Project: Nonlinear equation - finding the roots of polynomial

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1 Introduction

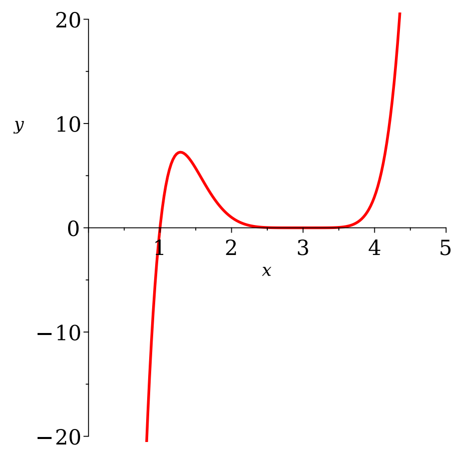

Figure 1: Polynomial $f(x) = (x-1)(x-3)^6$.

Figure 1 show nonlinear function which, in fact, is polynomial

$$f(x) = (x-1)(x-3)^6 \tag{1}$$

It has two roots $x = 1$ and $x = 3$, the first has odd parity and can be determined iteratively by any method designated to finding roots of nonlinear function while the second has even parity and only Secant and Newton-Raphson methods are appropriate. Our aim is to use the polynomial given in Eq.1 in a synthetic test of above-mentioned three iterative methods.

2 Practical part

1. Write computer program implementing algorthms for Bissection, Secant and Newton-Raphson methods (pseudocodes are given in Sec.3).
2. Find iteratively the odd parity root $x = 1$ using these three methods. In caclulations assume parameters: $itmax = 30$ - maximal number of iteration, $\varepsilon = 10^{-12}$ - stopping criterion and starting points for:
   - bissection method: $x_a = 0.1$ and $x_b = 1.5$
   - secant method: $x_a = 0.1$ and $x_b = 0.5$

- Newton-Raphson method: $x = 0.1$

In each iteration write to file: index of iteration $k+1$, approximated solution x_{k+1} and difference between two last approximations $\Delta = x_{k+1} - x_k$

3. Repeat calculations for second root of even parity $x = 3$ using only secant and Newton-Raphson methods. Increase the the total number of iterations to $itmax = 300$. Starting points for:
   - secant method: $x_a = 4.0$ and $x_b = 6.0$
   - Newton-Raphson method: $x = 6.0$
4. At home prepare the report including your results. Try to answear to the following questions:
   - Based on results of calculations, can you assess which method is most efficient and which is the worst?
   - Explain why for the second root $x = 3$ there are needed much more iterations than for the first one?
   - What will hapen, i.e. how change the number of iteration needed to get convergence, if you use following modified Newton-Raphson iterative equation?

$$x_{k+1} = x_k - r\frac{f(x_k)}{f'(x_k)}, \quad r = n \tag{2}$$

where n is the degree of the root ($n = 6$ in second case).

3 Computational hints

- algorithm of bissection method

```
input: x_a, x_b, itmax, ε
        k=0
do
    k ← k + 1

    x_c ← (x_a + x_b)/2

    if f(x_a) · f(x_c) < 0 then
        x_b ← x_c
    else
        x_a ← x_c
    end if

    !save approximated solution
    x ← x_c

    Δ ← |x_a − x_b|

while k < itmax & Δ > ε
```

- algorithm of Secant method

```
input: x_0, x_1, itmax, ε
        k=0
```

```
do
    k ← k + 1

    x_2 ← x_1 − f(x_1) (x_1 − x_0) / (f(x_1) − f(x_0))

    !save data for next iteration
    x_0 ← x_1
    x_1 ← x_2

    Δ ← |x_1 − x_0|

    !save approximated solution
    x ← x_1

while k < itmax & Δ > ε
```

- algorithm of Newton-Raphson method

```
input: x, itmax, ε
       k=0
do
    k ← k + 1
    x_1 ← x − f(x) / f'(x)

    Δ ← |x − x_1|

    !save approximate solution
    x ← x_1

while k < itmax & Δ > ε
```

4 Example results

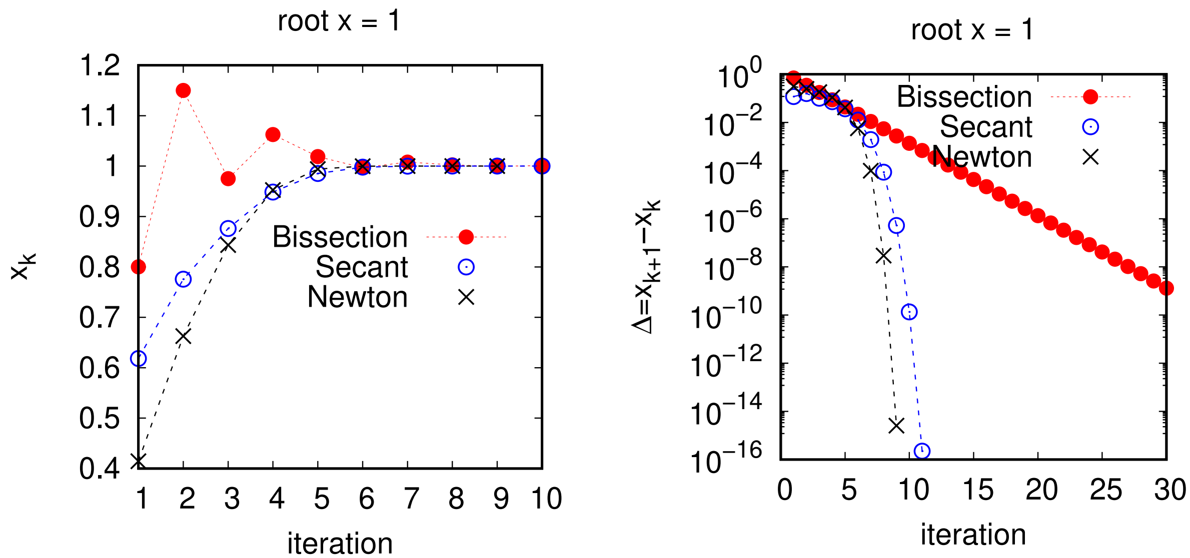

Figure 2: Bissection, Secant and Newton methods used for finding first $x = 1$ root. Approximated position of root (left) and distance between two consecutive approximation (right).

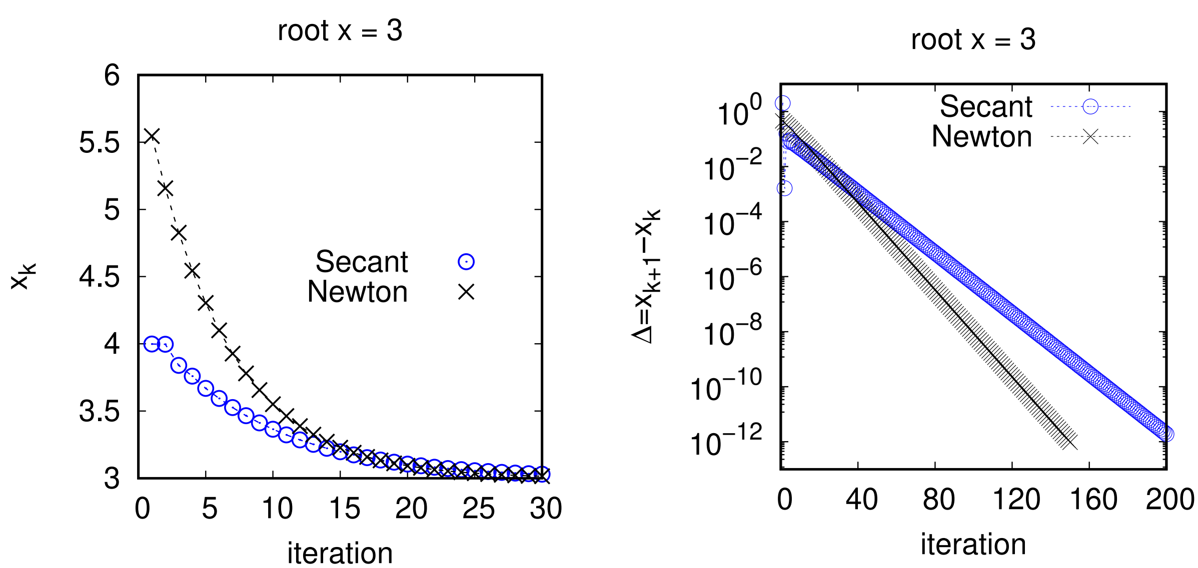

Figure 3: Secant and Newton methods used for finding second $x = 3$ root. Approximated position of root (left) and distance between two consecutive approximation (right).