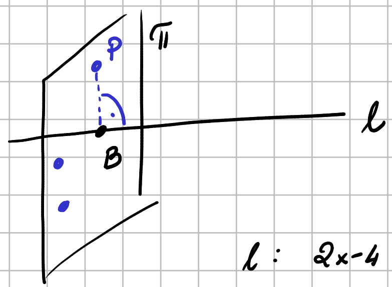
ALGEBRA - Zadanie domowe nr 3
Geometria analityczna

Zad.1 Punkty, których rzutem prostokątnym na $l: 2x-4 = 2-y = 2z+4$ jest $B=(3,0,-1)$.



$B \in l$, istotnie

$2\cdot 3-4=2 \quad 2-0=2 \quad 2\cdot(-1)+4=2$

$\Pi \perp l$, $B\in\Pi$ $\quad \forall P\in\Pi$ rzut P na l to B

$l: 2x-4 = 2-y = 2z+4$

$l: \frac{x-2}{\frac{1}{2}} = \frac{y-2}{-1} = \frac{z-(-2)}{\frac{1}{2}}$ $\qquad \vec{a}=[1,-2,1] \parallel l \Rightarrow \vec{a}\perp\Pi$

$\vec{a}\perp\Pi \wedge B\in\Pi$ $\qquad \Pi: 1\cdot(x-3)-2(y-0)+1\cdot(z+1)=0$

$\Pi: x-2y+z-2=0$

Zad. 2

a) $\Pi: \begin{cases} x=t \\ y=t-s \\ z=-1+2t+s \end{cases}$; $t,s\in\mathbb{R}$ $\qquad$ Znaleźć równanie ogólne Π.

$t=0, s=0 \Rightarrow P_0=(0,0,-1)\in\Pi$ $\qquad \vec{a}=[1,1,2]\parallel\Pi$ $\qquad \vec{b}=[0,-1,1]\parallel\Pi$

$\vec{n}:=\vec{a}\times\vec{b}$ $\qquad \vec{n}\perp\Pi$ $\qquad \vec{n}=\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 2 \\ 0 & -1 & 1 \end{vmatrix} = [3,-1,-1]$ $\qquad \Pi: 3(x-0)-1\cdot(y-0)-1(z+1)=0$

$\Pi: 3x-y-z-1=0$

b) $\Pi: 3x-y-z-1=0$ $\qquad$ Znaleźć parametryzację Π.

Niech $y=\alpha$, $z=\beta$. Wówczas $3x-\alpha-\beta-1=0$, $x=\frac{1}{3}(\alpha+\beta+1)$

$\Pi: \begin{cases} x=\frac{1}{3}(\alpha+\beta+1) \\ y=\alpha \\ z=\beta \end{cases}$; $\alpha,\beta\in\mathbb{R}$ $\leftarrow$ Inna parametryzacja niż w podpunkcie a)

Zad. 3

$l_1: x+4=2y+2=z$ $\qquad l_2: \begin{cases} x=-2s \\ y=-13+6s \\ z=0 \end{cases}$; $s\in\mathbb{R}$

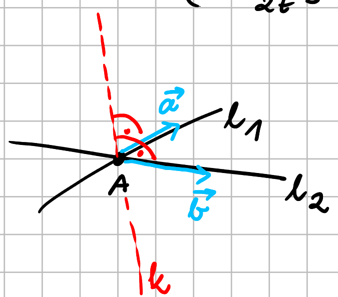
$\{A\}=l_1\cap l_2$

$k=?$ t. że $k\perp l_1 \wedge k\perp l_2 \wedge A\in k$

$l_1: \frac{x-(-4)}{1}=\frac{y-(-1)}{\frac{1}{2}}=\frac{z-0}{1}$ $\qquad \vec{a}=[2,1,2]\parallel l_1$ $\qquad l_1: \begin{cases} x=-4+2t \\ y=-1+t \\ z=2t \end{cases}$; $t\in\mathbb{R}$

$\vec{b}=[-2,6,0]\parallel l_2$

$A=?$ $\begin{cases} -4+2t=-2s \\ -1+t=-13+6s \\ 2t=0 \to t=0 \end{cases}$ $\to$ $\begin{cases} -4=-2s \to s=2 \\ -1=-13+6s \text{ spełnia} \end{cases}$ $\qquad A=(-4,-1,0)$



$\vec{n}:=\vec{a}\times\vec{b}=\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & 2 \\ -2 & 6 & 0 \end{vmatrix} = [-12,-4,14]\parallel k$

$A\in k \Rightarrow k: \begin{cases} x=-4-12t \\ y=-1-4t \\ z=14t \end{cases}$; $t\in\mathbb{R}$

Algebra – Zad. domowe nr. 3 – Geometria analityczna

Zad. 4 $l_1: \frac{x-9}{8} = \frac{y-5}{3} = \frac{z-2}{1}$ $l_2: \frac{3x+9}{4} = y+1 = \frac{24-3z}{7}$

$l_2: \frac{x+3}{\frac{4}{3}} = \frac{y+1}{1} = \frac{z-8}{-7/3}$

a) $l_1 \cap l_2 = \{P_0\}$ $P_0 = ?$

$l_1: \begin{cases} x = 9+8t \\ y = 5+3t \\ z = 2+t \end{cases}$ $t \in \mathbb{R}$ $l_2: \begin{cases} x = -3+\frac{4}{3}s \\ y = -1+s \\ z = 8-7/3 s \end{cases}$

$\begin{cases} 9+8t = -3+4/3 s \\ 5+3t = -1+s \\ 2+t = 8-7/3 s \end{cases}$

$s = 6+3t$

$2+t = 8 - 7/3(6+3t) =$

$= 8 - 14 - 7t$

$8t = -8$, $t = -1$

$s = 3$

$9+8 \cdot (-1) = 1 = -3+4/3 \cdot 3$ ok.

$P_0 = (1, 2, 1)$

b) $\pi = ?$ $l_1, l_2 \subset \pi$

$\vec{a} = [8,3,1] \parallel l_1$, $\vec{b} = [4/3, 1, -7/3] \parallel l_2$

równ. param.

$\pi: \begin{cases} x = 9 + 8t + 4/3 s \\ y = 5 + 3t + s \\ z = 2 + t - 7/3 s \end{cases}$

$\vec{n} = \vec{a} \times \vec{b} \perp \pi$

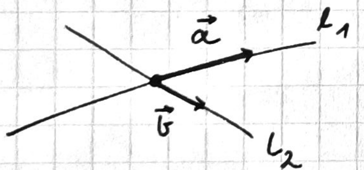
$\vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 8 & 3 & 1 \\ 4/3 & 1 & -7/3 \end{vmatrix} = [-8, 20, 4]$

$\pi: -8(x-9) + 20(y-5) + 4(z-2) = 0$

$-2(x-9) + 5(y-5) + z - 2 = 0$

$-2x + 5y + z - 9 = 0$

c) l_4, l_3 dwusieczne kątów między l_1 i l_2



$\vec{v_3} = \frac{\vec{a}}{|\vec{a}|} + \frac{\vec{b}}{|\vec{b}|}$ $v_4 = \frac{\vec{a}}{|\vec{a}|} - \frac{\vec{b}}{|\vec{b}|}$

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dwusieczne kątów to przekątne

$|\vec{a}| = \sqrt{64+9+1} = \sqrt{74}$

$|\vec{b}| = \sqrt{\frac{16}{9} + 1 + \frac{49}{9}} = \frac{1}{3}\sqrt{16+9+49}$

$= \frac{\sqrt{74}}{3}$

$\vec{v_3} = \frac{1}{\sqrt{74}} \left([8,3,1] + 3[4/3, 1, -7] \right) =$

$= \frac{1}{\sqrt{74}} \cdot [12, 6, -20]$

$\vec{v_4} = \frac{1}{\sqrt{74}} [(8,3,1) - (4,3,-21)] = \frac{1}{\sqrt{74}} [4, 0, 22]$

$P_0 \in l_3$, $P_0 \in l_4$

$l_3: \begin{cases} x = 1 + 12t \\ y = 2 + 6t \\ z = 1 - 20t \end{cases}$ $t \in \mathbb{R}$

$l_4: \begin{cases} x = 1 + 4s \\ y = 2 \\ z = 1 + 22s \end{cases}$ $s \in \mathbb{R}$

Zad. 5 $P = P_{\Delta ABC} = ?$ $A = (1,2,3)$ $l: 3x-3 = 6-y = 2z-8$

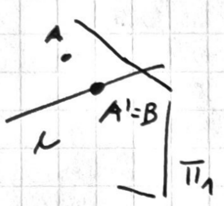
$\pi: -3x+2y-2z-29=0$

B – rzut prostokątny A na l

C – p. symetryczny do A wzgl. π

$l: \frac{x-1}{\frac{1}{3}} = \frac{y-6}{-1} = \frac{z-4}{\frac{1}{2}}$ $\vec{v} = [\frac{1}{3}, -1, \frac{1}{2}] \parallel l$

$l_1: \begin{cases} x = 1 + \frac{1}{3}t \\ y = 6 - t \\ z = 4 + \frac{1}{2}t \end{cases}$



$\pi_1 \ni A$ $\vec{v} \perp \pi_1$ $\pi_1: \frac{1}{3}(x-1) - (y-2) + \frac{1}{2}(z-3) = 0$

$\pi_1 \perp l$

$\{B\} = \pi_1 \cap l$

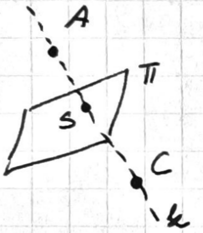
$2(x-1) - 6(y-2) + 3(z-3) = 0$

$2 \cdot \frac{1}{3}t - 6(4-t) + 3(1 + \frac{1}{2}t) = 0$

$\frac{2}{3}t - 24 + 6t + 3 + \frac{3}{2}t = 0$

$\frac{4+36+9}{6}t = 21$ $t = 21 \cdot \frac{6}{49} = \frac{18}{7}$

$B = (1 + \frac{6}{7}, \frac{42-18}{7}, 4 + \frac{9}{7}) = (\frac{13}{7}, \frac{24}{7}, \frac{37}{7})$



$k \perp \pi$

$A \in k$

$\vec{m} = [-3,2,-2] \perp \pi$

$\vec{m} \parallel k$

$k: \begin{cases} x = 1 - 3t \\ y = 2 + 2t \\ z = 3 - 2t \end{cases}; t \in \mathbb{R}$

$\{S\} = \pi \cap k$

$-3(1-3t) + 2(2+2t) - 2(3-2t) - 29 = 0$

$17t = 34$

$t = 2$

$S = (-5, 6, -1)$

$C = T_{\vec{AS}}(S)$ $\vec{AS} = [-6, 4, -4]$ $C = (-5-6, 6+4, -1-4)$

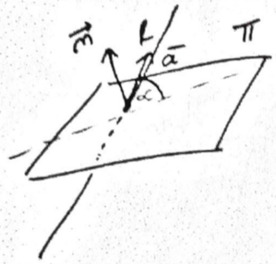
$C = (-11, 10, -5)$

$P_\Delta = \frac{1}{2}|\vec{AB} \times \vec{AC}|$ $\vec{AB} = [\frac{6}{7}, \frac{10}{7}, \frac{16}{7}]$ $\vec{AC} = [-12, 8, -8]$

$\vec{AB} \times \vec{AC} = \frac{1}{7}\begin{vmatrix} \hat{i} & \hat{j} & k \\ 6 & 10 & 16 \\ -12 & 8 & -8 \end{vmatrix} = \frac{1}{7}[-208, -144, 168]$

$P_\Delta = \frac{1}{2} \cdot \sqrt{\frac{208^2 + 144^2 + 168^2}{7^2}} = \frac{1}{14}\sqrt{208^2 + 144^2 + 168^2}$

Zad. 6 Oblicz miarę kąta między płaszczyzną $\pi: x - z = 0$ a prostą $l: \begin{cases} x = 3+2t \\ y = 1 \\ z = -2-3t \end{cases}; t \in \mathbb{R}$.



$\vec{m} = (1,0,-1) \perp \pi$

$\vec{a} = (2,0,-3) \parallel l$

$\alpha = \angle(\pi, l) = \frac{\pi}{2} - \angle(\vec{m}, \vec{a})$ (β) $\alpha + \beta = \pi/2$

$\cos\beta = \cos(\frac{\pi}{2} - \alpha) = \sin\alpha = \frac{|\vec{m} \circ \vec{a}|}{|\vec{m}| \cdot |\vec{a}|} = \frac{|2+0+3|}{\sqrt{2} \cdot \sqrt{4+9}} = \frac{5}{\sqrt{26}}$

$\alpha = \arcsin \frac{5}{\sqrt{26}}$

Zad. 7

$$\begin{cases} ax+y=1 \\ 2x-y=a \\ x+y=a \end{cases}$$

$$U=\left[\begin{array}{cc|c} a & 1 & 1 \\ 2 & -1 & a \\ 1 & 1 & a \end{array}\right]$$

$\det U = -a^2 + a + 2 + 1 - a^2 - 2a = -2a^2 - a + 3 = -(2a+3)(a-1)$

$a \in \mathbb{R} \setminus \{1, -\frac{3}{2}\}$ $\det U = 3$ → ukł. sprzeczny

$a \in \mathbb{R}\{1, -\frac{3}{2}\}$ $r(U) = r(A) = 2$ → oznaczony

(P) a) $\forall a < -2$ sprzeczny

(P) b) ukł. ozn. $\Rightarrow a = 1 \vee a = -\frac{3}{2}$

(F) c) Można wybrać a, by był nieoznaczony