Introduction

Task:

Design medium-temperature resistance heating elements in the form of a spiral (Fig.1) for a 3-phase resistance furnace of a given power $P$ (value of power given individually to each student). The furnace is supplied from a 3-phase 230/400 V line. The heating elements should be made of a Kanthal A-1 round wire. Assume the temperature of the heating element equal to 1200 $^\circ$C.

It should be assumed that three equal spiral elements will be used, each supplied with 400 V. Assuming additionally the supply line voltages symmetry and load symmetry, the task is limited to making calculations only for a single heating element generating power $P/3$.

Parameters of Kanthal A-1 round wires are given in Fig.2 and Fig.3.

Fig.1. Spiral heating element

I. Determination of the heating element parameters

1. Resistance $R$ of a (single) heating element

On the one hand

\[ R = \frac{U^2}{P_1} . \]
where

\[ P_1 = \frac{P}{3} \]

is the power of one heating element. On the other hand

\[ R = \frac{\rho l}{S} = \frac{\rho l}{\pi d^2} = \frac{4\rho l}{\pi d^2} \tag{2} \]

where

- \( \rho \) – resistivity of the Kanthal A-1 wire at its maximum temperature,
- \( l \) – length of the wire,
- \( d \) – diameter of the wire,
- \( S \) – cross-section of the wire.

Comparing (1) and (2) length \( l \) and diameter \( d \) of the wire can be computed. But note that there are many solutions and a question arises: which pair of \( d \) and \( l \) computed is the right one?

Advice: Heating element surface load \( p_o \) has to be taken into account.

2. **Heating element surface load**

Surface load \( p_o \), of a heating element, W/m\(^2\), is determined by

\[ p_o = \frac{P_1}{A_o} = \frac{P_1}{\pi d l} \tag{3} \]

where

\[ A_o = \pi d l \tag{4} \]

is the side surface of the wire.

The correct value of the heating element surface load depends on the type of the wire used, on the structure of the heating element and its ceramic supports, on the structure of the furnace and its working conditions (temperature, atmosphere, the rate of the furnace on-offs, etc.).

Admissible ranges of the surface load can be found in proper data sheets and the choice of its value is done experimentally basing on working conditions of the heating element. At a higher load surface:

- less material is needed to build the heating element,
- durability of the heating element is reduced.

**Note:** The surface load \( p_o \) of the heating element and its resistivity should be read from Figs. 3 and 2 respectively, taking into account its working temperature.

3. **Determination of the wire diameter**

Comparing (1) with (2) and taking into account \( l \) determined from (3) gives

\[ d = \sqrt[3]{\frac{4\rho P_1^2}{\pi^2 p_o U^2}} \tag{5} \]

From the data sheet in Fig.2 we choose a diameter \( d_z \) possibly close to the value \( d \) calculated from (5).

4. **Determination of the wire length**

After comparing (1) with (2) and replacing \( d \) with \( d_z \) the wire length \( l \) can be calculated
\[ l = \frac{\pi d_z^2 U^2}{4 \rho P_i} \]  
(6)

The obtained value of \( l \) should be rounded to whole centimeters and marked as \( l_z \).

**Note:** The computed value of length \( l_z \) should not be increased to take the heating element terminals into account etc.

5. **Determination of the spiral heating element diameter \( D \)**

The relation between the spiral heating element external diameter \( D \) and the wire diameter \( d_z \) should be chosen in the range (round \( D \) to whole millimeters):

\[
D = (5 \div 7) \ d_z \quad \text{for} \quad d_z > 1 \ \text{mm}
\]

\[
D = (4 \div 10) \ d_z \quad \text{for} \quad d_z < 1 \ \text{mm}.
\]

(7a)

(7b)

A too small heating element diameter \( D \) makes it difficult to wind the spiral (a too small radius of curvature); a too big heating element diameter \( D \) decreases the heating element endurance.

6. **Determination of the spiral heating element pitch \( s \)**

The spiral pitch \( s \) is chosen in the range (round \( s \) to whole millimeters):

\[
s = (2 \div 3) \ d_z
\]

(8)

7. **Determination of the number of the turns of the spiral heating element**

The number \( n \) of the turns of the spiral heating element is determined from

\[
n = \frac{l_z}{\pi(D - d_z)}
\]

(9)

where \( \pi(D - d_z) \) is the average length of one turn of the spiral heating element.

8. **Determination of the spiral heating element length**

The spiral heating element length \( l_s \) is determined as

\[
l_s = ns
\]

(10)

**Note:** In case of using Excel for calculations, its function \( PI() \) can be used.

II. **Verification calculations**

Basing on the input data and the calculated wire diameter \( d_z \) and wire length \( l_z \) determine the following parameters of the heating element: (reminder - our calculations concern only one phase):

a) heating element resistance,

b) current through the heating element,

c) current density,

d) power dissipated in the heating element,

e) surface load of the heating element.
Note: In the verification calculations above use \( d_z \) and \( l_z \) (obtained after rounding) – not \( d \) and \( l \), calculated directly from (5) and (6).

III. Report contents

1. Place a summary of the following quantities obtained during execution of p. I quoting also the formula used:
   a) resistance furnace assumed power \( P \), kW,
   b) power \( P_1 \) of a single phase, kW,
   c) calculated heating element resistance, equal to the wire resistance, \( \Omega \),
   d) surface load of the heating element assumed,
   e) wire diameter \( d \) calculated from (5), mm,
   f) wire diameter \( d_z \) chosen from the catalogue (Fig.2), mm,
   g) wire length \( l \) calculated from (6), m,
   h) wire length \( l_z \) rounded to whole centimeters, m,
   i) spiral heating element external diameter \( D \), mm,
   j) spiral heating element pitch \( s \), mm,
   k) number of the turns of the spiral heating element,
   l) spiral heating element length \( l_s \), m.

2. Place the results of the verification calculations from p. II. Make sure that the surface load calculated here is within the permissible area (Fig.3).

3. Place a summary of the following quantities assumed or calculated in p.1 of the report and verified in p.2 of the report adding also relative errors:
   a) heating element resistance,
   b) heating element power,
   c) heating element surface load.

Make sure that the relative errors are small.

IV. Bibliography

[2] KANTHAL datasheet:

m/Resistance%20heating%20wire%20and%20strip/S-KA026-B-ENG-2012-01.pdf
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Fig. 3. Surface load of round wires used in industry furnaces. Consider Kanthal A-1.