Detail-preserving Regularization Based Removal of Impulse Noise from Highly Corrupted Images

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Abstract. This paper proposes a new filtering scheme for eliminating random-valued impulse noise from gray images. In the first phase a noise detector is utilized to extract the noise candidates. Next, the algorithm applies a connected component analysis in order to gather the neighboring noisy pixels into separate sets of connected noise candidates. The corrupted pixels are restored using a detail preserving regularization method. The main idea of the proposed approach is to gather the noisy candidate pixels into separate sets of connected pixels and solve the minimization functional over these pixels. Experimental results illustrate the efficiency and effectiveness of the algorithm.

1 Introduction

Impulse noise can corrupt images due to noisy sensors or channel transmission errors. Typical median filters, which are usually utilized invariantly across the whole images to remove noise, tend to modify both noise pixels and undisturbed pixels. To achieve a good compromise between the image-detail preservation and the noise reduction an impulse detector can be utilized prior to filtering [1]. The filtering is then selectively applied to regions where there is impulse noise. In such decision-based filters the possible noise pixels are first detected and then replaced through a median filter, while all other pixels are unchanged. The adaptive center-weighted median filter (ACWMF) [2] can effectively discover the noise even when its ratio is high. The main drawback is that each noisy pixel is replaced by a median value of neighboring pixels without considering the local structure of the image. The replacement of the noisy pixels by the median involves blurring of edges, which is evidently visible when the noise ratio is high. A recently proposed detail-preserving variational method [3][4] first detects noisy pixels and then uses a non-smooth data filtering term along with edge preserving regularization to restore the corrupted pixels. The minimization of a convex functional is conducted on the set consisting of all noisy pixels [4]. The nonlinear equation is solved by Newton's method with a suitable initial guess [5].

Our approach detects noisy pixels and additionally applies a connected component analysis in order to gather the neighboring noisy pixels into separate sets of connected noise candidates. To minimize the functional over each set of connected noise candidates we utilize the Levenberg-Marquardt (LM) algorithm. LM can be considered as a combination of steepest descent and the Gauss-Newton method. The steepest descent that is utilized first, guarantees the convergence of the algorithm and the faster Gauss-Newton is utilized finally to achieve the desired tolerance. The ACWMF filter is used to extract noise candidates and its output is utilized as an initial guess for the optimization algorithm. The novelty of our algorithm lies in the use of connected component analysis to gather the noisy candidate pixels into separate sets and to perform a local optimization over these sets. The optimization is then easier. This makes our algorithm several times faster than the algorithm proposed in [3].

The paper is organized as follows. In the next section we briefly review ACWMF filter. In Section 3 we present all ingredients of our method and discuss how our algorithm differs from relevant algorithms. In Section 4 we demonstrate the efficiency and effectiveness of the algorithm using various test images. Some conclusions are drawn in the last section.

2 The Adaptive Center-weighted Median Filter

Let $x_{i,j}$ be the gray level in a noisy M-by-N image at pixel location $(i,j) \in \mathcal{A} \equiv \{1,...,M\} \times \{1,...,N\}$. The general expression of the ACWMF filter is as follows:

$$y_{i,j}^{2k} = \text{median}\{x_{i-u,j-v}(2k) \diamond x_{i,j} \mid -h \leq u,v \leq h\} \quad (1)$$

where $(2h+1)^2$ is the window size, and $\diamond$ represents the repetition operation. For $k = 0,1,...,J-1$, where $J = 2h(h+1)$, we can determine the differences $d_k = \mid y_{i,j}^{2k} - x_{i,j} \mid$. They satisfy the condition $d_k \leq d_{k-1}$ for $k \geq 1$. To determine if the considered pixel (i,j) is noisy a set of thresholds T_k is utilized, where $T_{k-1} > T_k$ for $k = 0,1,...,J-1$. The output of the filter is defined in the following manner:

$$y_{ACWMF} = \begin{cases} y_{i,j}^0, & \text{if } \exists k, d_k > T_k \\ x_{i,j}, & \text{otherwise} \end{cases} \quad (2)$$

where $y_{i,j}^0$ is the output of the standard median filter. For a window of size 3×3 four thresholds T_k, $k = 0,...,3$ are needed. Using the median of the absolute deviations from the median $MAD = \text{median}\left\{\mid x_{i-u,j-v} - y_{i,j}^0 \mid : -h \leq u,v \leq h\right\}$ which is robust estimation of dispersion, we can define the thresholds T_k as $T_k = s * MAD + \delta_k$ where $0 \leq s \leq 0.6$, $\sigma_0 = 40$, $\sigma_1 = 25$, $\sigma_2 = 10$, and $\sigma_3 = 5$ [2].

3 Our Filter

Our method consists of two steps, which are applied alternatively. The ACWMF filter is utilized to extract noise candidates as well as to provide an initial guess for the optimization procedure in each iteration $l = 1, ..., L$. Denote by $\tilde{y}^{(l)}$ the image obtained by applying the ACWMF to the noisy image $y^{(l-1)}$. The noise candidate set is extracted through the ACWMF filter and it is extracted on the basis of the following formula:

$$\mathcal{N}_{(l)} = \left\{ (i,j) \in \mathcal{A} : \tilde{y}^{(l)}_{i,j} \neq y^{(l-1)}_{i,j}, \text{and } y^{()}_{i,j} \in \{0, 1, ..., 255\} \right\}. \tag{3}$$

The set of all uncorrupted pixels in iteration l is $\mathcal{N}^C_{(l)} \in \mathcal{A} \backslash \mathcal{N}_{(l)}$ and we keep their original values. Let $\mathcal{B}_{(l)}$ be a binary image indicating the candidates of noisy pixels. A labeling procedure applied to the image $\mathcal{B}_{(l)}$ produces the connected components $\mathcal{C}^{(k)}_{(l)}$, where $k = 1, ..., K$. Let $\mathcal{N}^{(k)}_{(l)}$ be a subset of the set $\mathcal{N}_{(l)}$ whose pixels belong to $\mathcal{C}^{(k)}_{(l)}$. Let us now consider a noise candidate at position $(i,j) \in \mathcal{N}^{(k)}_{(l)}$. Each of its 4-connected [6] neighbors $(m,n) \in \mathcal{V}_{i,j}$ is either an undistorted pixel, i.e. $(m,n) \in \mathcal{N}^C_{(l)}$ or is another noise candidate, i.e. $(m,n) \in \mathcal{N}^{(k)}_{(l)}$. The corrupted pixels are then restored by minimizing a convex objective function $F_{y|\mathcal{N}^{(k)}_{(l)}} : \mathcal{R}^{M \times N} \rightarrow \mathcal{R}$ of the following form:

$$\mathcal{F}_{\mathbf{y}|\mathcal{N}^{(k)}_{(l)}}(\mathbf{u}) = \sum_{(i,j) \in \mathcal{N}^{(k)}_{(l)}} \left\{ \mid u_{i,j} - y_{i,j} \mid + \frac{\beta}{2}(S_1 + S_2) \right\} \tag{4}$$

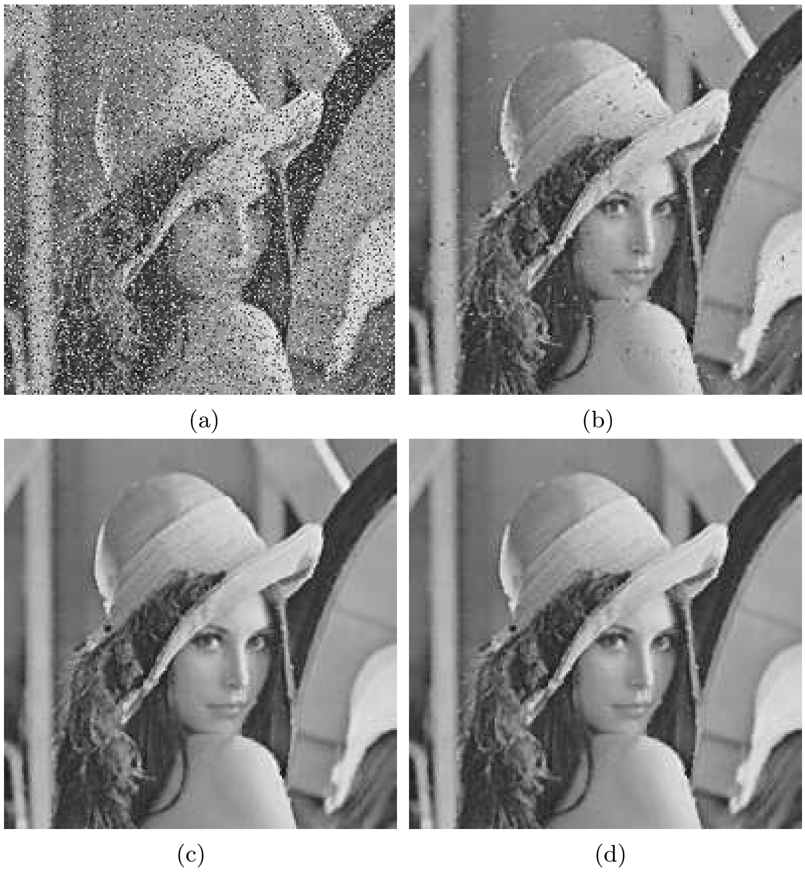
$$S_1 = \sum_{(m,n) \in \mathcal{V}_{i,j} \cap \mathcal{N}^C_{(l)}} 4\phi(u_{i,j} - y_{m,n})$$

$$S_2 = \sum_{(m,n) \in \mathcal{V}_{i,j} \cap \mathcal{N}^{(k)}_{(l)}} \phi(u_{i,j} - u_{m,n})$$

where β is a regularization factor, ϕ is an edge preserving potential function [7][8]. Examples of such functions are: $\phi(t) = \sqrt{\alpha + t^2}$ where $\alpha > 0$ and $\phi(t) = \mid t \mid^{\alpha}$, $1 < \alpha \leq 2$. In the output image $y^{(l)}$ the corrupted pixels are set to values generated by the optimization procedure, whereas all undistorted pixels are copied from the $y^{(l-1)}$. The data-fitting term $\mid u_{i,j} - y_{i,j} \mid$ prevents the wrongly detected undistorted pixels from being modified to other values, whereas the regularization term $(S_1 + S_2)$ accomplishes the edge-preserving smoothing of corrupted pixels [3][4]. The regularization factor balances the effects of the data-fitting term and the *a priori* term. In our approach the noise candidates are restored by minimizing the functionals $\mathcal{F}_{\mathbf{y}|\mathcal{N}^{(k)}_{(l)}}(\mathbf{u})$, $k = 1, .., K$, whereas [3][4] restore the noise candidates by minimizing a single functional that is restricted to the noise candidate set $\mathcal{N}_{(l)}$.

4 Tests

In this section we compare our method with ACWMF [2] and detail-preserving regularization [3] in terms of restoration errors and computation time. In all images, 30% or 50% pixels were corrupted with random-valued impulse noise, see Fig. 1. and Fig. 2. The peak signal to noise ratio (PSNR) and mean absolute error (MAE) [9] have been utilized to measure restoration errors. The potential function $\phi(t) = \mid t \mid^{1.3}$ has been applied in all experiments.



(a) (b) (c) (d)

Fig. 1. Image with 30% noise (a). Restored images by ACWMF with $s = 0.3$ (b), our method in 3 iterations (c), and in 4 iterations (d) with $\beta = 3.0$, $s = 0.6$ in 1-st it., $s = 0.5$ in 2-nd it., and $s = 0.2$ in the next iterations.

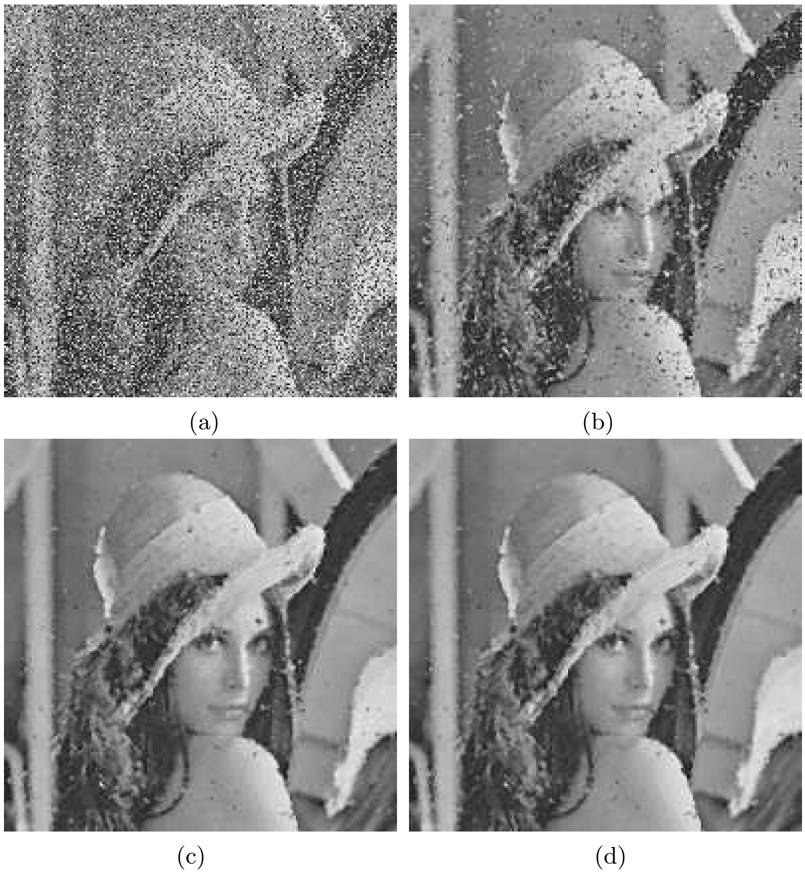


Fig. 2. Image with 50% noise (a). Restored images by ACWMF with $s = 0.1$ (b), our method in 3 iterations (c), and in 4 iterations (d) with $\beta = 3.0$, $s = 0.6$ in 1-st iteration and $s = 0.2$ in the next iterations.

The results from Tab. 1-2 indicate that errors obtained by our method in 3 iterations are quite comparable with errors that have been obtained by method [3] in four iterations. The method is several times faster than the mentioned above method, see Tab. 3 where computation time of the LM procedure is related to processing time of ACWMF. The algorithms were implemented in C and run on a PC workstation with a Pentium IV 2.4 GHz processor. The work [3] reports that for 30% noise the minimization procedure takes 30 times more CPU time than ACWMF. In our approach the optimization procedure takes about 3 times more CPU time than ACWMF. In comparison with ACWMF the proposed algorithm yields superior subjective quality with respect to impulse noise cancellation and image detail preservation. The SolvOpt optimization procedure [10] that allows for minimization nonlinear, possibly non-smooth nonlinear functions has also been tested in our algorithm. However, the computation time of this procedure is far longer than processing time of LM.

Table 1. Restoration errors at 30% noise

		bridge	*camera*	*goldhill*	*lena*
PSNR	Noisy image	14.37	13.69	14.37	14.60
	ACWMF	23.82	23.32	25.03	27.03
	Our method	25.27	24.75	27.42	30.16
MAE	Noisy image	22.24	23.42	21.87	21.42
	ACWMF	6.47	5.06	4.90	3.35
	Our method	5.92	3.97	4.11	2.31

Table 2. Restoration errors at 50% noise

		bridge	*camera*	*goldhill*	*lena*
PSNR	Noisy image	12.00	11.54	12.23	12.40
	ACWMF	19.19	18.11	20.02	20.98
	Our method	22.68	22.26	24.46	25.93
MAE	Noisy image	37.00	38.56	36.03	35.52
	ACWMF	14.11	13.88	11.90	9.80
	Our method	9.77	7.22	7.16	4.99

Table 3. Computation time [sec.]

	ACWMF	LM-1st. it.	LM-2nd. it.	LM-3rd. it.
30% noise	0.34	0.81	0.15	0.07
50% noise	0.35	1.04	0.46	0.21

In order to test how good the noise cancellation is we performed an optimization-based restoration of noisy images assuming that the noise detector is perfect. The LM procedure employing such perfect noise indicator and operating on connected noise candidates restores in one iteration the image *lena* corrupted by 30% and 50% impulse noise with PSNR=32.8 dB and PSNR=29.0 dB, respectively.

Next, we compared our method with recently proposed techniques. In [11], Luo reports restoration results in PSNR for images corrupted by 30% random-valued impulse noise. For example, for standard image *lena* of size 256×256 this work reports the following restoration results: ACWMF - 27.18 dB, iterative procedure [3] - 28.33 dB, algorithm-based on alpha-trimmed mean [11] - 28.48 dB. Taking into account results from Tab. 1 it is evident that our method provides significant improvement over all other approaches.

5 Conclusion

This paper considers the 2-phase methods in removal of impulse noise from highly corrupted images. We propose a new method for eliminating random-valued impulse noise from gray images. The main idea of our approach is to gather the noisy candidate pixels into individual sets of connected pixels and solve the minimization functional over these pixels. To minimize the functional over each set of connected noise candidates we utilize the Levenberg-Marquardt algorithm. The ACWMF filter is used to extract noise candidates and its output is utilized as an initial guess for the optimization algorithm. Our method can speed up the computations and the restored images are better. Experimental results indicate that the images are restored with satisfactory quality even at very high level of impulse noise. In our experiments with highly corrupted images the proposed algorithm performed better on all test-images than other relevant 2-phase algorithms.

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