ANISOTROPIC 2D MESHES IN FINITE ELEMENT MODELLING

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ABSTRACT

Summary The paper focuses on the problem of generation of anisotropic meshes for simulation of processes using the finite element method. The technique based on the Delaunay triangulation has been modified in order to enable stretching of elements in the selected direction. Application of these meshes allows obtaining a sufficient accuracy of calculations while maintaining the number of elements as low as possible.

A remarkable attention has been recently paid to the problem of an adaptation of meshes to the considered physical model based on the finite element method. The goal of research is a construction of mesh, which ensures a sufficient precision of calculations for the selected physical model, and which minimizes the computation cost (i.e. reduces the number of degrees of freedom of the discretization). The number of degrees of freedom is dependent on the dimension of the problem, the order of its approximation and the number of nodes and elements (i.e. the density of the mesh). A uniform, global increase of the number of degrees of freedom may not lead to an improvement of the obtained solution, e.g. because of the round-off errors resulting from the limited numerical precision of a computer.

The control of the error resulting from the discretization of the problem requires an adequate adaptation scheme. Thus, the objective of the present work was development of an adaptation technique, which enables generation of the anisotropic meshes.

The procedure of a mesh adaptation can be described as follows:

- generate an initial mesh, respecting the geometry of the domain only,
- calculate the first approximated solution,
- while the solution for the given discretization of the domain is unsatisfactory:
  1. perform the adaptation of the mesh,
  2. calculate the subsequent, approximate solution.

The adaptation of a mesh (step 1) can be performed by sequence of modifications of the mesh e.g. by local increase of number of degrees of freedom while maintaining the discretization in other regions. In the alternative approach, a completely new discretization is created on the basis of the information acquired from the previous iteration of an adaptation procedure. The second of the adaptation schemes is considered in the paper.

In order to describe criteria used for supervising the construction of an adapted mesh, a control space is utilized. The control space is a cover of the modelled domain, divided into sub-domains.
containing descriptions of criteria that must be satisfied in theirs interiors. Two variations of the representation of the control space were implemented: a uniform rectangular grid and a quadtree structure. The control space being considered allows additionally influencing locally the size and the aspect ratio of mesh elements in the given direction. It gives the opportunity of generation of anisotropic meshes. The problem of construction of elongated elements was reduced to the generation of an isotropic mesh by means of an adjustment of a metric in subdomains of the control space. This method is similar to those proposed in [1, 2, 3].

In our method, the Hessian $H$ is applied to determine the metric throughout the control space. With this approach, the mesh is refined and/or stretched considering the curvature of chosen representative variable of the problem. The Hessian is initially approximated in the nodes of the mesh by differential quotients. Then, a discrete operator $\tilde{H}$ and consequently a discrete matrix $\tilde{M}$ can be determined. In order to define the metric globally, the matrix $\tilde{M}$ should be properly interpolated throughout the entire domain.

In subsequent steps of the adaptation procedure, a generation of a new mesh is required. The mesh generator developed by the Authors [4, 5] is based on the well-known Delaunay triangulation. The density and the shape of elements within the domain can be influenced by an adequate distribution of boundary nodes, the selection of the procedure of the introduction of inner nodes and by altering the values of various parameters controlling the entire process of the triangulation. Created meshes can be anisotropic when proper metric is applied. Additionally, a possibility of conversion to quadrilateral mesh exists for both isotropic and anisotropic meshes.

The proposed method of the adaptation of finite element meshes is employed in the simulation of temperature distribution in the two-dimensional process of heat transfer.

REFERENCES


