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Materiał niedozwolony do użytku na kolokwium praktycznym

1 AR(2)->MA(inf)

$$x_t = 0.3 \cdot x_{t-1} - 0.7 \cdot x_{t-2} + \epsilon_t \qquad \epsilon_t \sim N(0,1) \tag{1}$$

Pierwsze wyrazy: $\psi_0 = 1$ i $\psi_1 = \varphi_1 = 0.3$

$$\varphi_2 = \varphi_1 \cdot \psi_1 + \varphi_2 \cdot \psi_0 = 0.3 \cdot 0.3 + (-0.7) \cdot 1 = -0.61 \tag{2}$$

$$\varphi_3 = \varphi_1 \cdot \psi_2 + \varphi_2 \cdot \psi_1 = 0.3 \cdot (-0.61) + (-0.7) \cdot 0.3 = -0.183 - 0.21 = -0.393 \tag{3}$$

$$\varphi_4 = \varphi_1 \cdot \psi_3 + \varphi_2 \cdot \psi_2 = 0.3 \cdot (-0.393) + (-0.7) \cdot (-0.61) = -0.1179 + 0.427 = 0.3091 \tag{4}$$

$$x_t = \epsilon_t + 0.3\epsilon_{t-1} - 0.61\epsilon_{t-2} - 0.393\epsilon_{t-3} + 0.3091\epsilon_{t-4} + ... \tag{5}$$

```
close all; clear; clc;
N = 20000; M = 50;
epsil = sqrt(1)*randn(1,N);
xAR = epsil;  xMA = epsil;
xAR(2) = xAR(2) + 0.3* xAR(1);
for k = 3: N
    xAR(k) = xAR(k) + 0.3 * xAR(k-1) - 0.7*xAR(k-2);
end
psi = ones(1, M);
psi(2) = 0.3;
for k = 3 : M
    psi(k) = 0.3*psi(k-1)-0.7*psi(k-2);
end
for k = 2 : N
    for m = 1 : min(k-1, M-1)
        xMA(k) = xMA(k)+epsil(k-m)*psi(m+1);
    end
end
[mean(xAR) mean(xMA)]
[var(xAR) var(xMA)]
 x = [xAR' xMA'];
```

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Materiał niedozwolony do użytku na kolokwium praktycznym

2 MA(1) -> AR(inf)

$$x_t = \epsilon_t + 0.5 \cdot \epsilon_{t-1} \tag{6}$$

Sprawdzamy warunek $|z| > 1$:
$1 + 0.5 \cdot z = 0 \quad \Rightarrow 0.5z = -1 \quad \Rightarrow z = -2 \quad \Rightarrow |-\mathbf{2}| > \mathbf{1}$ :: OK

$$\varphi_k = (-\theta_1)^k \quad \Rightarrow \quad x_t = \epsilon_t - \sum_{k=1}^{\infty} (-\theta_k)^k \cdot x_{t-k} \tag{7}$$

UWAGA: dla MA(1) jest "minus" przed sumą, dla MA(2) jest plus.
Czyli AR to:

$$x_t = \epsilon_t + 0.5 \cdot x_{t-1} - 0.25 \cdot x_{t-2} + 0.125 \cdot x_{t-3} - ... \tag{8}$$

```
close all; clear; clc;
N = 20000; M = 4*50;
epsil = sqrt(1)*randn(1,N); % var=1 -> std = 1
xAR = epsil;  xMA = epsil;
for k = 2 : N
    xMA(k) = epsil(k) + 0.5*epsil(k-1);
end
theta = (-0.5).^(1:M);
for k = 2 : N
    for m = 1 : min(k-1, M)
        xAR(k) = xAR(k)-xAR(k-m)*theta(m); % tu minus
    end
end
[mean(xAR) mean(xMA)]
[var(xAR) var(xMA)]
```

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Materiał niedozwolony do użytku na kolokwium praktycznym

3 MA(2)-> AR(inf)

$$x_t = \epsilon_t + 0.5 \cdot \epsilon_{t-1} + 0.2 \cdot \epsilon_{t-2} \tag{9}$$

1) sprawdzamy warunek odwracalności, tj. pierwiastki z: $1+0.5z+0.2z^2 = 0$ muszą mieć moduł $|z| > 1$;

$$2z^2 + 5z + 10 = 0 \quad \Rightarrow \Delta = -55 \quad \Rightarrow z_{1,2} = \frac{-5 - i\sqrt{55}}{4} \quad \Rightarrow |z_{1,2}| = \frac{\sqrt{80}}{4} > 1 \tag{10}$$

2) Wyliczamy współczynniki iteracyjnie:

$$\psi_k = -\theta_1 \cdot \psi_{k-1} - \theta_2 \cdot \psi_{k-2} \quad przy: \psi_1 = \theta_1 \quad \psi_2 = \theta_2 - \theta_1 \cdot \psi_1 \tag{11}$$

$$\psi_1 = 0.5 \qquad \psi_2 = 0.2 - 0.5 \cdot (0.5) = -0.05 \tag{12}$$

$$\psi_3 = -\theta_1 \cdot \psi_2 - \theta_2 \cdot \psi_1 = -0.5 \cdot (-0.05) - 0.2 \cdot 0.5 = 0.025 - 0.1 = -0.075 \tag{13}$$

$$\psi_4 = -\theta_1 \cdot \psi_3 - \theta_2 \cdot \psi_2 = -0.5 \cdot (-0.075) - 0.2 \cdot -0.05 = 0.0375 + 0.01 = 0.0475 \tag{14}$$

$$x_t = \epsilon_t + 0.5 \cdot x_{t-1} - 0.05 \cdot x_{t-2} - 0.075 \cdot x_{t-3} + 0.0475 \cdot x_{t-4} + ... \tag{15}$$

```
 close all; clear; clc;
N = 20000; M = 60;
epsil = sqrt(1)*randn(1,N); % var=1 -> std = 1
xAR = epsil;  xMA = epsil;
xMA(2) = xMA(2)+0.5*epsil(1);
for k = 3 : N
    xMA(k) = xMA(k) + 0.5*epsil(k-1) + 0.2*epsil(k-2);
end
psi = 0.5*ones(1, M);
psi(2) = -0.5*psi(1)+0.2;
for k = 3 : M
    psi(k) = -0.5*psi(k-1) - 0.2*psi(k-2);
end
psi(1:6)
for k = 2 : N
    for m = 1 : min(k-1, M)
        xAR(k) = xAR(k)+xAR(k-m)*psi(m); % tu plus
    end
end
[mean(xAR) mean(xMA)]
[var(xAR) var(xMA)]
```