

Recurrence relations - exercises

1. Find a recurrence relation and give initial conditions for the number of bit strings of length n that do not have two consecutive 0s. How many such bit strings are there of length five?
2. A vending machine dispensing books of stamps accepts only one-dollar coins, \$1 bills, and \$5 bills.
 - a) Find a recurrence relation for the number of ways to deposit n dollars in the vending machine, where the order in which the coins and bills are deposited matters.
 - b) What are the initial conditions?
 - c) How many ways are there to deposit \$10 for a book of stamps?
 - d) Find the solution of the recurrence relation.
3.
 - a) Find a recurrence relation for the number of ways to completely cover a $2 \times n$ checkerboard with 1×2 dominoes. [Hint: Consider separately the coverings where the position in the top right corner of the checkerboard is covered by a domino positioned horizontally and where it is covered by a domino positioned vertically.]
 - b) What are the initial conditions for the recurrence relation in part (a)?
 - c) Find the solution of the recurrence relation.
4.
 - a) Find a recurrence relation for the number of ways to lay out a walkway with slate tiles if the tiles are red, green, or gray, so that no two red tiles are adjacent and tiles of the same color are considered indistinguishable.
 - b) What are the initial conditions for the recurrence relation in part (a)?
 - c) How many ways are there to lay out a path of seven tiles as described in part (a)?
 - d) Find the solution of the recurrence relation.
5. Solve these recurrence relations together with the initial conditions given:
 - a) $a_n = a_{n-1} + 6a_{n-2}$ for $n \geq 2$, $a_0 = 3$, $a_1 = 6$.
 - b) $a_n = 7a_{n-1} - 10a_{n-2}$ for $n \geq 2$, $a_0 = 2$, $a_1 = 1$.
 - c) $a_n = 6a_{n-1} - 8a_{n-2}$ for $n \geq 2$, $a_0 = 4$, $a_1 = 10$.
 - d) $a_n = 2a_{n-1} - a_{n-2}$ for $n \geq 2$, $a_0 = 4$, $a_1 = 1$.
 - e) $a_n = a_{n-2}$ for $n \geq 2$, $a_0 = 5$, $a_1 = -1$.
 - f) $a_n = -6a_{n-1} - 9a_{n-2}$ for $n \geq 2$, $a_0 = 3$, $a_1 = -3$.
 - g) $a_{n+2} = -4a_{n+1} + 5a_n$ for $n \geq 0$, $a_0 = 2$, $a_1 = 8$.
 - h) $a_n = 2a_{n-1} + a_{n-2} - 2a_{n-3}$ for $n \geq 3$, $a_0 = 3$, $a_1 = 6$, and $a_2 = 0$.
 - i) $a_n = 7a_{n-2} + 6a_{n-3}$ for $n \geq 3$, $a_0 = 9$, $a_1 = 10$, and $a_2 = 32$.
 - j) $a_n = 5a_{n-2} - 4a_{n-4}$ for $n \geq 4$, $a_0 = 3$, $a_1 = 2$, $a_2 = 6$, and $a_3 = 8$.