

Zestaw zadań – CIĄGI LICZB RZECZYWISTYCH

1. Zbadaj zbieżność ciągów o wyrazie ogólnym:

$$a_n = \begin{cases} (-1)^n & n \leq 100 \\ \frac{n^2}{n^2+1} & n > 100 \end{cases}, \quad a_n = \left(1 + \frac{n}{n+1} \cos\left(\frac{n\pi}{2}\right)\right), \quad a_n = \sqrt{n(n - \sqrt{n^2 - 1})},$$

$$a_n = \sqrt[3]{n^3 + 2n^2} - n, \quad a_n = \frac{1 + 3 + \dots + (2n - 1)}{2 + 4 + \dots + 2n},$$

$$a_n = \frac{1 - 2 + 3 - 4 + \dots + (2n - 1) - 2n}{\sqrt{n^2 + 1}}, \quad a_n = \frac{(2n + 1)^4 - (n - 1)^4}{(2n + 1)^4 + (n + 1)^4},$$

$$a_n = \frac{2\sqrt{n} + \sqrt[3]{n} + 3\sqrt[5]{n+1}}{\sqrt{3n-2} + \sqrt[3]{n+1}}, \quad a_n = \frac{\sqrt{n^3 - 2n^2 + 1} + \sqrt[3]{n^4 + 1}}{\sqrt[4]{n^6 + 6n^5 + 2} + \sqrt{n^7 + 3n^2 + 1}},$$

$$a_N = \frac{1 + \frac{1}{2} + \dots + \frac{1}{2^N}}{1 + \frac{1}{3} + \dots + \frac{1}{3^N}}, \quad a_n = \frac{(n+2)! + (n+1)!}{(n+2)! - (n+1)!}, \quad a_n = n(\ln(n+1) - \ln n),$$

$$a_n = \frac{\log_2(n+1)}{\log_3(n+1)}, \quad a_n = \left(\frac{1+i}{1-i}\right)^{n!}, \quad a_n = \frac{\sqrt[3]{n^2} \sin(n!)}{n+1},$$

$$a_n = \frac{1}{2n} \cos(n^3) - \frac{3n}{6n+1}, \quad a_n = \frac{(-2)^n + 3^n}{(-2)^{n+1} + 3^{n+1}}, \quad a_n = 5^n - 4^n - 3^n - 2^n,$$

$$a_n = \left(\frac{n+7}{n-5}\right)^{2n+3}, \quad a_n = \left(\frac{n+3}{4n+1}\right)^n, \quad a_n = \left(\frac{n^2+2}{n^2+4}\right)^{3n^2},$$

$$a_n = \frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \dots + \frac{1}{n(n-1)}, \quad a_N = \sqrt[N]{5^N + 7^N + 9^N},$$

$$a_N = \sqrt[N]{100 + 2^N + \sin N}, \quad a_N = \sqrt[2N+1]{3N+2}, \quad a_N = \sqrt[N^2]{N+1},$$

$$a_n = \frac{1}{\sqrt{n^2+1}} + \frac{1}{\sqrt{n^2+2}} + \dots + \frac{1}{\sqrt{n^2+n}}, \quad a_N = \sqrt[N]{\frac{1}{2} + \frac{2}{3} + \frac{3}{4} + \dots + \frac{N}{N+1}},$$

$$a_N = \sqrt[N]{\frac{3^N + 2^N}{5^N + 4^N}}, \quad a_n = \frac{2n^2 + \sin(n!)}{4n^2 - 3 \cos(n^2)}, \quad a_n = \frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \dots + \frac{1}{\sqrt{n}}.$$

2. Oblicz granice ciągów:

$$0, 2; \quad 0, 23; \quad 0, 233; \quad 0, 2333; \dots$$

$$\sqrt{2}; \quad \sqrt{2\sqrt{2}}; \quad \sqrt{2\sqrt{2\sqrt{2}}}; \dots$$

3. Znajdź \liminf i \limsup ciągów o wyrazie ogólnym:

$$a_n = 1 - \frac{1}{n}, \quad a_n = [(-1)^n - 2]^{n+1},$$

$$a_N = \sqrt[N]{1 + 2^{N(-1)^N}}, \quad a_n = (-1)^n \left(1 + \frac{1}{n}\right)^n + \sin\left(\frac{n\pi}{4}\right).$$