Fizyka techniczna – ALGEBRA – Kartkówka 2 (20pkt)

Zad.1

$$[A|B]=\left[\begin{array}{cccc|c}4&2&-2&-p&2\\8&5&-2&2-p&5\\4&2&p+1&-p&p+5\\8&4&p-1&-2p&p+7\end{array}\right]\xrightarrow[w_4-2w_1]{w_2-2w_1,\ w_3-w_1}\left[\begin{array}{cccc|c}4&2&-2&-p&2\\0&1&2&p+2&1\\0&0&p+3&0&p+3\\\cancel{0}&\cancel{0}&\cancel{p+3}&\cancel{0}&\cancel{p+3}\end{array}\right]$$

- $p\neq-3 \Rightarrow$ Układ nieoznaczony, ∞ wiele rozw. zależnych od 1 parametru
  $r(A)=r(U)=3<n=4$
- $p=-3$ nieozn.
  $$\left[\begin{array}{cccc|c}4&2&-2&3&2\\0&1&2&-1&1\end{array}\right]\xrightarrow{w_1+w_2}\left[\begin{array}{cccc|c}4&3&0&2&3\\0&1&2&-1&1\end{array}\right]\Rightarrow\begin{cases}x_1=\frac14(3-3x_2-2x_4)\\x_3=\frac12(1-x_2+x_4)\\x_2,x_4\in\mathbb{R}\end{cases}$$ 2 parametry
  $r(U)=r(A)=2<n=4$

Zad.2

a) $A\in M_6(\mathbb{R})\quad A^3-4A^T=0$

$A^3=4A^T \Rightarrow \det(A^3)=\det(4A^T) \Rightarrow (\det A)^3=4^6\det A \iff \det A\cdot(4^6-(\det A)^2)=0$

$\det A=0 \lor \det A=4^3=64 \lor \det A=-64$

b) $B,C\in M_4(\mathbb{R})\quad \det B=16,\quad B\to\begin{pmatrix}k_1\leftrightarrow k_4\\ \frac14 w_3\\ w_1+w_2\end{pmatrix}\to C\qquad \det C=(-1)\cdot\frac14\det B=-4$

c) $E^4(X-4I)^T=\frac12E^3F^3D^{-1}D^T$
$$E=\begin{bmatrix}1&3&1&0\\0&3&1&0\\4&2&0&0\\0&0&0&1/2\end{bmatrix}\qquad F=\begin{bmatrix}\sqrt[3]{2}&0&0&0\\0&2&0&0\\0&0&-2&0\\0&0&0&\sqrt[3]{2}\end{bmatrix}$$
$D,E,F\in M_4(\mathbb{R})$, nieosobliwe, D-symetryczna

$\Downarrow$

$D=D^T \Rightarrow E^4(X-4I)^T=\frac12E^3F^3\underbrace{D^{-1}\cdot D}_{I}=\frac12E^3F^3 \Rightarrow (X-4I)^T=E^{-4}\frac12E^3F^3=\frac12E^{-1}F^3$

$X-4I=\left(\frac12E^{-1}F^3\right)^T=\frac12(F^3)^T(E^{-1})^T=\frac12(F^T)^3\cdot(E^{-1})^T \Rightarrow X=4I+\frac12(F^T)^3(E^{-1})^T$

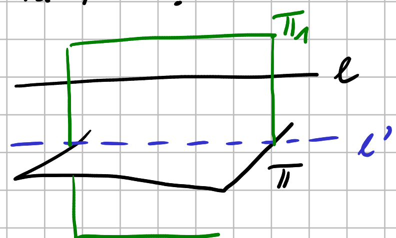
$E^{-1}=?$
$$\left[\begin{array}{cccc|cccc}1&3&1&0&1&0&0&0\\0&3&1&0&0&1&0&0\\4&2&0&0&0&0&1&0\\0&0&0&1/2&0&0&0&1\end{array}\right]\xrightarrow[w_4\cdot2]{w_3-4w_1,\ w_1-w_2}\left[\begin{array}{cccc|cccc}1&0&0&0&1&-1&0&0\\0&3&1&0&0&1&0&0\\0&-10&-4&0&-4&0&1&0\\0&0&0&1&0&0&0&2\end{array}\right]\xrightarrow{w_3+3w_2}\left[\begin{array}{cccc|cccc}1&0&0&0&1&-1&0&0\\0&3&1&0&0&1&0&0\\0&-1&-1&0&-4&3&1&0\\0&0&0&1&0&0&0&2\end{array}\right]\xrightarrow{w_3\cdot(-1),\ w_2\leftrightarrow w_3}\left[\begin{array}{cccc|cccc}1&0&0&0&1&-1&0&0\\0&1&1&0&4&-3&-1&0\\0&3&1&0&0&1&0&0\\0&0&0&1&0&0&0&2\end{array}\right]\xrightarrow{w_3-3w_2}$$
$$\to\left[\begin{array}{cccc|cccc}1&0&0&0&1&-1&0&0\\0&1&1&0&4&-3&-1&0\\0&0&-2&0&-12&10&3&0\\0&0&0&1&0&0&0&2\end{array}\right]\xrightarrow{w_3\cdot(-\frac12)}\left[\begin{array}{cccc|cccc}1&0&0&0&1&-1&0&0\\0&1&1&0&4&-3&-1&0\\0&0&1&0&6&-5&-3/2&0\\0&0&0&1&0&0&0&2\end{array}\right]\xrightarrow{w_2-w_3}\left[\begin{array}{c|cccc}I_4&1&-1&0&0\\&-2&2&1/2&0\\&6&-5&-3/2&0\\&0&0&0&2\end{array}\right]$$

F-diagonalna $\Rightarrow F^T=F \Rightarrow F^3=\begin{bmatrix}2&&&0\\&8&&\\&&-8&\\0&&&2\end{bmatrix}$
$$X=\begin{bmatrix}4&&&0\\&4&&\\&&4&\\0&&&4\end{bmatrix}+\begin{bmatrix}1&0&0&0\\0&4&0&0\\0&0&-4&0\\0&0&0&1\end{bmatrix}\cdot\begin{bmatrix}1&-2&6&0\\-1&2&-5&0\\0&1/2&-3/2&0\\0&0&0&2\end{bmatrix}$$
$$X=\begin{bmatrix}4&&&0\\&4&&\\&&4&\\0&&&4\end{bmatrix}+\begin{bmatrix}1&-2&6&0\\-4&8&-20&0\\0&-2&6&0\\0&0&0&2\end{bmatrix}=\begin{bmatrix}5&-2&6&0\\-4&12&-20&0\\0&-2&10&0\\0&0&0&6\end{bmatrix}$$

Zad.3

$l:\ x-3=10-2y=2z+12\qquad$ l' - rzut prostokątny l na Π ?

$\Pi:\ 2x+3y-z+1=0\qquad$ $\Pi_1\supset l,\ \Pi_1\supset l'$?



$l:\ \frac{x-3}{1}=\frac{y-5}{-\frac12}=\frac{z-(-6)}{\frac12} \Rightarrow \vec a=[1,-\frac12,\frac12]\parallel l,\ P_0=(3,5,-6)\in l$

$\vec n=[2,3,-1]\perp\Pi$

$\vec a\circ\vec n=2-\frac32-\frac12=0 \Rightarrow \vec a\perp\vec n \Rightarrow l\parallel\Pi$ ale $P_0\notin\Pi\quad 2\cdot3+3\cdot5-(-6)+1\neq0 \Rightarrow \Pi\cap l=\emptyset$

Znajdziemy $\Pi_1\perp\Pi,\ P_0\in\Pi_1\qquad \vec n_1:=\vec a\times\vec n\perp\Pi_1\qquad \vec n_1=\begin{vmatrix}\hat i&\hat j&\hat k\\1&-1/2&1/2\\2&3&-1\end{vmatrix}=[-1,2,4]$

$P_0\in\Pi_1 \Rightarrow \Pi_1:\ -1(x-3)+2(y-5)+4(z+6)=0$

$\Pi_1:\ -x+2y+4z+17=0 \leftarrow$ zawiera l i l'

$l'=\Pi\cap\Pi_1\qquad l':\begin{cases}2x+3y-z+1=0\\-x+2y+4z+17=0\end{cases}\qquad$ Rów. parametryczne ?

$\vec b:=\vec n\times\vec n_1\parallel l';\ \vec b=\begin{vmatrix}i&j&k\\2&3&-1\\-1&2&4\end{vmatrix}=[14,-7,7]=7[2,-1,1]$

np. $y=0\quad\begin{cases}2x-z+1=0\\-x+4z+17=0\end{cases}\qquad A=(-3,0,-5)\in l'\qquad l':\begin{cases}x=-3+2t\\y=0-t\\z=-5+t\end{cases},\ t\in\mathbb{R}$

$x=4z+17 \Rightarrow 8z+34-z+1=0,\ 7z+35=0,\ z=-5 \Rightarrow x=-3$