8. Model calibration

Historical vs market calibration (1)

- Calibration to historical data – current and future scenarios are predicted based on statistical patterns observed in the past.

- Calibration to market data – we aim to match prices observed in the market at the moment. *What value of the calibrated parameter enables to get 'correct' prices of simple, liquid products available in the market?*

Historical vs market calibration (2)

- Pricing models are usually calibrated to the market, as the arbitrage-free pricing requires the use of the risk-neutral measure (which is determined at each point of time by the market, i.e. by the sum of individual decisions made by market participants).
- Some parameters cannot be implied from the market (or it is very challenging), e.g. correlations between risk factors, mean-reversion parameters. In such cases historical calibration is used.
- Model calibrated to historical data may slowly adapt to changing market conditions and may produce prices inconsistent with the market. On the other hand, market calibration may be unstable during the periods of high market volatility. It may also contain bias coming from risk premiums, storage costs, etc.

Calibration to the market data (1)

- $p = (p_1, \ldots, p_n) \in \mathbb{R}^n$ – model parameter to be calibrated.
- $\mathcal{B} = \{B_1, \ldots, B_d\}$ – calibration basket such that for each product B_i, $i \in \{1, \ldots, d\}$, the market price $\textsc{Market Price}_i$ is available, and B_i can be priced within the model.
- $\textsc{Model price}_i(p)$ denotes the theoretical price of B_i, $i \in \{1, \ldots, d\}$, calculated in the model, using a given value of parameter p.
- Let

$$\textsc{Market Price}_{\mathcal{B}} = \Big(\textsc{Market Price}_1, \ldots, \textsc{Market Price}_d\Big),$$
$$\textsc{Model price}_{\mathcal{B}}(p) = \Big(\textsc{Model price}_1(p), \ldots, \textsc{Model price}_d(p)\Big).$$

Calibration to the market data (2)

Approach 1: minimization of the error function. We solve the problem of finding

$$\arg\min_{p \in \mathbb{R}^d} F(p),$$

where the error function F typically has the form

$$F(p) = \Big\| \textsc{Market Price}_{\mathcal{B}} - \textsc{Model price}_{\mathcal{B}}(p) \Big\|$$

for some norm $\|\cdot\|$ in $\mathbb{R}^d$ (which can be composed with an increasing function for the sake of computational simplicity).
For example, one may take squared Euclidean norm $\|\cdot\|_2^2$ as F.

Then $F(p) = \sum_{i=1}^{d} \big(\textsc{Market Price}_i - \textsc{Model price}_i(p)\big)^2$.

Calibration to the market data (2)

Approach 2: bootstrap.

- Assume that the parameter p has a term structure, i.e. $p(t) = p_i$ for $t \in [T_{i-1}, T_i)$, where $i \in \{1, \ldots, d\}$ and T_0.
- Assume that for $i \in \{1, \ldots, d\}$, B_i has maturity T_i (thus its theoretical price depends on $p_1, \ldots, p_i$ but not on $p_{i+1}, \ldots, p_d$).
- Bootstrapping procedure consists of d steps. In i-th step, we asume that $p_1, \ldots, p_{i-1}$ are already known. Then we choose p_i such that the equation

$$\textsc{Market Price}_i = \textsc{Model price}_i(p)$$

is satisfied. Note that $p_1, \ldots, p_{i-1}$ are fixed and the price does not depend on $p_{i+1}, \ldots, p_d$. Hence, at i-th step we need to solve an equation with the single unknown p_i. Typically we cannot deduce a closed formula for p_i from this equation, so we employ a numerical solver.