Algebra - Zad. domowe nr. 3 - Geometria analityczna

(1) $l_1: \frac{x-9}{8} = \frac{y-5}{3} = \frac{z-2}{1}$ $l_2: \frac{3x+9}{4} = y+1 = \frac{24-3z}{7}$

$l_2: \frac{x+3}{\frac{4}{3}} = \frac{y+1}{1} = \frac{z-8}{-7/3}$

a) $l_1 \cap l_2 = \{P_0\}$ $P_0 = ?$

$l_1: \begin{cases} x = 9+8t \\ y = 5+3t \\ z = 2+t \end{cases}$ $t \in \mathbb{R}$ $l_2: \begin{cases} x = -3+\frac{4}{3}s \\ y = -1+s \\ z = 8-7/3\, s \end{cases}$ $\begin{cases} 9+8t = -3+4/3\, s \\ 5+3t = -1+s \\ 2+t = 8-7/3\, s \end{cases}$

$s = 6+3t$

$2+t = 8 - 7/3(6+3t) =$
$= 8 - 14 - 7t$
$8t = -8$, $t = -1$
$s = 3$
$9+8(-1) = 1 = -3+4/3 \cdot 3$ ok.

$P_0 = (1, 2, 1)$

b) $\pi = ?$ $l_1, l_2 \subset \pi$

$\vec{a} = [8,3,1] \parallel l_1$, $\vec{b} = [4/3, 1, -7/3] \parallel l_2$

równ. param.

$\pi: \begin{cases} x = 9 + 8t + 4/3\, s \\ y = 5 + 3t + s \\ z = 2 + t - 7/3\, s \end{cases}$

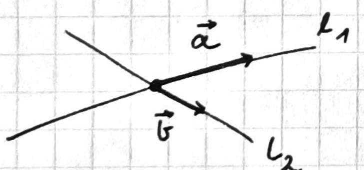
$\vec{n} = \vec{a} \times \vec{b} \perp \pi$

$\vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 8 & 3 & 1 \\ 4/3 & 1 & -7/3 \end{vmatrix} = [-8, 20, 4]$

$\pi: -8(x-9) + 20(y-5) + 4(z-2) = 0$

$-2(x-9) + 5(y-5) + z-2 = 0$
$-2x + 5y + z - 9 = 0$

c) l_4, l_3 dwusieczne kątów między l_1 i l_2



$\vec{v_3} = \frac{\vec{a}}{|\vec{a}|} + \frac{\vec{b}}{|\vec{b}|}$ $\vec{v_4} = \frac{\vec{a}}{|\vec{a}|} - \frac{\vec{b}}{|\vec{b}|}$

Romb

dwusieczne kątów to przekątne

$|\vec{a}| = \sqrt{64+9+1} = \sqrt{74}$
$|\vec{b}| = \sqrt{\frac{16}{9} + 1 + \frac{49}{9}} = \frac{1}{3}\sqrt{16+9+49}$
$= \frac{\sqrt{74}}{3}$

$\vec{v_3} = \frac{1}{\sqrt{74}}\left([8,3,1] + 3[4/3, 1, -7]\right) =$
$= \frac{1}{\sqrt{74}} \cdot [12, 6, -20]$

$\vec{v_4} = \frac{1}{\sqrt{74}}[(8,3,1) - (4,3,-21)] = \frac{1}{\sqrt{74}}[4, 0, 22]$ $P_0 \in l_3$, $P_0 \in l_4$

$l_3: \begin{cases} x = 1 + 12t \\ y = 2 + 6t \\ z = 1 - 20t \end{cases}$ $t \in \mathbb{R}$ $l_4: \begin{cases} x = 1 + 4s \\ y = 2 \\ z = 1 + 22s \end{cases}$ $s \in \mathbb{R}$

② $P = P_{\Delta ABC} = ?$ $\quad A=(1,2,3)$ $\quad l: 3x-3 = 6-y = 2z-8$

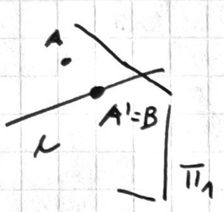
$\pi: -3x+2y-2z-29=0$

B – rzut prostokątny A na l

C – p. symetryczny do A wzgl. π

$l: \frac{x-1}{\frac{1}{3}} = \frac{y-6}{-1} = \frac{z-4}{\frac{1}{2}}$ $\quad \vec{v} = [\frac{1}{3}, -1, \frac{1}{2}] \parallel l$

$l_1: \begin{cases} x = 1 + \frac{1}{3}t \\ y = 6 - t \\ z = 4 + \frac{1}{2}t \end{cases}$



$\pi_1 \ni A$

$\pi_1 \perp l$

$\{B\} = \pi_1 \cap l$

$\vec{v} \perp \pi_1$ $\quad \pi_1: \frac{1}{3}(x-1) - (y-2) + \frac{1}{2}(z-3) = 0$

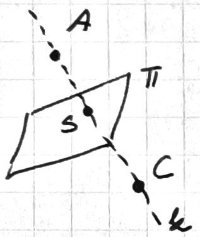
$2(x-1) - 6(y-2) + 3(z-3) = 0$

$2 \cdot \frac{1}{3}t - 6(4-t) + 3(1+\frac{1}{2}t) = 0$

$\frac{2}{3}t - 24 + 6t + 3 + \frac{3}{2}t = 0$

$\frac{4+36+9}{6}t = 21 \quad t = 21 \cdot \frac{6}{49} = \frac{18}{7}$

$B = (1+\frac{6}{7}, \frac{42-18}{7}, 4+\frac{9}{7}) = (\frac{13}{7}, \frac{24}{7}, \frac{37}{7})$



$k \perp \pi$

$A \in k$

$\vec{m} = [-3, 2, -2] \perp \pi$

$\vec{m} \parallel k$

$k: \begin{cases} x = 1 - 3t \\ y = 2 + 2t \\ z = 3 - 2t \end{cases} ; t \in \mathbb{R}$

$\{S\} = \pi \cap k$

$-3(1-3t) + 2(2+2t) - 2(3-2t) - 29 = 0$

$17t = 34$

$t = 2$

$S = (-5, 6, -1)$

$C = T_{\vec{AS}}(S)$ $\quad \vec{AS} = [-6, 4, -4]$

$C = (-5-6, 6+4, -1-4)$

$C = (-11, 10, -5)$

$P_\Delta = \frac{1}{2} |\vec{AB} \times \vec{AC}|$

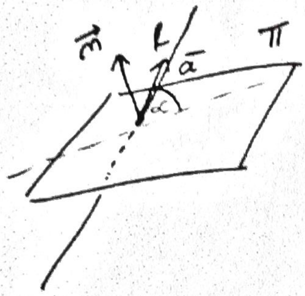
$\vec{AB} = [\frac{6}{7}, \frac{10}{7}, \frac{16}{7}]$ $\quad \vec{AC} = [-12, 8, -8]$

$\vec{AB} \times \vec{AC} = \frac{1}{7} \begin{vmatrix} i & j & k \\ 6 & 10 & 16 \\ -12 & 8 & -8 \end{vmatrix} = \frac{1}{7}[-208, -144, 168]$

$P_\Delta = \frac{1}{2} \cdot \sqrt{\frac{208^2 + 144^2 + 168^2}{7^2}} = \frac{1}{14}\sqrt{208^2 + 144^2 + 168^2}$

<u>Zad. 4</u> Oblicz miarę kąta między płaszczyzną

$\pi : x - z = 0$ a prostą $l : \begin{cases} x = 3+2t \\ y = 1 \\ z = -2-3t \end{cases}$; $t \in \mathbb{R}$.



$\vec{m} = (1,0,-1) \perp \pi$

$\vec{a} = (2,0,-3) \parallel l$

$\alpha = \measuredangle(\pi, l) = \frac{\pi}{2} - \overset{\beta}{\overset{\shortparallel}{\measuredangle(\vec{m},\vec{a})}}$ $\qquad \alpha + \beta = \pi/2$

$\cos\beta = \cos\left(\frac{\pi}{2} - \alpha\right) = \sin\alpha = \frac{|\vec{m} \circ \vec{a}|}{|\vec{m}| \cdot |\vec{a}|} = \frac{|2+0+3|}{\sqrt{2} \cdot \sqrt{4+9}} = \frac{5}{\sqrt{26}}$

$\alpha = \arcsin \frac{5}{\sqrt{26}}$