AGH

Akademia Górniczo-Hutnicza
im. Stanisława Staszica w Krakowie

Modern physics

1. Waves as particles and particles as waves

Outline

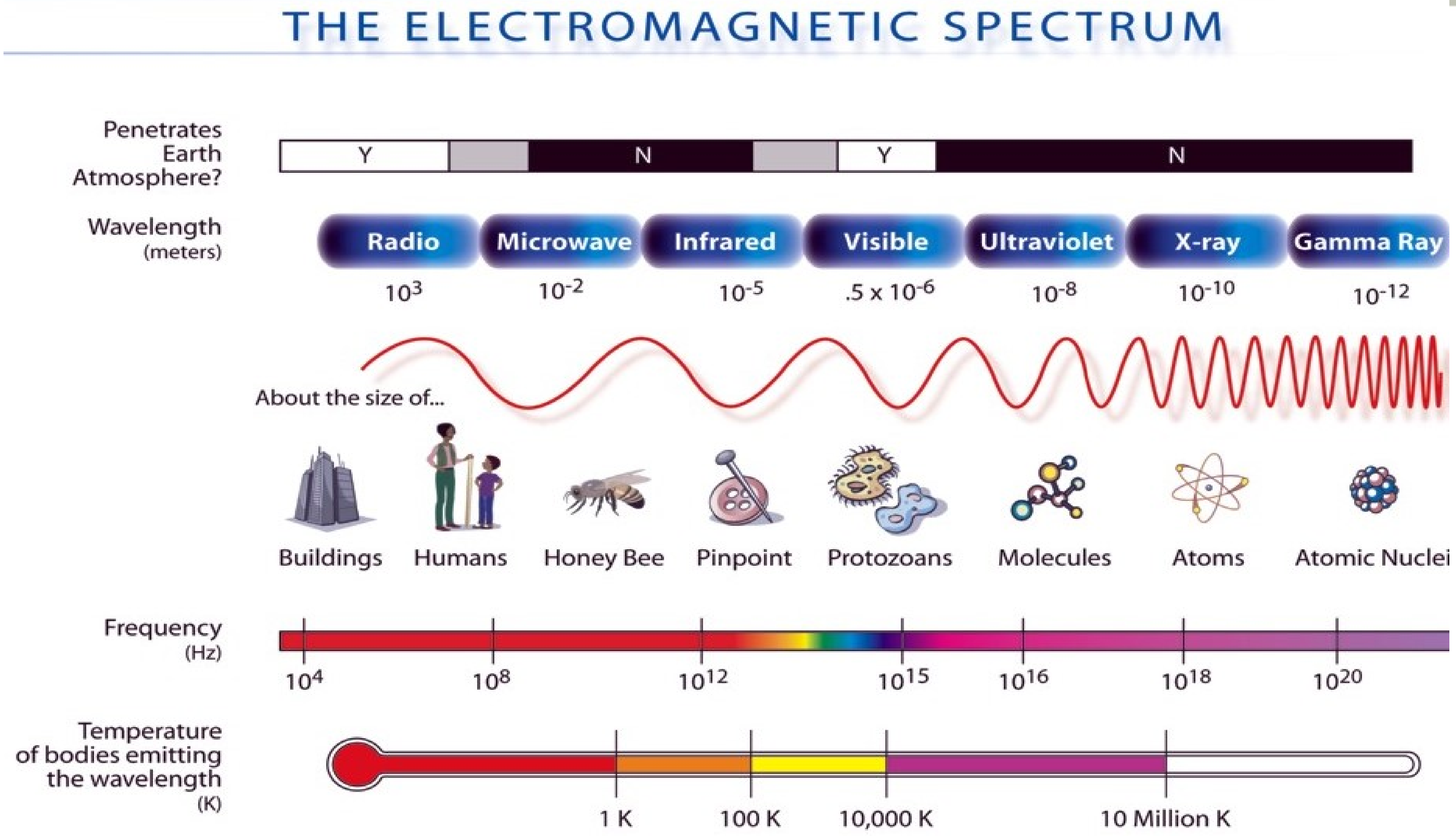
1.1. The nature of photons

1.2. The photoelectric effect

1.3. The Compton effect

1.4. Matter waves and their detection

1.1. The nature of photons



1.1. The nature of photons

Electromagnetic radiation is treated in terms of electromagnetic waves as predicted by Maxwell's equations. Interference, diffraction and polarization phenomena cannot be explained otherwise. But there are effects that require completely different approach to the nature of electromagnetic radiation.

This approach requires a quantum of radiation, a photon.

1.1. The nature of photons

Photon is a massless particle that travels with the speed of light $c \approx 3 \cdot 10^8$ m/s.

Its energy E and the momentum $\vec{p}$ are related by

$$\boxed{E = |\vec{p}|c}$$

The work of Planck and Einstein established a fact that the energy of a photon is linearly dependent on the frequency f of the light with which it is associated

$$\boxed{E = hf}$$

Constant introduced by Max Planck

$h = 6.63 \cdot 10^{-34}$ $J \cdot s$

1.1. The nature of photons

Using the wave relation:

$$\lambda f = c$$

where λ is the wavelength of light associated with the photon

we find that the momentum p of a single photon
is inversely proportional to the wavelength

$$\boxed{p = \frac{E}{c} = \frac{hf}{c} = \frac{h}{\lambda}}$$

1.1. The nature of photons

We note that the energy of a single photon given by *E=hf* can be alternatively expressed with the angular frequency ω:

$$\omega = 2\pi f$$

as:

$$\boxed{E = \hbar\omega}$$

where

$$\hbar = h/2\pi \approx 1.05 \cdot 10^{-34}\ \mathrm{J \cdot s}$$

Planck's constant

1.1. The nature of photons

This picture suggests that the intensity of radiation of a given frequency, i.e. the rate at which the radiation delivers energy per unit area, is a question only of the number of photons. The more intense the radiation, the larger is the number of photons.

Example: Suppose that a 60 W lightbulb radiates primary at a wavelength λ≈1000 nm, a number just above the optical range. Find the number of photons emitted per second.

Solution: If we divide the total energy per second by the energy per photon, we will have the number of photons per second. We know the total energy per second is 60 W. The frequency of the light is:

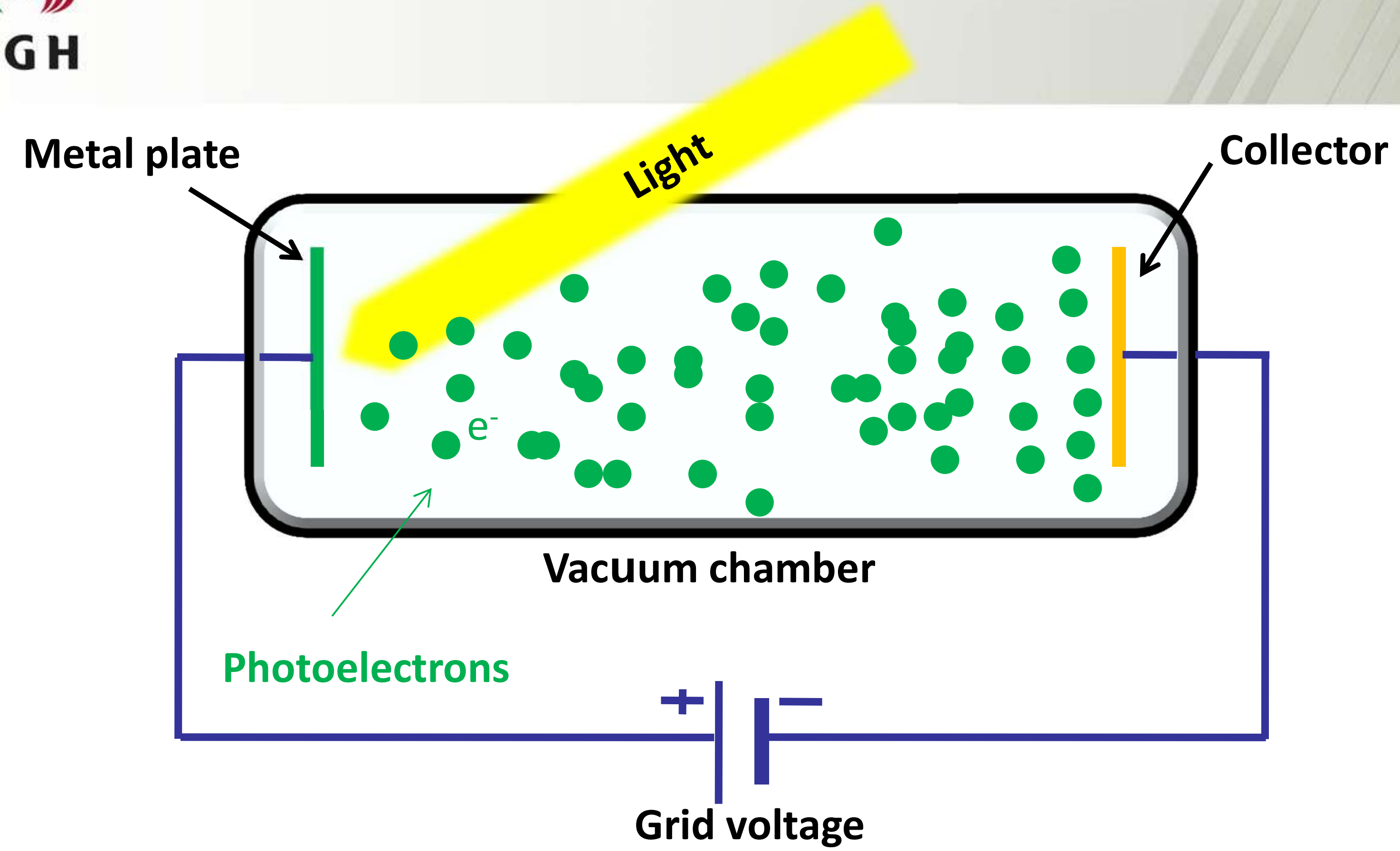
$$f = c/\lambda \approx 3 \cdot 10^{14}\, Hz$$

and the energy per photon is *E=hf*

Then the number of photons emitted per second is:

$$n = \frac{60W}{hf} = \frac{60W}{(6.63 \cdot 10^{-34}\, J \cdot s)(3 \cdot 10^{14}\, s^{-1})} = 3 \cdot 10^{20}\, photons / s$$

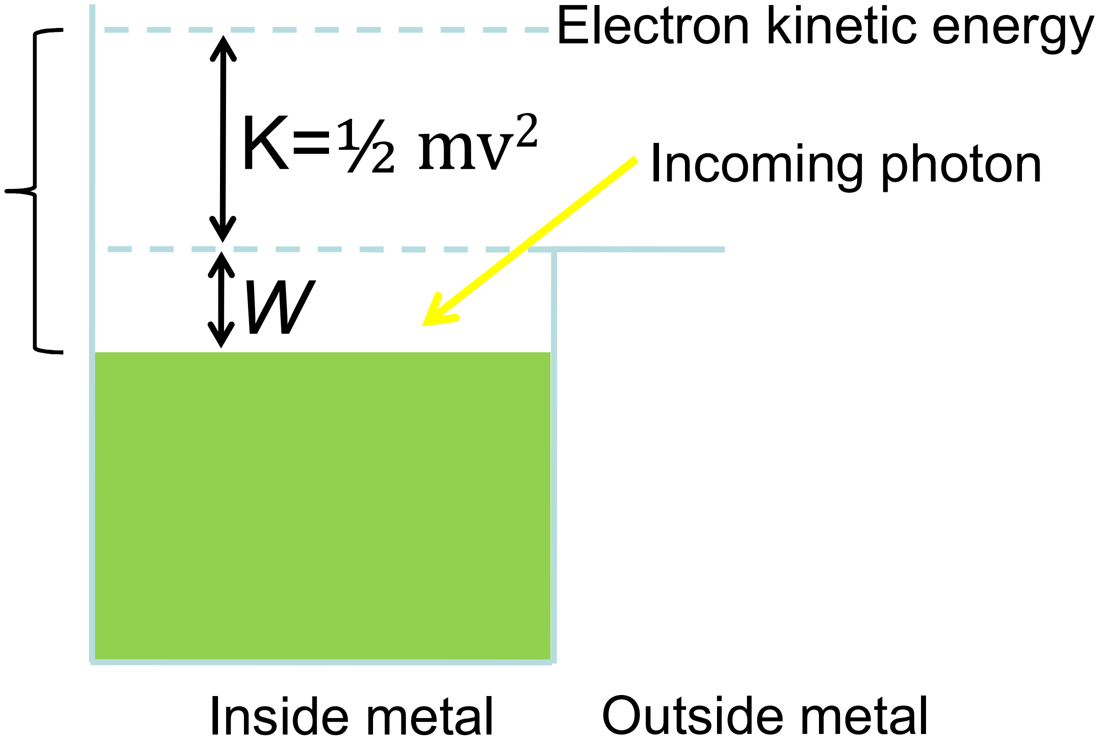
1.2. The photoelectric effect



Light strikes a metal plate in the vacuum chamber. The electron current is measured by a collector, and the kinetic energy is determined by the grid voltage needed to stop electrons from reaching the collector.

1.2. The photoelectric effect

Minimum
hf needed
for electron with
given
$K=\frac{1}{2}mv^2$



1.2. The photoelectric effect

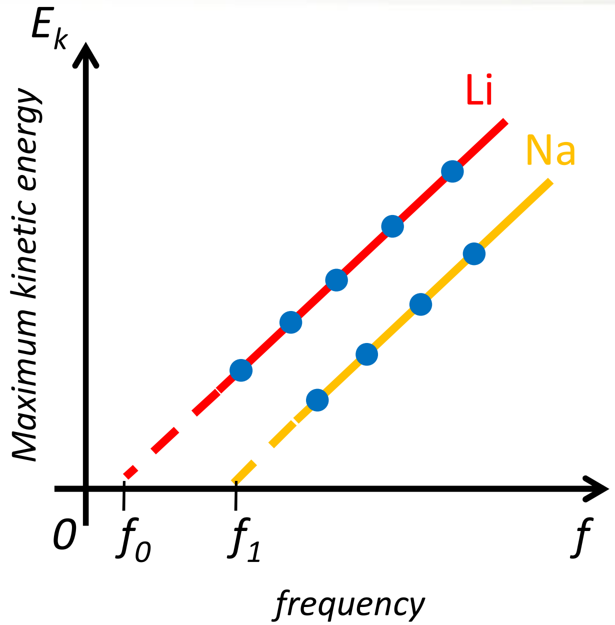
Metals contain a large number of free electrons (m_e - electron mass, $-e$ is the electron charge) about one or two per atom. These electrons are quasi-free which means they are not bound to atoms but are not free to leave the metal. It takes a certain amount of energy to get an electron out of a metal, this is exactly the work function W of the metal. The work function varies from metal to metal and depends on the conditions of the surface. Typical values of W range from 2 to 8 eV.

The mechanism of the photoelectric effect, proposed by Einstein assumes that photon is absorbed by an electron if the photon energy exceeds a certain value determined by the following condition:

$$\boxed{hf > W}$$

Energy is transferred to the electron which allows it to escape the metal. Electrons emitted by a metal subject to radiation are called photoelectrons.

1.2. The photoelectric effect



For some metals, a weak beam of blue light produces a photocurrent, while a very intense red light produces none. If *hf* is larger than *W*, then the electrons will emerge with a speed *v* such that

$$\boxed{\frac{1}{2}m_e v^2 = hf - W}$$

conservation of energy

1. The energy of photoelectrons from a particular metal depends only on the frequency of the radiation, and once the threshold frequency is exceeded, the dependence of the electron's kinetic energy on the frequency is linear. The kinetic energy of the photoelectron is ***independent*** of the intensity of the radiation, i.e. on the number of photons. Single photon is absorbed by a single electron.

1.2. The photoelectric effect

Contrast this picture with the classical one (**wrong** here), in which the energy carried by light depends on the square of the amplitude of the fields. No matter how small the frequency of the light, no matter how small the intensity, if one waits long enough, electrons will accumulate enough electromagnetic radiation to overcome the work function and escape from the metal.

2. The *number* of photoelectrons emitted is proportional to the intensity of radiation, i.e. to the number of photons that shine on the metal. This is not at all characteristic of the classical picture

3. There is no time interval between the impact of the photon beam on the metal and the beginning of the emission of photoelectrons. In the classical picture, the radiant energy arrives continuously and accumulates until there is enough energy to liberate an electron.

Remember that truly free isolated electron cannot absorb photon and remain an electron, since this would violate the conservation of energy or of momentum. But this is not a problem here, because the struck electron can transfer the momentum to the metal as a whole.

1.2. The photoelectric effect

Example: An experiment shows that when electromagnetic radiation of wavelength 270 nm falls on an aluminum surface, photoelectrons are emitted. The most energetic of these are stopped by a potential difference of 0.406 volts. Use this information to calculate the work function of aluminum.

Solution: The kinetic energy of the most energetic photoelectrons is given by the electron charge times the potential that stops the photoelectrons:

$$K = eV = (1.6\cdot 10^{-19}\ \mathrm{C})(0.405\ \ \mathrm{V}) = 0.65\cdot 10^{-19}\ \mathrm{J}$$

The photon energy is

$$E = hf = \frac{hc}{\lambda} = \frac{(6.63\cdot 10^{-34}\ \mathrm{J\cdot s})(3.00\cdot 10^{8}\ \mathrm{m/s})}{270\cdot 10^{-9}\ \mathrm{m}} = 7.37\cdot 10^{-19}\ \mathrm{J}$$

The difference is the work function:

$$W = E - K = 6.72\cdot 10^{-19} J = \frac{6.72\cdot 10^{-19} J}{1.6\cdot 10^{-19} J/eV} = 4.2\ eV$$

1.2. The photoelectric effect

The photoelectric effect has many important applications:

- camera exposure meter
- light-activated keys for automobiles
- distant television controls
- garage door openers
- photomultipliers

1.3. The Compton effect

If light consists of photons, collisions between photons and particles of matter (e.g. electrons) should be possible.

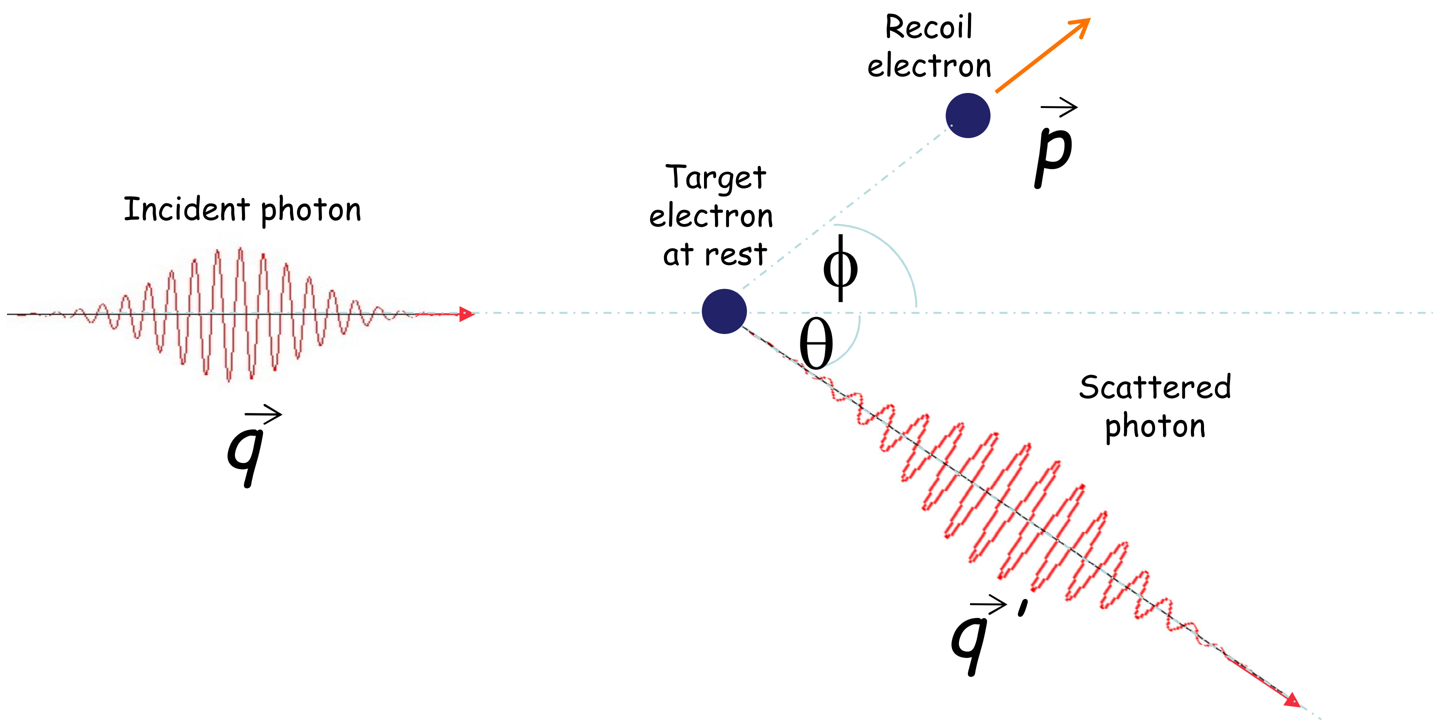
Compton effect is scattering of a photon γ on the quasi-free electron *e* in metal foil:

$$\gamma + e \rightarrow \gamma' + e'$$

Assume that:

- the initial electron is at rest, with zero momentum and relativistic energy $m_e c^2$
- initially, photon has energy *hf* and momentum $\vec{\mathbf{q}}$ whose magnitude *is hf/c*

1.3. The Compton effect



1.3. The Compton effect

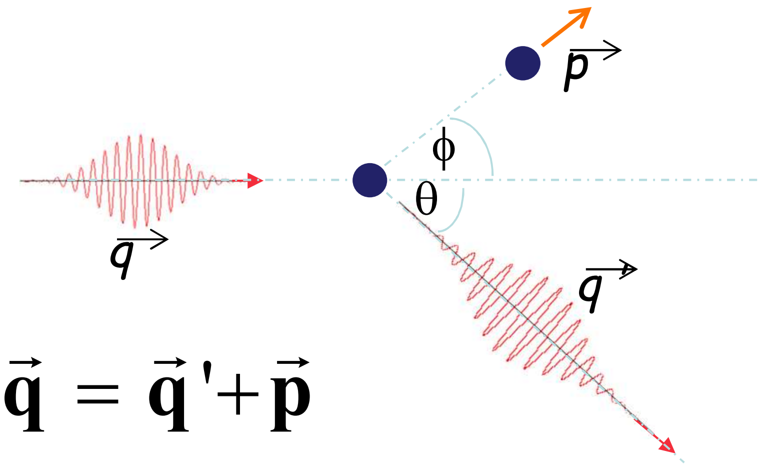
After the collision:

•the photon has energy *hf′* and momentum $\vec{\mathbf{q}}'$ whose magnitude *is hf′/c*

•the final electron momentum is $\vec{\mathbf{p}}$

•the final energy of electron (relativistic) is expressed as:

$$\sqrt{p^2c^2 + m_e^2c^4}$$



conservation of momentum

$$\vec{\mathbf{q}} = \vec{\mathbf{q}}' + \vec{\mathbf{p}}$$

conservation of energy

$$hf + m_ec^2 = hf' + \sqrt{p^2c^2 + m_e^2c^4}$$

1.3. The Compton effect

The energy shift $\Delta\lambda=\lambda'-\lambda$ between the wavelength of photon after (λ') and before (λ) scattering is given by:

$$\Delta\lambda = \lambda' - \lambda = \frac{h}{m_e c}(1 - \cos\theta)$$

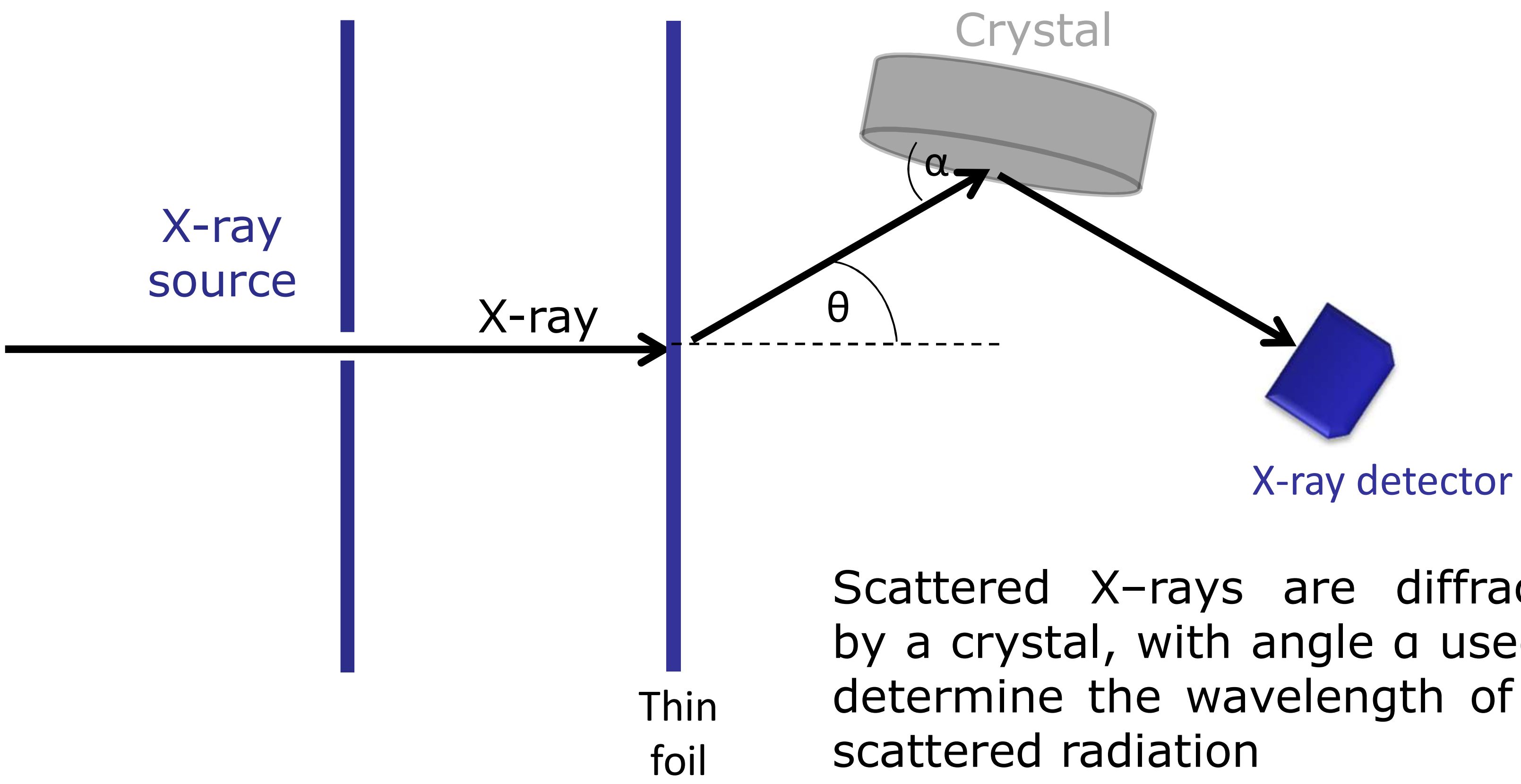
Compton wavelength

$2.4 \cdot 10^{-12}$m

scattering angle

Compton sent X-rays (high frequency photons) through thin metallic foils and looked for radiation scattered at different angles.

1.3. The Compton effect

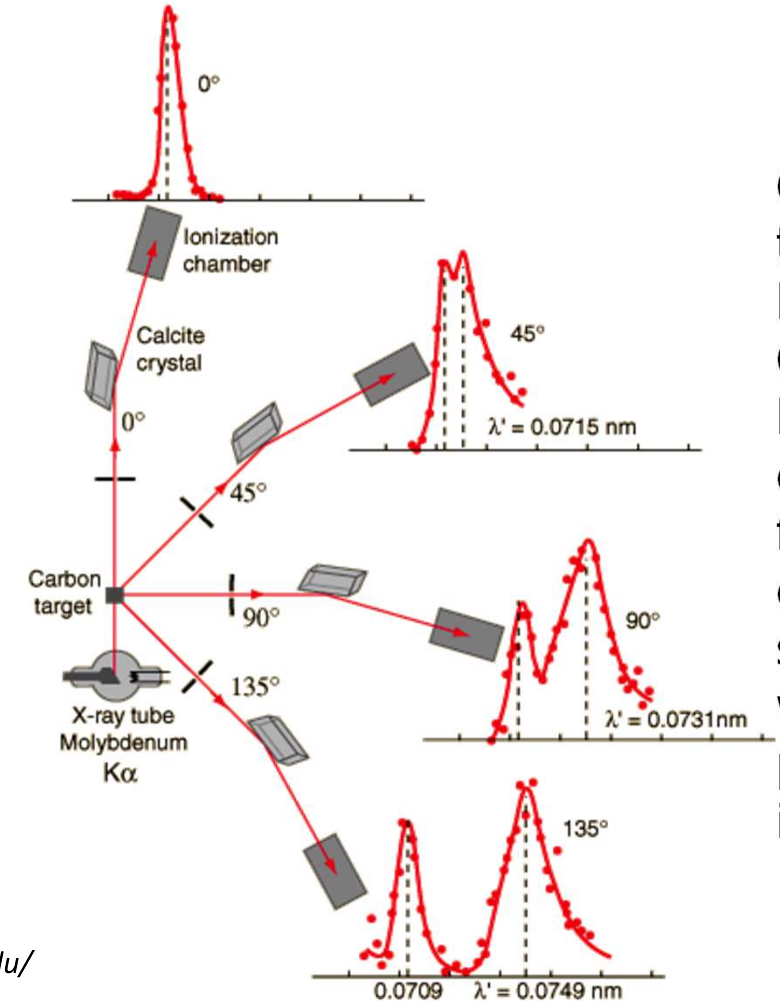


Scattered X–rays are diffracted by a crystal, with angle α used to determine the wavelength of the scattered radiation

Experimental set-up for Compton effect

1.3. The Compton effect

The data for Compton experiment. The peak, due to X –ray scattering by free electrons, becomes more pronounced as the scattering angle increases.



hyperphysics.phy-astr.gsu.edu/

Compton found that the scattered photons had two wavelengths. One set of photons has a wavelength shift exactly as predicted for scattering from electrons. A second set had an unshifted wavelength due to positively charged ions (larger mass).

1.3. The Compton effect

Example: In a Compton scattering experiment, an incoming X-ray of wavelength $\lambda = 5.53 \cdot 10^{-2}$ nm is scattered and detected at an angle of 35^o. Find the fractional shift in the wavelength of the scattered X-ray.

Solution: If λ is the incoming wavelength and λ' is the wavelength of the scattered X ray, then the fractional change in wavelength is given by:

$$\frac{\lambda' - \lambda}{\lambda} = \frac{h}{m_e c \lambda}(1 - \cos\theta)$$

$$= \frac{(6.63 \cdot 10^{-34}\, J \cdot s)\,(1 - \cos(35^o\,))}{(0.91 \cdot 10^{-30}\, kg)\,(3.00 \cdot 10^{8}\, m/s)\,(5.53 \cdot 10^{-11}\, m)} = 7.9 \cdot 10^{-3}$$

or about a 1% shift.

1.3. The Compton effect

The Compton effect can be a nuisance. It is only because X-ray films are thin that the recoil electrons from the Compton effect do not ruin the resolution of image.

On the positive side, the Compton effect does play an important role in cancer therapy. X-ray photons penetrate to a tumor, where they produce showers of electrons through Compton scattering. In this way, and through further scattering of these electrons, energy can be deposited in the core of tumor.

1.4. Matter waves and their detection

Louis de Broglie

In 1923, in a 16-page doctoral thesis, the French nobleman and physicist Louis de Broglie proposed that matter has wavelike properties. De Broglie suggested that the relation between the momentum and wavelength, true for photons is a perfectly general one and applies to radiation and matter alike.

$$\boxed{\lambda = \frac{h}{|\vec{p}|}}$$

de Broglie relation

λ is de Broglie wavelength of matter

1.4. Matter waves and their detection

Observe, that the momentum p:

$$p = mv$$ if the particle is nonrelativistic

and

$$p = \frac{m_o v}{(1 - v^2/c^2)^{1/2}}$$

Louis de Broglie

if the particle is relativistic

For photons, the relation $\lambda = \frac{h}{|\vec{p}|}$ is not new

De Broglie thesis attracted much attention, and suggestions were made to for verifying the existence of de Broglie waves through the observation of electron diffraction.

1.4. Matter waves and their detection

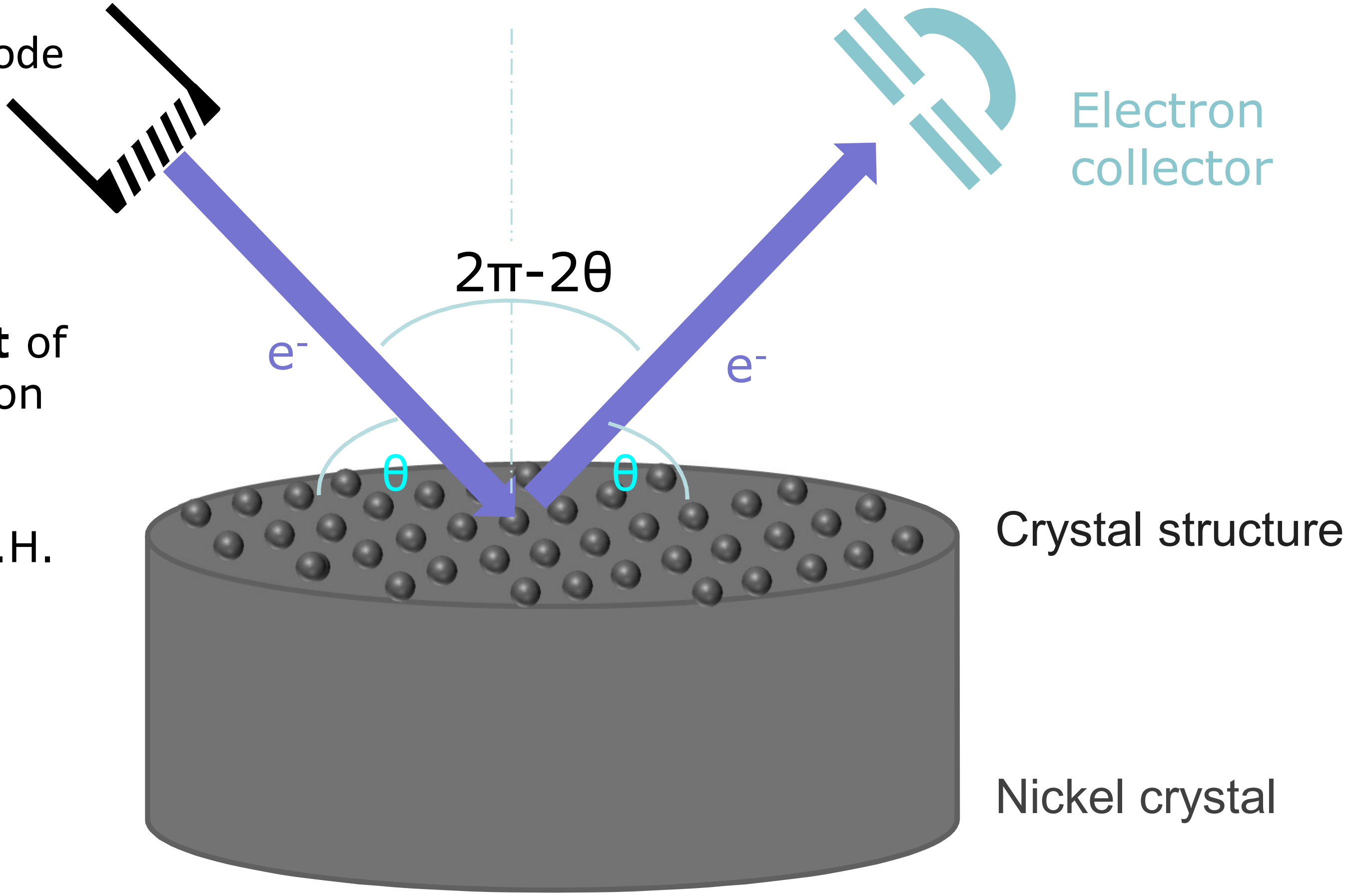
Interference maxima and minima appear when waves pass through gratings. The effects of diffraction are most evident when the wavelength of the wave is comparable to the spacing on the grating. Electron wavelength is hundreds of times shorter that the wavelengths of visible light.

Thus, a very different grating has to be used, it is the regular array of atoms making up a crystal.

The experiments consist in looking for preferential scattering in certain directions – diffraction maxima – when electrons are incident on the surface of a crystal.

The conditions for these interference effects are those of classical optics.

1.4. Matter waves and their detection



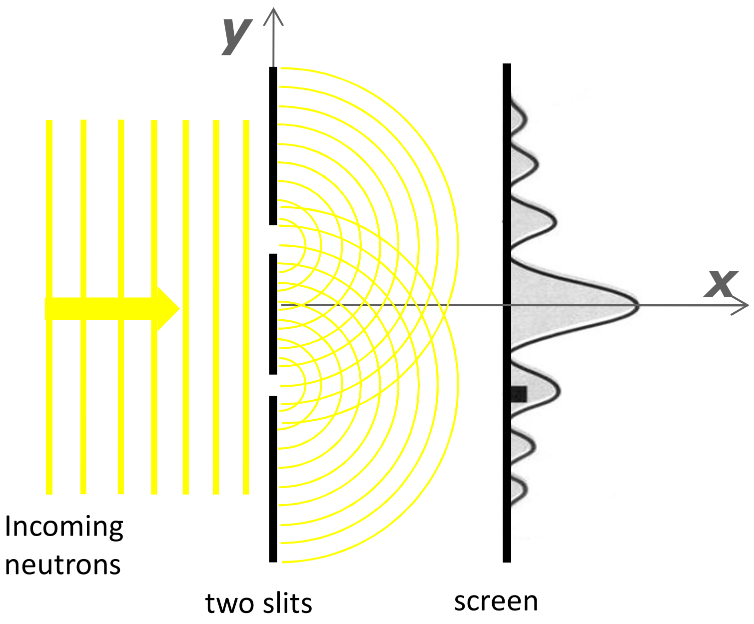
The **experiment** of electron diffraction was performed in 1927 by C.J. **Davisson** and L.H. **Germer** in US.

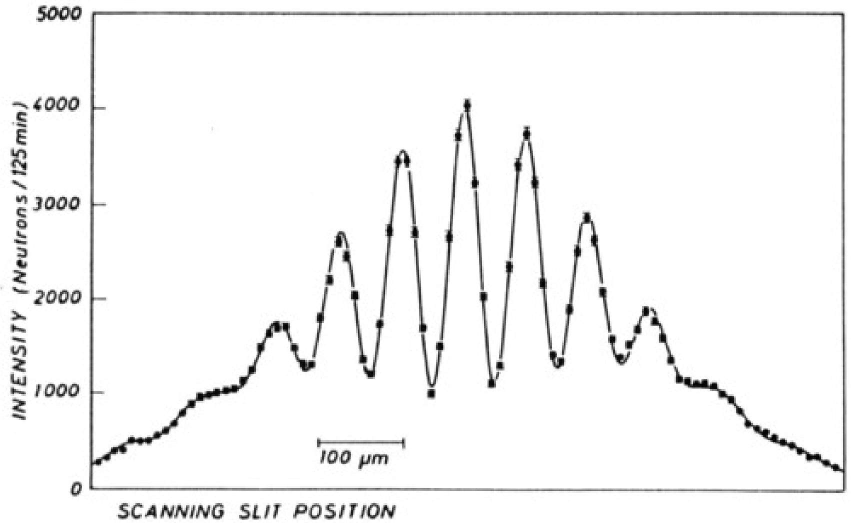
Electrons from the cathode strike a surface of a nickel crystal and are scattered to an electron collector. The balls represent the regular array of the nickel atoms that make up the crystal

1.4. Matter waves and their detection

Matter-diffraction experiments have been carried out with more massive particles, such as neutrons and helium atoms. In each case, the results agree with de Broglie's idea.

Neutrons produce a diffraction pattern when they pass through double slit





A. Zeilinger, R. Gähler, C.G. Shull, W. Treimer, and W. Mampe, Single -and double-slit interference pattern made with neutrons, *Reviews of Modern Physics, Vol.* ***60****, 1988.*

1.4. Matter waves and their detection

Just as the small size of *h* hides the fact that photons exist, so it hides the wave properties of matter from our everyday experience.

A dust particle of mass 10^{-4}g traveling at 1 m/s has a momentum of 10^{-6} kg ·m/s and a wavelength of

$$\lambda = \frac{h}{p} = \frac{6.6 \cdot 10^{-34} J \cdot s}{10^{-6} kg \cdot m / s} = 6.6 \cdot 10^{-28} m$$

This number is so small – the diameter of an atom is of about 10^{-10} m - that it is impossible to detect even with the finest instruments, let alone with our human senses.

Conclusions

- Quanta of light carry both energy and momentum, and these are proportional to each other. If we think of these quanta as particles, then special relativity implies that they are massless, always moving with the speed of light
- The quantum nature of light has been tested in the photoelectric effect. The quantum hypothesis suggests that the kinetic energy of photoelectrons is proportional to the frequency of the light, but does not depend on its intensity
- Compton effect was of great historical importance because it confirmed that photons are real particles with momentum as well as energy. Collisions between the energetic quanta of radiation and electrons obey relativistic energy and momentum conservation laws

Conclusions

- As light shows particle characteristics, so matter shows wave characteristics. The wavelength of a particle in motion is equal to Planck's constant divided by the momentum of the particle. For objects like baseballs, this wavelength and any associated wave properties are so small as to be unobservable, but for electrons in atoms the wave effects are quite visible. Electrons impinging on suitable diffraction gratings show diffraction patterns characteristic of waves