of the volume of a trunk and branches (left) was estimated based its solid of revolution and disc integration using a custom-designed 3D figure based on rotating a 2nd-order polynomial curve around an axis. Additionally, the computed biomass of tree leaves/needles (centre) was added to the total biomass of a tree in the thickness of its crown layer (right).

2.2. Sites and measurements

The investigated sites were established by the Department of Forest Engineering at the University of Agriculture in Krakow, Poland, during large research project on water exchange balance in forest communities, i.e. between the atmosphere, stands, and soil. The solar radiation data were recorded from the 1st of October 2015 to the 30th of June 2018 at two research sites located in the Beskid Sądecki Mountain Range (49°29′6.0″ N, 20°46′44.3″ E). The first plot, NKTP (49 30′51.39″ N, 20 55′11.32″ E, 702 m AMSL) was located inside a 135-year-old fir stand: fir (*Abies alba Mill.)* 99% and spruce *(Picea abies (L.) Karst.)* 1%; the stand at this plot was managed as a selection forest system. The second site, KKPS (49 27′23.08″N, 20 57′57.47″E, 741 m AMSL), was situated within a one-storied, 115-year-old stand of mainly fir (*Abies alba Mill.* 81%, *Picea abies (L.) Karst.* 19%).

The dendrometric measurements within the investigated tree stands were taken at circular surfaces around the investigated profiles with a radius of 30 m. The diameters at breast height (130 cm above ground, *DBH*) were gauged with a precision calliper. The tree heights, *H*, and the height of the stem layers in the studied tree stands were measured trigonometrically using a Zeiss Dahlta 010B tacheometer. The height of the stem layer was defined as the height of the lowest green branches. Thus, the thickness of the tree crowns, *CT*, was computed as the difference between tree height and the height of the stem layer. Secondly, the acquired data were used to calculate mean diameter at breast height and mean tree stand height by applying Lorey's formula, which gives slightly higher values than the traditional arithmetic average because it also takes the basal area into account. The results (standardized for 1 ha) are shown in Table 4 (Starzak et al. 2019). Vertical profiles of the volumetric biomass density of the tree stands at the research plots are presented in Fig. 3.

Due to the technical and organizational constraints of field research within tree stands, the global solar irradiance was only measured at 4 levels. Solar irradiance was measured by precision semiconductor sensors. These sensors had a spectral response from 300 to 1100 nm, which is equivalent to the spectrum of shortwave solar radiation. Silicon sensors are insensitive to temperature variation (error less than 0.04 % K^{-1}) and have excellent long-term stability. All sensors were set horizontally and faced upward. The sensors' field of view was roughly 170°. At each of the aforementioned research sites, a climb-up mast was installed for at least one full calendar year: NKTP – from 1st October 2015 to 25th September 2016; KKPS – from 1st October 2016 to 30th June 2018. Two solar radiation sensors were installed on horizontal boom extenders that could be retracted to the mast on small trolleys: the first was just above the treetops; the second was just under the canopy. Two additional solar sensors were positioned at heights of about 2.4 m and 0.6 m over the ground. A range of hydro-meteorological data, including wind velocity, solar insolation, temperature, humidity etc., was automatically recorded every 6 min. The scheme of the gauging section inside the tree stands is shown in Fig. 4.

Recording solar radiation data at 4 levels made it possible to distinguish 6 layers in each tree stand. On the basis of the biometric features recorded for each tree at the research sites, the total biomass over the ground was calculated for each layer. The mean values of the recorded solar insulation for whole research period as well as selected characteristic of distinguished layers inside tree stands are presented in Table 5. Values of VSD_L for all considered layers were computed based on averaging $VSD_{\Delta h}(h)$, Eq. (10), in the thickness of a given layer, i.e.

Table 4
Empirical data and calculated biometric features of the stands at the investigated sites, standardized for 1 ha.

Site	A	N	$\overline{DBH}$	$\overline{H}$	V_s	M_t	M_s	M_l	$\overline{VSD}$
NKTP	135	883	18.2	28.0	587.27	514.76	486.65	28.11	1.838
KKPS	115	318	38.8	32.1	732.57	629.88	599.95	29.93	1.962

A – age [year]; N – number of trees per 1 ha; $\overline{DBH}$ – average diameter at breast height [cm]; $\overline{H}$ – average stand height [m];
V_s – total volume of trunks and branches [m^3];
M_t – total mass of trees [t of fresh mass·ha^{-1}]; M_s – total mass of trunks and branches [t of fresh mass·ha^{-1}];
M_l – total mass of leaves [t of fresh mass·ha^{-1}]; $\overline{VSD}$ – average volumetric biomass density of a tree stand at the research site [kg·m^{-3}·ha^{-1}].

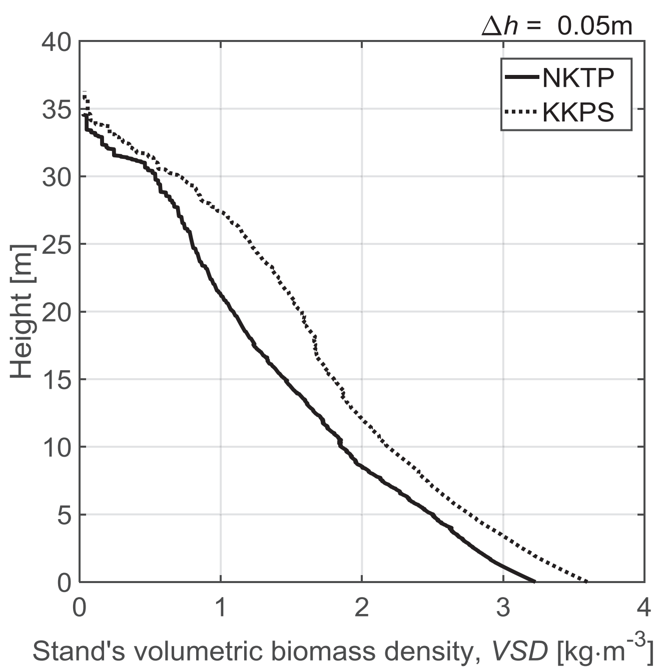

Fig. 3. Vertical distribution of volumetric biomass density at the research sites: NKTP (135-year-old fir stand managed as a selection forest system) and KKPS (one-storied, 115-year-old mainly fir stand). Stand's volumetric biomass density, VSD, is defined as the ratio of the total biomass of all the above-ground plant parts to the volume occupied by a given tree stand. All calculations were performed for a slice thickness of $\Delta h = 0.05$ m. Additionally, the total computed biomass of tree leaves (needles) was added to the total biomass of a tree in the thickness of its crown layer.

$VSD_L = \int_h^{h+\Delta H_L} VSD_{\Delta h}(h)dh$. Variation in the solar radiation transmittance at the research sites (hourly mean values calculated based only on daylight hours) for each individual layer are shown in Fig. 5. There are no noticeable relationships between the solar radiation transmittance and the time of day or year, both of which affect the intensity of solar radiation to a large extent. In contrast, solar radiation transmission differs significantly among the distinguish layers inside the investigated tree stands. It may be supposed that observed variations are largely determined by the volumetric biomass density of analysed layers (VSD_L, Table 5). Furthermore, it can be noticed that the amount of transmission depends on the solar insolation recorded at the top surface of a given layer, I_L, and also by the height of the top surface of a given layer over the ground. The highest variation can be observed for layers with small values of incident solar insolation, $I_L < 20$ [W·m^{-2}].

2.3. Attenuation of solar radiation within tree stand

Considering the specific conditions which determine the energy budget inside tree stands (Eagleson, 1970; Czarnowski, 1989) and the spatial distribution of solar sensors, it may be expected that variability in

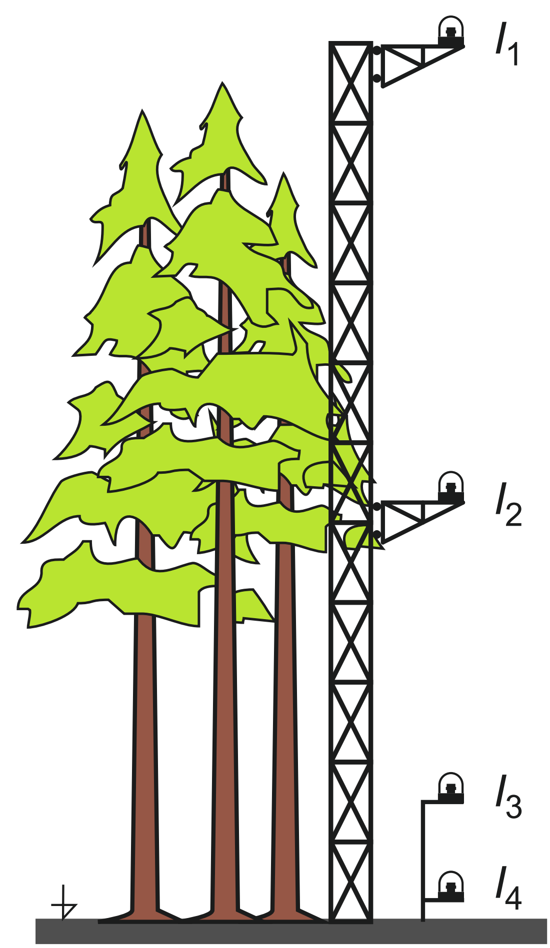

Fig. 4. Scheme of the gauging section at research plots. Silicon sensors were used to gauge global solar insolation at 4 levels: just above the treetops (I_1); just under the canopy (I_2); at heights of about 2.5 m (I_3) and 0.6 m (I_4) above the ground. Furthermore, a range of hydro-meteorological data, including wind velocity, temperature, humidity, and precipitation, was automatically recorded every 6 min.

solar radiation transmission could be modelled by:

- volumetric biomass density and the thickness of a given layer,
- the average solar insolation measured at the top surface of a given layer,
- the height of the top surface of a given layer above ground level.

The above assumptions were verified based on data recorded during field measurements. Hourly mean data of solar insulation calculated based only on daylight hours were analysed. A multi-way variance analysis (ANOVA) was used to determine the significance of the influence of the selected explanatory parameters on solar radiation transmission. Furthermore, these parameters' percentage contribution to the

Table 5
Recorded data and selected characteristic of distinguished layers inside tree stands at the research plots.

Site	Layer	Heights	ΔH_L	$HAGL_L$	VSD_L	I_L	W_L
NKTP	21	28.0 ÷ 10.7	17.3	28.0	0.952	237.5	0.148
	31	28.0 ÷ 2.3	25.7	28.0	1.293	226.1	0.042
	41	28.0 ÷ 0.7	27.3	28.0	1.371	210.0	0.031
	32	10.7 ÷ 2.3	8.4	10.7	2.260	26.1	0.285
	42	10.7 ÷ 0.7	10.0	10.7	2.370	25.4	0.203
	43	2.3 ÷ 0.7	1.6	2.3	2.950	6.0	0.750
KKPS	21	32.1 ÷ 12.8	19.3	32.1	1.126	220.1	0.154
	31	32.1 ÷ 2.4	29.7	32.1	1.530	205.8	0.075
	41	32.1 ÷ 0.6	31.5	32.1	1.622	205.7	0.061
	32	12.8 ÷ 2.4	10.4	12.8	2.443	16.0	0.473
	31	12.8 ÷ 0.6	12.2	12.8	2.574	16.0	0.425
	43	2.4 ÷ 0.6	1.8	2.4	3.313	11.7	0.803

ΔH_L – layer thickness [m]; $HAGL_L$ – height of the top surface of a given layer over the ground [m];
VSD_L – volumetric biomass density of a given layer [kg·m^{-3}]; I_L – average solar insolation measured at the top surface of a given layer [W·m^{-2}];
W_L – solar radiation transmittance for a given layer (mean value);

description of variability in solar radiation transmittance was assessed using component variance analysis. In conclusion, solar insolation may be treated as an equivalent of the weather conditions in which the process of solar transmission through a stand takes place. The time of day greatly affects the amount of solar radiation, I_L, at the top of all four layers, but it has no significant influence on solar transmission through these layers. Nevertheless, the lower a layer's height above ground level, the lower the solar insolation values, and the higher the solar radiation transmittance values (Table 5). This may substantiate a thesis that these both parameters should also be used to describe the relationship between solar radiation transmission through a stand and the biometric features of that stand (Fig. 6).

3. Results

Firstly, the unmodified Beer-Lambert law, i.e. the one-parameter negative exponential relationship, was analysed under the assumptions that material density, μ, was replaced by the mean volumetric biomass density for the considered layer, VSD_L, and the optical path length through the sample, x, was equal to layer thickness ΔH_L. For site NKTP, the goodness-of-fit statistics for this simple model showed that variation in the solar radiation transmittance can be explained with a precision of 92.7%, an average error of estimation of 27.5%, and a standard deviation of 0.067. For site KKPS, these values are 79.3%, 37.2% and 0.124, respectively. Therefore, it may be concluded that application of the Beer-Lambert law inside a stand is well-grounded. Nevertheless, it is essential to find additional explanatory variables which could considerably reduce the standard error of estimation. Thereafter, based on data presented in Fig. 6 (left), the average solar insolation measured at the top surface of a given layer, I_L, can be used as a supplementary variable. Thus, the solar radiation attenuation within a stand may be described by the following empirical formula:

$$W_L = e^{(-a\cdot VSD_L\cdot\Delta H_L)-(b\cdot I_L)} \tag{11}$$

where: VSD_L is a volumetric biomass density within a considered layer [kg·m^{-3}]; ΔH_L expresses layer thickness [m]. I_L denotes the average solar insolation measured at the top surface of a given layer [W·m^{-3}]; a, b are coefficients that are estimated in the process of formula identification. The goodness-of-fit statistics for Eq. (11) are presented in Table 6. Fig. 7 presents the relationships between the observed and predicted values for Eq. (11). Variation in solar radiation transmission may be explained with an extremely high precision of over 99% for both stands. The average errors of estimation are less than 5%.

4. Discussion

The paper presents a methodology to approximate solar radiation transmission through a tree stand at stand scale. Regarding the analysed data, the developed model can be used to estimate the amount of solar radiation on an arbitrarily chosen horizontal plane within a tree stand. In contrast to previously published works (Castro and Fetcher, 1998; Hale, 2001; Hardy et al., 2004; Sonohat et. al., 2004; Forrester et al., 2014), the developed model was based on stand's volumetric biomass density. It should be noted that the introduced notion of volumetric biomass density significantly differs from both the widely used stand density, i.e. Reineke's Stand Density Index (Reineke, 1933), and common physical density i.e. specific mass (a ratio of body mass to volume of this body). Stand's volumetric biomass density, VSD, expresses the ratio of the biomass of all plants above the ground to the volume occupied by a whole tree stand (including empty spaces between plants), but not just the volume of all plants. It could be also noted that in the presented study the total biomass of groundcover was omitted, M_g in Eq. (1), because vegetation in this layer practically did not exist at the research plots. Secondly, the lowest solar sensors were situated at height of approximately 0.6 m above the ground, i.e. over the soil cover. The developed model, based on the Beer-Lambert law Eq. (11), expresses solar radiation transmission through a tree stand with very high precision of almost 100% for both fir stands (Table 6). Therefore, it may be assumed that the model equations were properly constructed and the explanatory variables were correctly selected. Secondly, stand's volumetric biomass density, VSD, calculated based on the standard biometric features included in yield tables, may be interpreted as a reliable numerical value with ecological interpretation that can be used as a variable to determine the solar radiation transmission through a tree stand. Volumetric biomass density is calculated based on tree volume, average density of fresh wood, and the total biomass of leaves or needles. Furthermore, it is not species dependent. The presented empirical model of the volume of a singular tree, Eq. (3), was developed on the basis of a theory that the sum of all horizontal intersection areas of all tree shoots at a given height equals the area of a cylinder base with a diameter equivalent to the tree trunk base at ground level (Czarnowski, 1989). After a series of numerical experiments using different bodies, including cylinder, cone, truncated cone, half-barrel, and combinations of all these solids, the authors decided to create a custom-made 3D figure which also takes trunk taperness into account. A tree's narrowing toward the top is expressed by a bespoke parabola, Eq. (2). The developed formula has one general form for all the main forest-forming tree species in moderate climates, which is essential for modelling based on ecological assumptions. In addition, the presented calculation scheme based on Eq. (10) makes it possible, by integration, to compute a volumetric biomass density of any tree segment between arbitrarily chosen horizontal parallel planes located at any heights. The accuracy of the model was examined using datasets included in stand volume tables, and an extremely high precision of almost 100% was reached (Table 2). Hence, it may be concluded that the developed Eq. (3) can replace data sets included in these tables. Stand volume tables were created from data recorded during direct measurements of tree volumes (trunks and branches) in the field. Additionally, the formula found in a 2nd-order polynomial can be effortlessly integrated to calculate a volume of an arbitrarily chosen slice at a given height, Eq. (6), which is crucial to determine the vertical distribution of tree volume and volumetric biomass density (Fig. 2 and Fig. 3). The total biomass of leaves, Eq. (7), takes the theory of allometry into account, based on Czarnowski's suggestions (1989). The total biomass of leaves is expressed based on body volume with one scaling exponent for all species. The value of the scaling exponent refers to biomass distribution, which exploits a power law and focuses on how the mass of specific part of an organism relates to the total mass of this organism total mass (Poorter et al., 2015). The results of identification of the empirical formula are very good (Table 3).

Based on the presented goodness-of-fit statistics, it may be assumed

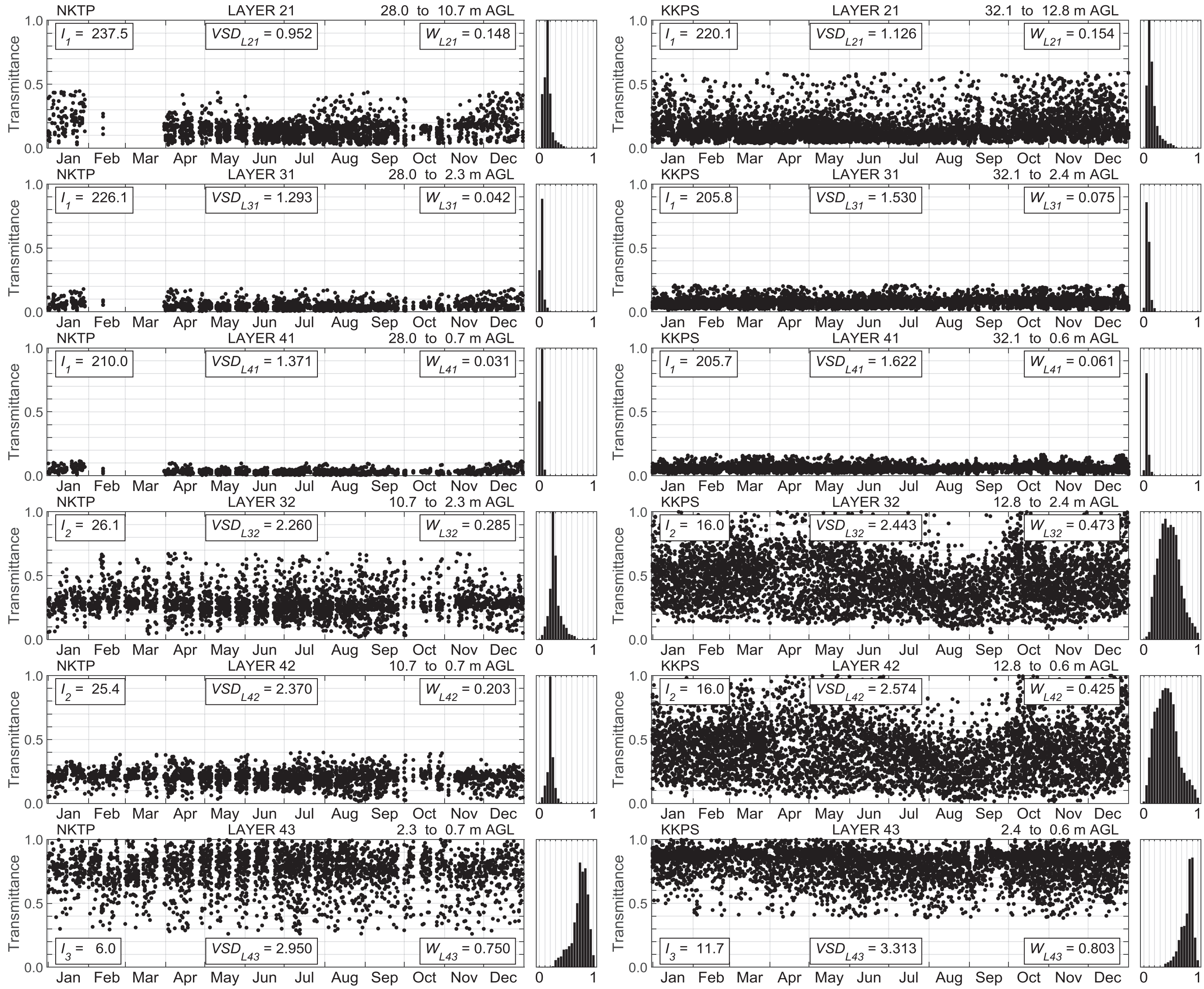

Fig. 5. Variation in solar radiation transmittance through tree stands at the research sites and their histograms. Hourly mean values of attenuation were calculated based on data recorded every 6 min in two coniferous stands: NKTP (135-year-old fir stand managed as a selection forest system) and KKPS (one-storied, 115-year-old mainly fir stand). Thus, global mean values of transmittance, W_L, and solar insolation at the top surface of a given layer, I_L [W·m^{-2}] were computed. It may be noticed that solar radiation transmission within a tree stand mainly depends on the volumetric biomass density of a given layer, VSD_L, layer thickness, and the amount of solar insolation recorded at the top surface of a given layer or by the height of the top surface of a given layer over the ground. The highest variation can be observed for layers with small values of incident solar radiation, $I_L < 20$ [W·m^{-2}].

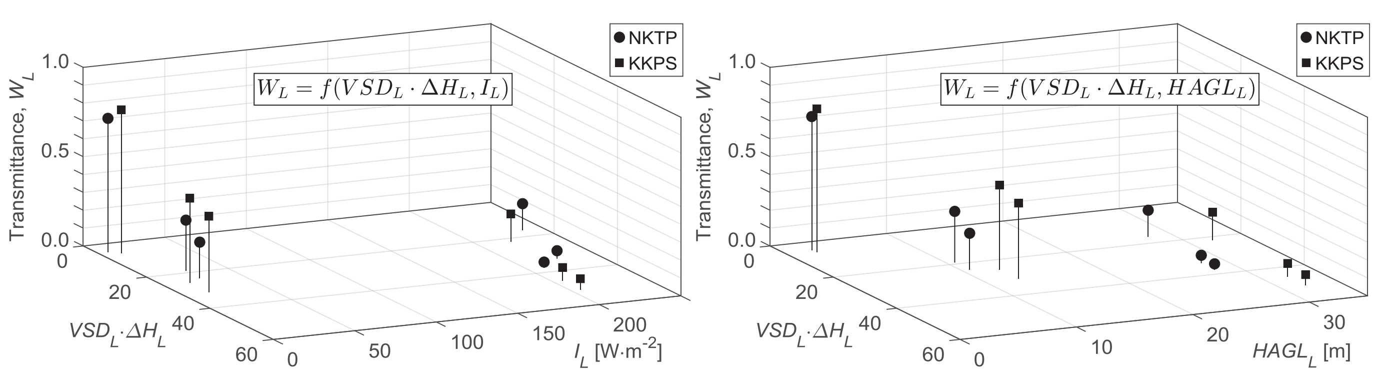

Fig. 6. Relationship between solar radiation transmittance, W_L, and tree stand volumetric biomass density in a given layer, VSD_L, layer thickness, ΔH_L, and average solar insolation measured at the top surface of a given layer, I_L, (left) or the height of the top surface of a given layer over the ground, $HAGL_L$, (right).

Table 6
Calculated coefficients and goodness-of-fit statistics for model (11).

Site	a	b	K	$100{\cdot}R^2$	σ	ν
NKTP	0.0611	0.0040	6	99.9	0.012	5.0
KKPS	0.0248	0.0064	6	99.9	0.011	3.4
Both sites	0.0364	0.0065	12	89.2	0.090	31.1

K – number of cases, R – correlation coefficient, σ – standard deviation of estimation, ν – average error of estimation [%].

that application of the Beer-Lambert law inside a tree stand is well justified. The unmodified Beer-Lambert law can explain variation in solar radiation transmittance with a precision of roughly 93% (site NKTP) and 80% (site KKPS), which is similar to models found in the literature. The accuracy of the solar radiation transmission model was significantly improved by adding the second term in the exponent in Eq. (11), i.e. average solar insolation measured at the top surface of a given layer, I_L. At site KKPS, the accuracy of the analysis of the solar radiation transmission variation was enhanced from 79.3% to 99.9%, and the average error of estimation decreased from 37.2% to 3.4%. In the tree stand at NKTP research site, the influence of the I_L parameter was a bit smaller: the aforementioned improvements were from 92.7% to 99.9% and from 27.5% to 5%, respectively. In forest practice, solar insulation is not gauged due to field constraints and may be only estimated over treetops (Sypka et al., 2016b). That is why, for practical purposes, this parameter should be replaced by other biometric features of a tree stand. Correct estimation of solar radiation transmittance without measuring solar insulation would be essential to contrast and compare solar absorption for different stands. Concerning the aforementioned analysis of the relationships between the parameters and solar radiation transmission, it may be suggested that solar insulation, I_L, could be replaced by the height of the top surface of a given layer above ground level, $HAGL_L$ (Fig. 6, right). In previous works (Suliński, 1993; Suliński and Sypka, 2000), the height of distinguished layers of biomass over the ground was also a key factor which affected the relationship between solar radiation attenuation and the amount of biomass. The previous model was developed by analogy to meteorological research: it was assumed that the height above ground level of a biomass layer may have a similar influence on the quantity of absorbed solar energy to the shading effect of clouds. Therefore, after replacing solar insolation, I_L, by the height of a given layer over the ground, $HAGL_L$, the presented model, Eq. (11), can be expressed by the following formula:

$$W_L = e^{(-p\cdot VSD_L\cdot \Delta H_L)-(q\cdot HAGL_L)} \qquad (12)$$

where: VSD_L represents volumetric biomass density within a considered layer [kg·m^{-3}]; ΔH_L denotes layer thickness [m]; $HAGL_L$ is the height of the top surface of a given layer above ground level [m]; p, q are coefficients that are estimated in the process of formula identification. The results of the formula identification (12) are presented in Table 7. The relationships between the observed and predicted values for Eq. (12) are shown in Fig. 8. Variation in solar radiation transmission can still be explained very accurately for the investigated stands: 99.4% and 96.7% for NKTP and KKPS, respectively. The average errors of estimation are acceptable: less than 14%.

Although the developed model was verified in two mainly fir stands (NKTP: fir 99%, spruce 1% and KKPS: fir 81%. spruce 19%), both the investigated stands' structures differ significantly (Table 4). Therefore, the presented computation scheme, based on only standard parameters like H, DBH and CT, can effectively be adaptable to other species or even mixed-species stands, while appropriate coefficients that account for differences between species are preserved for all evaluated trees in the investigated region (Table 2 and Table 3). For that reason, a significant question arises over whether the calculated coefficients in both models, Eqs. (11) and (12), Table 6 and Table 7, refer to specific features of a given mainly fir tree stand or have a more general character and may represent a general description of solar radiation transmission within forest complexes. The regression statistics for both models, Eqs. (11) and (12), simultaneously for both tree stands (12 cases in total), did not give an unambiguous conclusion. Although the values of the correlation coefficient, $100{\cdot}R^2$, are above 89% and the variability in solar radiation transmittance, W_L, may be explained with good accuracy, the values of the average error of estimation are quite high, from 31% to 33%. The large values of the mean estimation error may be mostly explained by the measurement conditions. Due to the technical limitations of field research, solar radiation was gauged only at four levels (six layers per stand), which is why the changeability of the explanatory variable, $VSD_{\Delta h}$, was relatively small. Furthermore, the silicon sensor used in the presented research had a photosensitive area of 6.6 mm^2 (2.8×2.4 mm); thus, the readings could have been biased by obstructions in the closest neighbourhood, for example, branches located just over a sensor. The location of the mast at each site was mainly determined by the surrounding arrangement of trees, i.e. small gaps in crowns were required to retract and mount the horizontal boom extenders (Fig. 4). Hence, better accuracy could be obtained by using, for example, sensor matrixes and averaged readings (Sonohat et al., 2004). Therefore, the aforementioned question about the general character of the model coefficients should be understood as a topic for future research. The vertical distributions of volumetric biomass density and the vertical profiles of max-normalized volumetric biomass density, $VSD'_{\Delta h} = VSD_{\Delta h}/\max(VSD_{\Delta h})$, in the investigated fir tree stands are presented in Fig. 9. All calculations were performed for a slice thickness of $\Delta h = 0.50$ m from ground level to the height of the tallest tree in a given plot. Although the tree stands in the research sites differ greatly in structure (Fig. 3, Table 4), some similarities may be observed in both presented curves. This is why the presented methodology may be useful to

Table 7
Calculated coefficients and goodness-of-fit statistics for model (12).

Site	p	q	K	$100{\cdot}R^2$	σ	ν
NKTP	0.0449	0.0437	6	99.7	0.020	8.1
KKPS	0.0022	0.0634	6	97.5	0.046	13.7
Both sites	0.0107	0.0703	12	88.1	0.094	32.8

K – number of cases, R – correlation coefficient, σ – standard deviation of estimation, ν – average error of estimation [%].

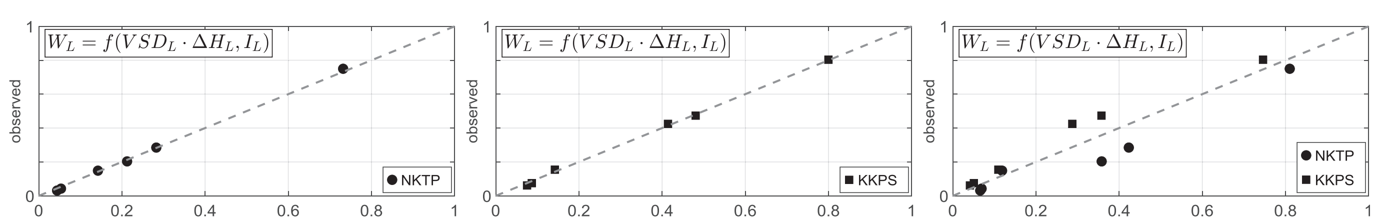

Fig. 7. The relationships between the observed and predicted values for the model of solar radiation transmission within the investigated tree stands. The model, which is based on the Beer-Lambert law, takes into account only volumetric biomass density within a considered layer, VSD_L, the thickness of a given layer, ΔH_L, as well as the average solar insolation measured at the top surface of a given layer, I_L.

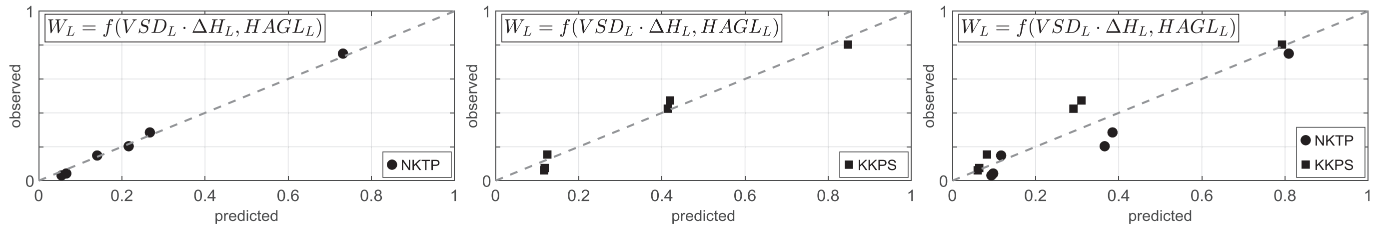

Fig. 8. The relationships within the investigated tree stands between observed and predicted values for the improved solar radiation transmission model, which can be used in forest practice. This model is still constructed based on the Beer-Lambert law; besides volumetric biomass density inside the considered layer, VSD_L, and the thickness of a given layer, ΔH_L, the height of the top surface of a given layer over the ground, $HAGL_L$, is taken into account.

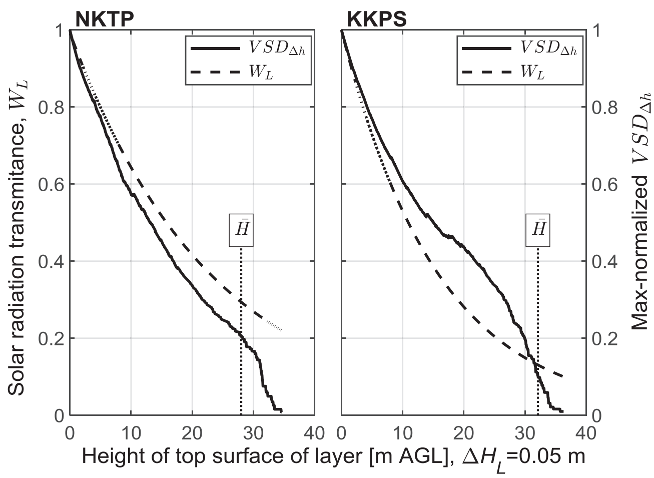

Fig. 9. Comparison of estimated transmission of solar radiation and vertical profiles of max-normalized volumetric biomass density, $\dot{VSD}_{\Delta h} = VSD_{\Delta h}/\max(VSD_{\Delta h})$, inside the investigated fir tree stands: NKTP (135-year-old stand managed as a selection forest system) and KKPS (one-storied, 115-year-old stand). All calculations were performed for a slice thickness of $\Delta h = 0.50$ m. $\bar{H}$ denotes mean tree stand height, calculated by applying Lorey's formula.

approximate solar radiation regimes in stands that differ in species, growth dynamics, vertical structure, silvicultural practices, or even some events caused by biotic and abiotic factors. Using data contained in forest management plans, this methodology could play a crucial role in constructing digital maps that depict the hydrological features of stands.

Generally, leaf area index, *LAI*, is used as an explanatory variable when modelling water exchange balance between the atmosphere, a stand and the ground. In contrast to *H*, *DBH*, *CT* or calculated VSD_L, measurements of *LAI* are difficult and inaccurate (Fassnacht et al., 1994; Weiss et al., 2004). Secondly, estimating *LAI* at a large spatial scale could be difficult to compute, especially in areas of stands where parts of trees of different species overlap. The application of other methods based on, for instance, hemispherical photography (Hardy et al., 2004; Ellis and Pomeroy, 2007) may be infeasible at a large spatial scale. A parameterization scheme based on volumetric biomass density can be applied to any chosen layer in a stand, not only to the canopy, where *LAI* or *LAD* is directly evaluated. In addition, unlike *LAI*-based models, such a scheme takes inherently woody tree parts, i.e. trunks and branches, into account. Correct estimation of VSD_L, based either on measurements that can easily be taken on the ground or on datasets included in forest management plans, creates the possibility of easy extrapolation of the model to large spatial scales, e.g. to a whole forest inspectorate with an area of approx. 10,000 ha. Besides, it should be noted that all biometric features of a stand, for example *N*, *DBH*, *H*, *LAI*, are dependent upon one another. In this case, the calculated total volumetric biomass density of a tree stand, $\overline{VSD}$, automatically includes all information about site quality, stand density (*SDI*), and *LAI*. Such relationships between these biometric features are a subject of interest in dendrometry (Turner et al., 2000). Secondly, models based on a tree stand's volumetric biomass density (Sypka and Starzak, 2013) or the total mass of the stand above the ground, M_t, (Sypka et al., 2016a) have successfully described wind or temperature regimes inside a tree stand. In the same way, there are solutions which take other biometric features of a stand into account, like basal area, stand age, or thinning intensity (Sonohat et al., 2004). The ongoing development of modern remote-sensing techniques such as airborne laser scanning may solve the abovementioned problem of measuring biometric stand features in large areas. Wide-scale forest information databases can provide detailed information about vegetation structure, volumetric biomass density, and even the position and height of every individual tree in the investigated area (Wężyk et al. 2013).

5. Conclusions

The paper shows that the Beer-Lambert law can be successfully applied to characterize solar radiation regimes for any given layer within a tree stand. Secondly, a methodology to estimate vertical distribution of biomass even for complex, uneven-aged, or mixed stands has been presented. A close relationship was observed between solar radiation transmission and volumetric biomass density, which describes the ratio of biomass accumulated on the ground to the volume occupied by a whole tree stand. Furthermore, empirical formulae to calculate tree volume and the biomass of leaves or needles for an arbitrarily chosen layer were introduced. The constructed models depend only upon standard stand biometric data which can typically be found in forest management plans or can be effortlessly measured in the field on the ground. Such a parameterization method should effectively be adaptable to other species, while appropriate coefficients that account for differences between species are preserved. In addition, such models may also be extrapolated to large spatial scales. Models that use biometric features are crucial in forest practice for predicting changes in stands due to management measures or stand development. The improvements that are necessary for the presented model may be possible after a series of measurements in areas that contain different types of single- and multi-species stands. Such thorough research should be done in a larger number of stands so that the variation of the explanatory parameters is larger.

CRediT authorship contribution statement

Józef Suliński: Conceptualization, Methodology, Formal analysis, Visualization, Supervision, Project administration, Funding acquisition. **Przemysław Sypka:** Software, Methodology, Investigation, Formal analysis, Data curation, Visualization, Writing - original draft, Writing - review & editing. **Rafał Starzak:** Formal analysis, Investigation, Supervision, Project administration, Funding acquisition.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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