1 Verification of isolating block conditions

We know that

$$f(q) = f(0) + C(q)q$$

where $C(q)$ is a matrix

$$C(q) \in [df].$$

In our case

$$C(q) = \begin{pmatrix} C_{11} & 0 \\ 0 & C_{22} \end{pmatrix} + \begin{pmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{pmatrix},$$

where C_{ii} are diagonal matrixes

$$\begin{aligned} C_{11} &= \operatorname{diag}(\lambda_1, ..., \lambda_n), \\ C_{22} &= \operatorname{diag}(\beta_1, ..., \beta_k). \end{aligned}$$

Let

$$\|A\|_b = \max_{v \neq 0} \left\{ \frac{1}{\|v\|^2} v^T A v \right\}.$$

If $\|q_1\| = r_1$ and $\|q_2\| \leq r_2$, then

$$\begin{aligned} (\pi_1 f(q) | \pi_1 q) &= (\pi_1 f(0) + C_{11} q_1 + R_{11} q_1 + R_{12} q_2 | q_1) \\ &= (\pi_1 f(0) | q_1) + (C_{11} q_1 | q_1) + (R_{11} q_1 | q_1) + (R_{12} q_2 | q_1) \\ &\geq \min \lambda_i \|q_1\| - \|R_{11}\|_b \|q_1\|^2 - \|\pi_1 f(0)\| \|q_1\| - \|R_{12}\| \|q_2\| \|q_1\| \\ &\geq r_1 (\min \lambda_i - \|R_{11}\|_b r_1 - \|\pi_1 f(0)\| - \|R_{12}\| r_2). \end{aligned}$$

Similarly, if $\|q_2\| = r_2$ and $\|q_1\| \leq r_1$ then

$$\begin{aligned} (\pi_2 f(q) | \pi_2 q) &= (\pi_2 f(0) + R_{21} q_1 + R_{22} q_2 + C_{22} q_2 | q_2) \\ &= (\pi_2 f(0) | q_2) + (R_{21} q_1 | q_2) + (R_{22} q_2 | q_2) + (C_{22} q_2 | q_2) \\ &\leq \max \beta_i \|q_2\| + \|R_{21}\| \|q_1\| \|q_2\| + \|\pi_2 f(0)\| \|q_2\| + \|R_{22}\|_b \|q_2\|^2 \\ &\leq r_2 (\max \beta_i + \|R_{21}\| r_1 + \|\pi_2 f(0)\| + \|R_{22}\|_b r_2). \end{aligned}$$

We also use the fact that for a 2x2 matrix we have

$$\left\| \begin{matrix} r_{11} & r_{12} + a \\ r_{21} - a & r_{22} \end{matrix} \right\|_b = \left\| \begin{matrix} r_{11} & r_{12} \\ r_{21} & r_{22} \end{matrix} \right\|_b,$$

and that $\|R\|_b \leq \|R\|$.