

## 1. Gramatyki i języki – zadania

Podać, jakie języki są generowane przez poniższe gramatyki. (Wielkie litery łacińskie oznaczają symbole nieterminalne, małe litery łacińskie, cyfry oraz znaki specjalne, jak np. nawiasy okrągłe lub kwadratowe, oznaczają symbole terminalne, symbolem początkowym jest nieterminal stojący w lewej stronie pierwszej produkcji.)

1.1.

$$\begin{aligned} S &\rightarrow aS \mid bS \mid aA \\ A &\rightarrow aA \mid bA \mid aB \\ B &\rightarrow aB \mid bB \mid \varepsilon \end{aligned}$$

1.2.

$$\begin{aligned} S &\rightarrow aS \mid bS \mid aA \\ A &\rightarrow aB \\ B &\rightarrow aB \mid bB \mid \varepsilon \end{aligned}$$

1.3.

$$\begin{aligned} S &\rightarrow aS \mid bS \mid aA \\ A &\rightarrow aB \mid bB \\ B &\rightarrow \varepsilon \end{aligned}$$

1.4.

$$\begin{aligned} S &\rightarrow aA \\ A &\rightarrow baA \mid aA \mid ba \mid a \end{aligned}$$

1.5.

$$\begin{aligned} S &\rightarrow SAB \mid AB \\ A &\rightarrow Aa \mid a \\ B &\rightarrow Bb \mid b \end{aligned}$$

1.6.

$$\begin{aligned} S &\rightarrow ABC \\ A &\rightarrow aAb \mid ab \\ B &\rightarrow bBa \mid ba \\ C &\rightarrow aCb \mid ab \end{aligned}$$

1.7.

$$\begin{aligned} S &\rightarrow AC \\ A &\rightarrow aAb \mid aBb \\ B &\rightarrow bBa \mid ba \\ C &\rightarrow aCb \mid ab \end{aligned}$$

**1.8.**

$$\begin{aligned} S &\rightarrow aAb \mid aSa \\ A &\rightarrow bAcc \mid ab \end{aligned}$$

**1.9.**

$$\begin{aligned} S &\rightarrow aSb \mid aAb \\ A &\rightarrow bAa \mid bBa \\ B &\rightarrow aBb \mid ab \end{aligned}$$

**1.10.**

$$\begin{aligned} S &\rightarrow aaSb \mid bAa \\ A &\rightarrow aAbb \mid ba \end{aligned}$$

Jakie języki są generowane przez poniższe gramatyki bezkontekstowe? Czy poniższe gramatyki są jednoznaczne? Uzasadnić.

**1.11**

$$\begin{aligned} S &\rightarrow AB \mid DC \\ A &\rightarrow aA \mid \varepsilon \\ B &\rightarrow bBc \mid \varepsilon \\ C &\rightarrow cC \mid \varepsilon \\ D &\rightarrow aDb \mid \varepsilon \end{aligned}$$

**1.12.**

$$\begin{aligned} S &\rightarrow A \mid CD \\ A &\rightarrow aAd \mid B \\ B &\rightarrow bBc \mid \varepsilon \\ C &\rightarrow aCb \mid \varepsilon \\ D &\rightarrow cDd \mid \varepsilon \end{aligned}$$

**1.13.**

$$E \rightarrow E+E \mid E^*E \mid (E) \mid a$$

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**1.14.**

$$\begin{aligned} S &\rightarrow bAb \\ A &\rightarrow aAb \mid acb \end{aligned}$$

**1.15.**

$$S \rightarrow aS \mid aSb \mid \varepsilon$$

**1.16.**

$$S \rightarrow SS \mid aSb \mid bSa \mid ab \mid ba$$

**1.17.**

$$S \rightarrow SAB \mid ABS \mid aabb$$

$$A \rightarrow Aaa \mid aa$$

$$B \rightarrow Bbb \mid bb$$

**1.18.**

$$S \rightarrow bSa \mid aSb \mid \varepsilon$$

**1.19.**

$$S \rightarrow aSa \mid bSb \mid aa \mid bb$$

**1.20.**

$$S \rightarrow SS \mid ab \mid ba$$

**1.21.**

$$S \rightarrow Sa \mid Aa$$

$$A \rightarrow aAb \mid ab$$

**1.22.**

$$S \rightarrow SA \mid SB \mid \varepsilon$$

$$A \rightarrow Aa \mid a$$

$$B \rightarrow Bb \mid b$$

**1.23.**

$$S \rightarrow abcc \mid aSc \mid aAc$$

$$A \rightarrow bAc \mid bc$$

**1.24.**

$$S \rightarrow A \mid B \mid C$$

$$A \rightarrow ab \mid aaaA$$

$$B \rightarrow bbc \mid bbbB$$

$$C \rightarrow ccca \mid cccC$$

**1.25.**

$$S \rightarrow a \mid ab \mid abc \mid abcS$$

**1.26.**

$$S \rightarrow aA$$

$$A \rightarrow baA \mid aA \mid ba \mid a$$

**1.27.**

$$S \rightarrow abb \mid abbA$$

$$A \rightarrow aab \mid aabS$$

1.28.

$$S \rightarrow SS \mid aSb \mid ab$$

1.29.

$$\begin{aligned} S &\rightarrow A \mid B \\ A &\rightarrow a \mid ab \mid abA \\ B &\rightarrow b \mid ba \mid baB \end{aligned}$$

1.30.

$$S \rightarrow SS \mid (S) \mid \varepsilon$$

1.31.

$$\begin{aligned} S &\rightarrow SS \mid [S] \mid A \\ A &\rightarrow AA \mid (A) \mid \varepsilon \end{aligned}$$

1.32.

$$S \rightarrow SS \mid ({}_1 S)_1 \mid ({}_2 S)_2 \mid ({}_3 S)_3 \mid \varepsilon$$

gdzie:  $\Sigma = \{ ({}_1, )_1, ({}_2, )_2, ({}_3, )_3 \}$

1.33.

$$S \rightarrow Sa \mid Sb \mid \varepsilon$$

1.34.

$$\begin{aligned} S &\rightarrow aSa \mid bAb \\ A &\rightarrow cAc \mid bab \end{aligned}$$

1.35.

$$\begin{aligned} S &\rightarrow aSb \mid bAc \\ A &\rightarrow Ac \mid cb \end{aligned}$$

1.36.

$$\begin{aligned} S &\rightarrow aAbBc \\ A &\rightarrow bAa \mid ba \\ B &\rightarrow cBb \mid cb \end{aligned}$$

1.37.

- (a)  $S \rightarrow aA \mid bB \mid \varepsilon$   
 $A \rightarrow aaC$   
 $B \rightarrow bbD$   
 $C \rightarrow aC \mid bB \mid \varepsilon$   
 $D \rightarrow aA \mid bD \mid \varepsilon$
- (b)  $S \rightarrow aA \mid bB \mid \varepsilon$   
 $A \rightarrow aC \mid bB \mid \varepsilon$   
 $B \rightarrow aA \mid bD \mid \varepsilon$   
 $C \rightarrow bB \mid \varepsilon$   
 $D \rightarrow aA \mid \varepsilon$

**1.38.**

$$\begin{aligned} \text{(a)} \quad S &\rightarrow aS / bB \\ B &\rightarrow bB / cC / \varepsilon \\ C &\rightarrow cC / \varepsilon \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad S &\rightarrow aA \\ A &\rightarrow aA / bB / cC \\ B &\rightarrow bB / cC \\ C &\rightarrow cC / \varepsilon \end{aligned}$$

$$\begin{aligned} \text{(c)} \quad S &\rightarrow aA / bB \\ A &\rightarrow aA / bB / cC \\ B &\rightarrow bB / cC / \varepsilon \\ C &\rightarrow cC / \varepsilon \end{aligned}$$

**1.39.**

$$\begin{aligned} S &\rightarrow aAB / aBA / bAA \\ A &\rightarrow a / aS / bAAA \\ B &\rightarrow b / bS / aABB / aBAB / aBBA \end{aligned}$$

**1.40.**

$$\begin{aligned} S &\rightarrow aB / bA \\ A &\rightarrow a / aS / bAA \\ B &\rightarrow b / bS / aBB \end{aligned}$$

**1.41.**

$$\begin{aligned} S &\rightarrow aSa / bSb / aTb / bTa \\ T &\rightarrow aT / bT / \varepsilon \end{aligned}$$

**1.42.**

$$\begin{aligned} S &\rightarrow aSb / bY / Ya \\ Y &\rightarrow bY / aY / \varepsilon \end{aligned}$$

**1.43.**

$$\begin{aligned} S &\rightarrow XSX / W \\ W &\rightarrow aTb / bTa \\ T &\rightarrow XTX / X / \varepsilon \\ X &\rightarrow b / a \end{aligned}$$

**1.44.**

$$\begin{aligned} S &\rightarrow D)A / A(D \\ D &\rightarrow (D) / DD / \varepsilon \\ A &\rightarrow (A / )A / \varepsilon \end{aligned}$$

**1.45.**

$S \rightarrow XcC \mid AaY$   
 $X \rightarrow aaXb \mid aaAb$   
 $Y \rightarrow bYcc \mid bCcc$   
 $C \rightarrow cC \mid \varepsilon$   
 $A \rightarrow aA \mid \varepsilon$

**1.46.**

$S \rightarrow XC \mid AY$   
 $X \rightarrow aaXb \mid aaBb \mid aBb$   
 $Y \rightarrow bYcc \mid bBcc \mid bBc$   
 $B \rightarrow cB \mid \varepsilon$   
 $A \rightarrow aA \mid a$   
 $C \rightarrow cC \mid c$

**1.47.**

$S \rightarrow A \mid B \mid C$   
 $A \rightarrow aAb \mid bb \mid Ab$   
 $B \rightarrow aBb \mid \varepsilon$   
 $C \rightarrow aCb \mid aa \mid aC$

**1.48.**

$S \rightarrow A \mid B \mid C$   
 $A \rightarrow aAb \mid abb \mid Ab$   
 $B \rightarrow aBb \mid aaBb \mid aaabb$   
 $C \rightarrow aaCb \mid aaab \mid aC$

**1.49.**

$S \rightarrow AB \mid CD$   
 $A \rightarrow aA \mid \varepsilon$   
 $B \rightarrow bBc \mid E \mid cD$   
 $C \rightarrow aCb \mid E \mid aA$   
 $D \rightarrow cD \mid \varepsilon$   
 $E \rightarrow bE \mid b$

**1.50.**

$S \rightarrow A \mid B$   
 $A \rightarrow aAb \mid aa \mid aA$   
 $B \rightarrow aBb \mid b \mid Bb$

**1.51.**

$S \rightarrow AIB$   
 $A \rightarrow 0A \mid \varepsilon$   
 $B \rightarrow 0B \mid 1B \mid \varepsilon$

**1.52.**

$S \rightarrow aSd / aXc / bYd / bZc \mid \varepsilon$

$X \rightarrow aXc / bZc \mid \varepsilon$

$Y \rightarrow bYd / bZc \mid \varepsilon$

$Z \rightarrow bZc \mid \varepsilon$

**1.53.**

$S \rightarrow AB \mid BA \mid A \mid B \mid \varepsilon$

$A \rightarrow CAC / a$

$B \rightarrow CBC / b$

$C \rightarrow a / b$

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