Energetyka - Kartkówka 2 — 32,5 pkt GRUPA 1

Zad. 1 9

a) Czy działanie o określone w $\mathbb{R}$ ma el. neutralny? 2

$a \circ b = a \cdot |b|$

$a \circ e = a \iff a \cdot |e| = a \Rightarrow |e| = 1$

$e \circ a = a \iff e|a| = a \rightarrow a \geq 0 \quad e = 1$ NIE

$a < 0 \quad e = -1$

b) Czy działanie $*$ w $\mathbb{R}$ jest łączne?

$x * y = x + y + xy$

$L = (x*y)*z = (x+y+xy)*z = x+y+xy+z+xz+yz+xyz$ $L = P$ TAK

$P = x*(y*z) = x*(y+z+yz) = x+y+z+yz+xy+xz+xyz$

Zad 2

$$\underset{A}{\begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix}} \cdot X \cdot \underset{B}{\begin{bmatrix} 1 & 3 \\ 1 & 2 \end{bmatrix}} = \underset{C}{\begin{bmatrix} 3 & 3 \\ 2 & 2 \end{bmatrix}}$$

$AXB = C$

$XB = A^{-1}C$

$X = A^{-1}CB^{-1}$

$\det A \neq 0$, $\det B \neq 0 \Rightarrow \exists A^{-1}, B^{-1}$

$$A^{-1} = ? \quad \left[\begin{array}{cc|cc} 3 & 1 & 1 & 0 \\ 2 & 1 & 0 & 1 \end{array}\right] \xrightarrow{w_1 - w_2} \left[\begin{array}{cc|cc} 1 & 0 & 1 & -1 \\ 2 & 1 & 0 & 1 \end{array}\right] \xrightarrow{w_2 - 2w_1} \left[\begin{array}{cc|cc} 1 & 0 & 1 & -1 \\ 0 & 1 & -2 & 3 \end{array}\right]$$

A^{-1}

$$B^{-1} = ? \quad \left[\begin{array}{cc|cc} 1 & 3 & 1 & 0 \\ 1 & 2 & 0 & 1 \end{array}\right] \rightarrow \left[\begin{array}{cc|cc} 1 & 3 & 1 & 0 \\ 0 & -1 & -1 & 1 \end{array}\right] \rightarrow \left[\begin{array}{cc|cc} 1 & 0 & -2 & 3 \\ 0 & -1 & -1 & 1 \end{array}\right] \rightarrow \left[\begin{array}{c|cc} I & -2 & 3 \\ & 1 & -1 \end{array}\right]$$

$$X = \begin{bmatrix} 1 & -1 \\ -2 & 3 \end{bmatrix} \begin{bmatrix} 3 & 3 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} -2 & 3 \\ 1 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} -2 & 3 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} -1 & 2 \\ 0 & 0 \end{bmatrix}$$

Zad. 3

$$A = \begin{bmatrix} 1 & 2 & 0 & 3 \\ 2 & p & p & 1 \\ 3 & 0 & 0 & 4 \\ 1 & 0 & 0 & 1 \end{bmatrix}$$

$p \in \mathbb{R}$ dla jakich p $\det A = 10$

$$\det A = p \cdot (-1)^6 \cdot \begin{vmatrix} 1 & 2 & 3 \\ 3 & 0 & 4 \\ 1 & 0 & 1 \end{vmatrix} = p(8-6) = -2p$$

$-2p = 10$

$p = -5$

Zad. Metodą Gaussa rozwiąż

$$\left[\begin{array}{cccc|c} 2 & 1 & -1 & 1 & 1 \\ 0 & 1 & 3 & -3 & 1 \\ 1 & 1 & 1 & -1 & 1 \end{array}\right] \rightarrow \left[\begin{array}{cccc|c} 1 & 1 & 1 & -1 & 1 \\ 0 & 1 & 3 & -3 & 1 \\ 2 & 1 & -1 & 1 & 1 \end{array}\right] \rightarrow \left[\begin{array}{cccc|c} 1 & 1 & 1 & -1 & 1 \\ 0 & 1 & 3 & -3 & 1 \\ \cancel{0} & \cancel{-1} & \cancel{-3} & \cancel{3} & \cancel{-1} \end{array}\right]$$

$$\rightarrow \begin{array}{c} \begin{array}{cccc} x & y & z & t \end{array} \\ \left[\begin{array}{cc|cc|c} 1 & 0 & -2 & 2 & 0 \\ 0 & 1 & 3 & -3 & 1 \end{array}\right] \end{array}$$

$x = 2z - 2t$

$y = 1 - 3z + 3t$

$z, t \in \mathbb{R}$

Zad. 1

a) $U = \{(x, 2x-1), x \in \mathbb{R}\}$ nie podprzestrzeń, $(0,0) \notin \mathbb{R}^2$

b)

$u = (1,2,0)$

$v = (0,1,1)$

$w = (1,0,3)$

$$\det \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 3 \end{bmatrix} \underset{w_3 - w_1}{=} \det \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \\ 0 & -2 & 3 \end{bmatrix} \overset{w_3 + 2w_2}{=} \det \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 5 \end{bmatrix} = 5 \neq 0 \Rightarrow$$ są liniowo niezależne