Project: Lagrange polynomial interpolation

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1 Introduction

From the lecture on polynomial interpolation we know that for the set of (n+1) interpolation nodes (x_i) and tabluated functions values (y_i)

$$\{(x_0,y_0),\,(x_1,y_2),\,(x_3,y_2),\,\ldots,(x_n,y_n)\},\quad x\in[x_{min},x_{max}] \tag{1}$$

the Lagrange formula for interpolation polynomial $W_n(x)$ is following

$$W_n(x)=\sum_{j=0}^{n} y_j \frac{\prod_{\substack{i=0\\ i\neq j}}^{n}(x-x_i)}{\prod_{\substack{i=0\\ i\neq j}}^{n}(x_j-x_i)} \tag{2}$$

In project we assess the quality of Lagrange polynomial interpolation for test function

$$y(x)=\frac{1}{1+x^2} \tag{3}$$

and two sets of nodes, namely, the equidistant nodes

$$\Delta=\frac{x_{max}-x_{min}}{n} \tag{4}$$

$$x_i=x_{min}+\Delta\cdot i,\qquad i=0,1,2,\ldots,n \tag{5}$$

and for Chebyshev nodes

$$x_i=\frac{x_{max}-x_{min}}{2}\cos\left(\frac{2.0\cdot i+1.0}{2.0\cdot n+2.0}\pi\right)+\frac{x_{min}+x_{max}}{2} \tag{6}$$

2 Practical part

1. Write a computer program to calculate the value of Lagrange polynomial for given set of (n+1) tabulated data.
2. Assume the interpolation interval is defined by its endpoints $x_{min}=-5$ and $x_{max}=5$. For $n=5$, 10, 20 generate sets of equidistant nodes (Eq.5) and values of function (Eq.3). For each n draw separate figure showing test function and interpolation polynomial (Eq.2).
3. Repeat calculations for $n=20$, 50 and Chebyshev nodes defined in (Eq. 6). Draw figures showing values of test and interpolation functions and two separate figures displaying an interpolation error

$$\varepsilon(x)=y(x)-W_n(x) \tag{7}$$

4. At home prepare the report. Comment on interpolation error for equidistant and Chebyshev nodes reffering to the number of nodes and their spatial distribution.

3 Computational hints

In order to calculate the Lagrange polynomial's value we use Eq.2 which is summation over ratios of two number products scaled with y(x) function's values. When evaluating the summation remember to initialize the variable which accumulates the result with value 0 while the variables which retain the values of the nominator and the denominator must be initialized with 1. The code below show the procedure of calculating the value of $W_n(x)$

```
input: n,  x,     x⃗ = [x_0, x_1, ..., x_n],  y⃗ = [y_0, y_1, ..., y_n]
   W ← 0
for j=0 to n by 1 do
   for i=0 to n by 1 do
      α ← 1
      β ← 1
      if  |i-j|>0 then
         α ← α·(x − x_i)
         β ← β·(x_j − x_i)
      end if
   end do
   W ← W + y_j α/β
end do
```

4 Example results

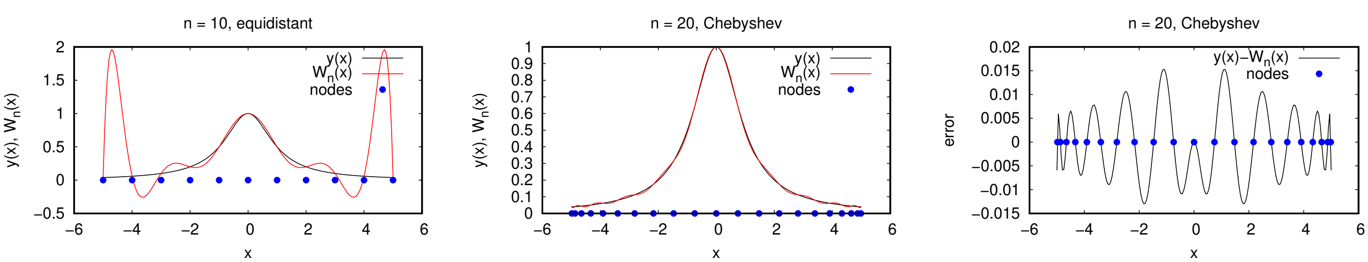

Figure 1: Polynomial interpolation for: equidistant nodes (left) and Chebyshev nodes (center). Right figure show interpolation error for Chebyshev nodes. Number of nodes equals $n+1$, the index n is displayed on top of each subfigure.