

## Zadanie domowe nr 6

Zad. 1 Czy to iloczynowy skalarny?

a)  $g: \mathbb{R}^2 \times \mathbb{R}^2 \rightarrow \mathbb{R} \quad g(x, y) = 3x_1y_1 - 5x_1y_2 - 5x_2y_1 + 7x_2y_2$   
 $x = (x_1, x_2), y = (y_1, y_2)$

NIE, bo nie jest  $g(x, x) = 3x_1^2 - 10x_1x_2 + 7x_2^2 = x_1^2 + 2(x_1 - 2x_2)^2 - 2x_1x_2 - x_2^2$   
 $2(x_1^2 - 4x_1x_2 + 4x_2^2)$

$g(1, 1) = 1 + 2 - 2 - 1 = 0$

Nieprawda, że  $g(x, x) = 0 \Leftrightarrow x = (0, 0)$

b)  $g: (\mathbb{R}_1 \setminus \{x\}) \times (\mathbb{R}_1 \setminus \{x\}) \rightarrow \mathbb{R} \quad g(p, q) = p(1)q(1) + 2p(2)q(2)$

TAK linijność na pierwszej zmiennej  
 $\forall p_1, p_2, q \in \mathbb{R}_1 \setminus \{x\} \quad \forall \alpha, \beta \in \mathbb{R} \quad g(\alpha p_1 + \beta p_2, q) = (\alpha p_1 + \beta p_2)(1)q(1) + 2(\alpha p_1 + \beta p_2)(2)q(2) =$   
 $= [\alpha p_1(1) + \beta p_2(1)]q(1) + 2[\alpha p_1(2) + \beta p_2(2)]q(2) =$   
 $= \alpha [p_1(1)q(1) + 2p_1(2)q(2)] + \beta [p_2(1)q(1) + 2p_2(2)q(2)] = \alpha g(p_1, q) + \beta g(p_2, q)$

symetria  $g(q, p) = q(1)p(1) + 2q(2)p(2) = p(1)q(1) + 2p(2)q(2) = g(p, q)$

dodatnia określoność  $g(p, p) = [p(1)]^2 + 2[p(2)]^2 \geq 0$

$g(p, p) = 0 \Leftrightarrow p(1) = 0 \wedge p(2) = 0 \quad p = \alpha x + b$

$\begin{cases} p(1) = \alpha + b = 0 \\ p(2) = 2\alpha + b = 0 \end{cases} \Rightarrow \begin{matrix} \alpha = 0 \\ b = -\alpha = 0 \end{matrix} \Rightarrow p = 0$

Zad. 2 Norma

a)  $f \in C([0, 2\pi], \mathbb{R}), f(x) = 2\sin x - \cos x \quad \forall f, g \in C([0, 2\pi], \mathbb{R}) \quad \langle f, g \rangle = \int_0^{2\pi} f(x)g(x)dx$

$\|f\| = \sqrt{\langle f, f \rangle} = \sqrt{\int_0^{2\pi} (2\sin x - \cos x)^2 dx} = \dots$

$\int (2\sin x - \cos x)^2 dx = \int (4\sin^2 x - 4\sin x \cos x + \cos^2 x) dx = \int (1 + 3\sin^2 x - 2\sin 2x) dx = \int (1 + \frac{3}{2}(1 - \cos 2x) - 2\sin 2x) dx$   
 $= \frac{5}{2} \int dx - \frac{3}{2} \int \cos 2x dx - 2 \int \sin 2x dx =$   
 $= \frac{5}{2}x - \frac{3}{2} \cdot \frac{1}{2} \sin 2x - 2(-\frac{1}{2} \cos 2x) + C = \frac{5}{2}x - \frac{3}{4} \sin 2x + \cos 2x + C$

$\|f\|^2 = \left[ \frac{5}{2}x - \frac{3}{4} \sin 2x + \cos 2x \right]_0^{2\pi} = 5\pi + 1 - 1 = 5\pi \Rightarrow \|f\| = \sqrt{5\pi}$

b)  $C \in M_2(\mathbb{R}), C = \begin{bmatrix} -1 & 0 \\ -1 & 1 \end{bmatrix} \quad \forall A, B \in M_2(\mathbb{R}) \quad \langle A, B \rangle = \text{tr}(A \cdot B^T)$

$\|C\|^2 = \langle C, C \rangle = \text{tr}(C \cdot C^T)$

$\begin{array}{c|cc} & -1 & -1 \\ \hline 0 & 1 & 1 \\ -1 & 1 & 2 \end{array} \quad \begin{matrix} C^T \\ C \cdot C^T \end{matrix}$   
 $\|C\| = \sqrt{\text{tr}(C \cdot C^T)} = \sqrt{1+2} = \sqrt{3}$

Zad. 3  $W = \text{lin}\{w_1 = (3, 2, 0, 1, -4), w_2 = (1, 2, -2, 0, 1), w_3 = (3, -2, 6, -2, 5)\} \subset \mathbb{R}^5$

Baza  $W^\perp$  ?

$r \begin{bmatrix} 3 & 2 & 0 & 1 & -4 \\ 1 & 2 & -2 & 0 & 1 \\ 3 & -2 & 6 & -2 & 5 \end{bmatrix} \xrightarrow{H_1 - H_2, H_3 - H_2} r \begin{bmatrix} 0 & -4 & 6 & 1 & -7 \\ 1 & 2 & -2 & 0 & 1 \\ 0 & -8 & 12 & -2 & 2 \end{bmatrix} \xrightarrow{H_3 - 2H_2} r \begin{bmatrix} 1 & 2 & -2 & 0 & 1 \\ 0 & -4 & 6 & 1 & -7 \\ 0 & 0 & 0 & -4 & 16 \end{bmatrix} = 3$

$\{w_1, w_2, w_3\}$  baza  $W$

$v \in W^\perp \Leftrightarrow v \perp W \Leftrightarrow \begin{cases} v \perp w_1 \\ v \perp w_2 \\ v \perp w_3 \end{cases} \Leftrightarrow \begin{bmatrix} 3 & 2 & 0 & 1 & -4 \\ 1 & 2 & -2 & 0 & 1 \\ 3 & -2 & 6 & -2 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ t \\ u \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \rightarrow \begin{cases} 3x + 2y - 2z + t - 4u = 0 \\ -4y + 6z + t - 7u = 0 \\ -4t + 16u = 0 \end{cases}$

### Zad. 3 - avg dalez

$$t=4u \Rightarrow \begin{cases} x+2y-2z+u=0 \\ -4y+6z-3u=0 \end{cases} \quad \begin{matrix} x=-2y+2z-u \\ u=-\frac{4}{3}y+2z \end{matrix} \quad \begin{matrix} x=-2y+2z+\frac{4}{3}y-2z=\frac{2}{3}y \\ t=4u=-\frac{16}{3}y+8z \end{matrix}$$

$$v \in W^\perp \Rightarrow v = \left( \frac{2}{3}y, y, z, -\frac{16}{3}y+8z, -\frac{4}{3}y+2z \right)$$

$$W^\perp = \text{lin} \left\{ \left( \frac{2}{3}, 1, 0, -\frac{16}{3}, -\frac{4}{3} \right), (0, 0, 1, 8, 2) \right\} = \text{lin} \left\{ \underset{\substack{\uparrow \\ \text{Baza } W^\perp}}{(2, 3, 0, -16, -4)}, (0, 0, 1, 8, 2) \right\}$$

### Zad. 4

$$\mathbb{R}_3[x] \quad \forall p, q \in \mathbb{R}_3[x] \quad \langle p, q \rangle = a a_1 + (b-c)(b_1-c_1) + (2c-b)(2c_1-b_1) + d d_1$$

$$p = ax^3 + bx^2 + cx + d$$

$$q = a_1x^3 + b_1x^2 + c_1x + d_1$$

•  $\mathcal{B} = (b_1=2, b_2=x+x^2, b_3=x+2x^2, b_4=3x^3)$  *baza ortogonalna?*

$\mathcal{C} = (1, x, x^2, x^3)$  *baza kanoniczna*  $\mathbb{R}_3[x]$

$$\begin{matrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{matrix} \text{ w } \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 3 \end{bmatrix} \xrightarrow{u_3-u_2} \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 3 \end{bmatrix} = 4 \Rightarrow \mathcal{B} \text{ to baza } \mathbb{R}_3[x]$$

d   c   b   a

$$\langle b_1, b_2 \rangle = 0 + 0 \cdot 0 + 0 \cdot 1 + 2 \cdot 0 = 0$$

$$\langle b_1, b_3 \rangle = 0 \cdot 0 + 0 \cdot 1 + 1 \cdot 0 + 0 \cdot 0 = 0$$

*baza  
jest ortogonalna*

$$\langle b_1, b_4 \rangle = 0 + 0 \cdot 1 + 0 \cdot 0 + 2 \cdot 0 = 0$$

$$\langle b_2, b_4 \rangle = 0 \cdot 3 + 0 \cdot 0 + 1 \cdot 0 + 0 \cdot 0 = 0$$

$$\langle b_3, b_4 \rangle = 0 \cdot 3 + 0 \cdot 0 + 0 \cdot 0 + 2 \cdot 0 = 0$$

$$\langle b_3, b_4 \rangle = 0 \cdot 3 + 1 \cdot 0 + 0 \cdot 0 + 0 \cdot 0 = 0$$

•  $v(x) = x^2 - x + 1$

$$v = [1, -1, 1, 0]_{\mathcal{C}} \quad v = [\alpha, \beta, \gamma, \delta]_{\mathcal{B}}$$

$$\|b_1\|^2 = 0+0+0+4 \quad \|b_2\|^2 = 0+0+1+0=1 \quad \|b_3\|^2 = 0+1+0+0=1 \quad \|b_4\|^2 = 9$$

$$\alpha = \frac{\langle v, b_1 \rangle}{\|b_1\|^2} = \frac{1}{4} (0+0+0+2) = \frac{1}{2} \quad \beta = \frac{\langle v, b_2 \rangle}{\|b_2\|^2} = \langle v, b_2 \rangle = 0+0+1 \cdot (-3) + 0 = -3$$

$$\gamma = \frac{\langle v, b_3 \rangle}{\|b_3\|^2} = \langle v, b_3 \rangle = 0+1 \cdot 2 + 0+0=2 \quad \delta = \frac{\langle v, b_4 \rangle}{\|b_4\|^2} = \frac{1}{9} (0+0+0+0) = 0$$

$$v = \left[ \frac{1}{2}, -3, 2, 0 \right]_{\mathcal{B}}$$

### Zad. 5

$$\mathbb{R}^4 \supset U = \text{lin} \{ b_1 = (1, 1, -1, 0), b_2 = (0, 2, -1, 1), b_3 = (1, 5, -3, 0) \}$$

a) Metoda G-S :  $\mathcal{C} = (c_1, c_2, c_3)$  - szukana baza ortogonalna

•  $c_1 := b_1 = (1, 1, -1, 0)$

•  $c_2 \in \{c_1\}^\perp \wedge \text{lin} \{b_1, b_2\} = \text{lin} \{c_1, c_2\} \Rightarrow \begin{cases} c_2 = b_2 + \alpha c_1 \\ c_2 \circ c_1 = 0 \end{cases}$

$$c_2 \circ c_1 = (b_2 + \alpha c_1) \circ c_1 = b_2 \circ c_1 + \alpha \cdot \|c_1\|^2 = (0, 2, -1, 1) \circ (1, 1, -1, 0) + \alpha \cdot (1+1+1) = (2+1) + 3\alpha - 3 + 3\alpha = 0, \underline{\underline{\alpha = -1}}$$

$$c_2 = b_2 - c_1 = (-1, 1, 0, 1)$$

•  $c_3 \in \{c_1, c_2\}^\perp \wedge \text{lin} \{b_1, b_2, b_3\} = \text{lin} \{c_1, c_2, c_3\} \Rightarrow c_3 = b_3 + \alpha c_1 + \beta c_2 \wedge c_3 \circ c_1 = 0 \wedge c_3 \circ c_2 = 0$

$$0 = c_3 \circ c_1 = (b_3 + \alpha c_1 + \beta c_2) \circ c_1 = b_3 \circ c_1 + \alpha \cdot \|c_1\|^2 + \beta \cdot 0 = (1, 5, -3, 0) \circ (1, 1, -1, 0) + \alpha \cdot 3 = 1+5+3+3\alpha = 9+3\alpha$$

$$0 = c_3 \circ c_2 = b_3 \circ c_2 + \beta \|c_2\|^2 = (1, 5, -3, 0) \circ (-1, 1, 0, 1) = -1+5 = 4 + 3\beta \Rightarrow \beta = -\frac{4}{3}$$

$$\underline{\underline{\alpha = -3}}$$

$$c_3 = (1, 5, -3, 0) - 3 \cdot (1, 1, -1, 0) - \frac{4}{3} \cdot (-1, 1, 0, 1) = \left( -\frac{2}{3}, \frac{2}{3}, 0, -\frac{4}{3} \right) = \frac{2}{3} (-1, 1, 0, -1)$$

Zad. 5 b) Rzut ortogonalny  $v = (1, 0, 1, 0) \in \mathbb{R}^4$  na  $U$

$$u = \Pi_U(v) = ? \quad U = v - u \perp U = \text{lin}\{c_1, c_2, c_3\} \Leftrightarrow$$

$$u = [\alpha, \beta, \gamma]_C \quad \alpha = \frac{\langle v, c_1 \rangle}{\|c_1\|^2} = \frac{1}{3} \cdot (1, 0, 1, 0) \cdot (1, 1, -1, 0) = \frac{1}{3} \cdot 0 = 0$$

$$\beta = \frac{\langle v, c_2 \rangle}{\|c_2\|^2} = \frac{1}{3} \cdot (1, 0, 1, 0) \cdot (-1, 1, 0, 1) = -\frac{1}{3}$$

$$\gamma = \frac{\langle v, c_3 \rangle}{\|c_3\|^2} = \frac{(1, 0, 1, 0) \cdot (-1, 1, 0, -2) \cdot \frac{2}{3}}{(\frac{2}{3})^2 \cdot 6} = (-1) \cdot \frac{3}{2} \cdot \frac{1}{6} = -\frac{1}{4}$$

$$v = [0, -\frac{1}{3}, -\frac{1}{4}]_C = -\frac{1}{3}c_2 - \frac{1}{4}c_3 = -\frac{1}{3}(-1, 1, 0, 1) - \frac{1}{4}(1, 1, 0, -2) \cdot \frac{2}{3} = (\frac{1}{3}, -\frac{1}{3}, 0, -\frac{1}{3}) + (\frac{1}{6}, -\frac{1}{6}, 0, \frac{1}{3}) \\ = (\frac{1}{2}, -\frac{1}{2}, 0, 0)$$

Zad. 6.  $(u_1, u_2)$  uzupełnić do bazy  $\mathbb{R}^4$   $u_1 = (1, 1, 1, 0)$ ,  $u_2 = (0, 1, -1, 1)$

$$u_1 \circ u_2 = 0 + 1 - 1 + 0 = 0 \quad u_1 \perp u_2$$

$$\alpha \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = 4 \quad u_3 = c_3, u_4 = c_4 \Rightarrow (u_1, u_2, u_3, u_4) \text{ baza } \mathbb{R}^4 \\ u_3 \perp u_4 \quad u_1 \circ u_3 = 1 \neq 0 \quad \text{nie ortogonalna}$$

$C = (c_1, c_2, c_3, c_4)$  szukana baza ortogonalna

$$c_1 = u_1, \quad c_2 = u_2 \quad \text{bo } u_1 \circ u_2 = 0$$

$$c_3 = u_3 + \alpha c_1 + \beta c_2 \quad \wedge \quad 0 = c_1 \circ c_3, \quad 0 = c_2 \circ c_3$$

$$0 = c_1 \circ c_3 = c_1 \circ (u_3 + \alpha c_1 + \beta c_2) = c_1 \circ u_3 + \alpha \cdot \|c_1\|^2 = (1, 1, 1, 0) \cdot (0, 0, 1, 0) + \alpha \cdot 3 = 1 + 3\alpha$$

$$0 = c_2 \circ c_3 = c_2 \circ u_3 + \beta \cdot \|c_2\|^2 = (0, 1, -1, 1) \cdot (0, 0, 1, 0) + 3\beta = -1 + 3\beta \Rightarrow \beta = \frac{1}{3}$$

$$c_3 = u_3 - \frac{1}{3}c_1 + \frac{1}{3}c_2 = (0, 0, 1, 0) + (-\frac{1}{3}, -\frac{1}{3}, -\frac{1}{3}, 0) + (0, \frac{1}{3}, -\frac{1}{3}, \frac{1}{3}) = (-\frac{1}{3}, 0, \frac{1}{3}, \frac{1}{3})$$

$$c_4 = u_4 + \alpha c_1 + \beta c_2 + \gamma c_3$$

$$0 = c_4 \circ c_1 = u_4 \circ c_1 + \alpha \cdot \|c_1\|^2 = (0, 0, 0, 1) \cdot (1, 1, 1, 0) + 3\alpha = 3\alpha, \quad \alpha = 0$$

$$0 = c_4 \circ c_2 = u_4 \circ c_2 + \beta \cdot \|c_2\|^2 = (0, 0, 0, 1) \cdot (0, 1, -1, 1) + 3\beta = 3\beta + 1, \quad \beta = -\frac{1}{3}$$

$$0 = c_4 \circ c_3 = u_4 \circ c_3 + \gamma \cdot \|c_3\|^2 = (0, 0, 0, 1) \cdot (-\frac{1}{3}, 0, \frac{1}{3}, \frac{1}{3}) + \frac{2}{3}\gamma = -\frac{1}{3} + \frac{2}{3}\gamma, \quad \gamma = \frac{1}{2}$$

$$c_4 = (0, 0, 0, 1) - (-\frac{1}{3}, 0, \frac{1}{3}, \frac{1}{3}) + (0, -\frac{1}{3}, \frac{1}{3}, -\frac{1}{3}) = (\frac{1}{3}, -\frac{1}{3}, 0, \frac{1}{3})$$

Zad. 7  $\varphi = \varphi_3 \circ \varphi_1 \circ \varphi_2 \sim \mathbb{R}^2$

$\varphi_1$  - obrót o  $\frac{\pi}{6}$  ↺

$\varphi_2$  - odbicie względem prostej tworzącej z O x kąt  $\frac{\pi}{3}$

$\varphi_3$  - obrót o  $\frac{\pi}{6}$  ↻

$$\varphi_1 \sim A_1 = \begin{bmatrix} \cos \frac{\pi}{6} & -\sin \frac{\pi}{6} \\ \sin \frac{\pi}{6} & \cos \frac{\pi}{6} \end{bmatrix} = \begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$$

$$\varphi_2 \sim A_2 = \begin{bmatrix} \cos \frac{\pi}{3} & \sin \frac{\pi}{3} \\ \sin \frac{\pi}{3} & \cos \frac{\pi}{3} \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$$

$$\varphi_3 \sim A_3 = \begin{bmatrix} \cos \frac{\pi}{6} & -\sin \frac{\pi}{6} \\ \sin \frac{\pi}{6} & \cos \frac{\pi}{6} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$$

$$\varphi \sim A = A_3 A_2 A_1 = \begin{bmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} \end{bmatrix}$$

Zad. 8

$$A = \begin{bmatrix} -\frac{4}{5} & -\frac{3}{5} \\ -\frac{3}{5} & \frac{4}{5} \end{bmatrix} \in M_2(\mathbb{R})$$

Jaką izometrię liniową reprezentuje?

$$A \cdot A^T = \frac{1}{5} \begin{bmatrix} -4 & -3 \\ -3 & 4 \end{bmatrix} \cdot \frac{1}{5} \begin{bmatrix} -4 & -3 \\ -3 & 4 \end{bmatrix} = \frac{1}{25} \begin{bmatrix} 25 & 0 \\ 0 & 25 \end{bmatrix} = I$$

A-ortogonalna  $\Rightarrow$  reprezentuje izom. liniową

$$\det A = -\frac{16}{25} - \frac{9}{25} = -1 \Rightarrow A \text{ reprezentuje}$$

Punkty  $(x, y)$  ma osi symetrii spełniając

$$A \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix} \Rightarrow \begin{cases} -4x - 3y = 5x \\ -3x + 4y = 5y \end{cases} \Rightarrow \begin{cases} -9x - 3y = 0 \\ -3x - y = 0 \end{cases} \Rightarrow \begin{cases} 3x + y = 0 \\ y = -3x \end{cases}$$

os symetrii

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$$u = \Pi_U(v) \text{ mit } v \text{ na } U, w = v - u \perp U$$

$$u = ? \quad v = (1, 0, 1, 0) \quad u \in U, u = \alpha \cdot b_1 + \beta \cdot b_2 + \gamma \cdot b_3$$

$$= (\alpha + \gamma, \alpha + 2\beta + 5\gamma, -\alpha - \beta - 3\gamma, \beta)$$

$$w = (1 - \alpha - \gamma, -\alpha - 2\beta - 5\gamma, 1 + \alpha + \beta + 3\gamma, 1 - \beta)$$

$$0 = b_1 \circ w = (1, 1, -1, 0) \circ w = 1 - \alpha - \gamma - \alpha - 2\beta - 5\gamma - 1 + \alpha + \beta + 3\gamma = -3\alpha - 3\beta - 9\gamma \Rightarrow \alpha + \beta + 3\gamma = 0$$

$$0 = b_2 \circ w = (0, 2, -1, 1) \circ w = -2\alpha - 4\beta - 10\gamma - 1 + \alpha + \beta + 3\gamma - \beta = -1 - 3\alpha - 6\beta - 13\gamma \Rightarrow 3\alpha + 6\beta + 13\gamma = -1$$

$$0 = b_3 \circ w = (1, 5, -3, 0) \circ w = 1 - \alpha - \gamma - 5\alpha - 10\beta - 25\gamma - 3 - 3\alpha - 3\beta - 9\gamma = -2 - 9\alpha - 13\beta - 35\gamma \Rightarrow 9\alpha + 13\beta + 35\gamma = -2$$

$$\left[ \begin{array}{ccc|c} 1 & 1 & 3 & 0 \\ 3 & 6 & 13 & -1 \\ 9 & 13 & 35 & -2 \end{array} \right] \rightarrow \left[ \begin{array}{ccc|c} 1 & 1 & 3 & 0 \\ 0 & 3 & 4 & -1 \\ 0 & 4 & 8 & -2 \end{array} \right] \rightarrow \left[ \begin{array}{ccc|c} 1 & 1 & 3 & 0 \\ 0 & 3 & 4 & -1 \\ 0 & 2 & 4 & -1 \end{array} \right] \rightarrow \left[ \begin{array}{ccc|c} 1 & 1 & 3 & 0 \\ 0 & 3 & 4 & -1 \\ 0 & 0 & 4/3 & -4/3 \end{array} \right]$$

$$4/3 \gamma = -1/3 \quad \gamma = -1/4$$

$$\beta = -1/3 - 4/3 \gamma = -1/3 + 1/3 = 0$$

$$\alpha = -\beta - 3\gamma = -3 \cdot (-1/4) = 3/4$$

$$u = [\alpha, \beta, \gamma]_B = [3/4, 0, -1/4]_B =$$

$$= \frac{3}{4}(1, 1, -1, 0) - \frac{1}{4}(1, 5, -3, 0) = \frac{1}{4}(2, -2, 0, 0) = \underline{\underline{(\frac{1}{2}, -\frac{1}{2}, 0, 0)}}$$

Zadanie  $C([0,1], \mathbb{R})$   $\langle f, g \rangle = \int_0^1 f(x)g(x)dx$   
 szukam ortogonalny  $f$  na  $U$

$$U = \text{lin} \{1, x^2\} \quad f(x) = e^x$$

$$b_1 = x^2 \quad b_2 = 1$$

$g$  - szukam prostokątny

$$g \in U \Rightarrow \exists a, b \in \mathbb{R} : g = ax^2 + b$$

$$f - g \perp U \Rightarrow \begin{cases} \langle f - g, b_1 \rangle = 0 \\ \langle f - g, b_2 \rangle = 0 \end{cases}$$

$$\bullet \quad 0 = \langle f - g, b_1 \rangle = \int_0^1 (e^x - ax^2 - b) x^2 dx = \int_0^1 (x^2 e^x - ax^4 - bx^2) dx = \dots$$

$$\int_0^1 x^2 e^x dx = x^2 e^x - \int 2x e^x dx = x^2 e^x - 2 \int x e^x dx = x^2 e^x - 2 [x e^x - \int e^x dx] =$$

$$= x^2 e^x - 2x e^x + 2e^x + C$$

$$\dots = x^2 e^x - 2x e^x + 2e^x - a \frac{x^5}{5} - b \frac{x^3}{3} \Big|_0^1 = e - 2e + 2e - \frac{1}{5}a - \frac{1}{3}b - 2 = e - 2 - \frac{1}{5}a - \frac{1}{3}b = 0$$

$$\bullet \quad 0 = \langle f - g, b_2 \rangle = \int_0^1 (e^x - ax^2 - bx) \cdot 1 dx = e^x - a \frac{x^3}{3} - b \frac{x^2}{2} \Big|_0^1 = e - \frac{1}{3}a - \frac{1}{2}b - 1 = 0$$

$$\begin{cases} \frac{1}{5}a + \frac{1}{3}b = e - 2 & / \cdot 15 \\ \frac{1}{3}a + \frac{1}{2}b = e - 1 & / \cdot 6 \end{cases}$$

$$3a + 5b = 15e - 30$$

$$2a + 3b = 6e - 6 \Rightarrow a = \frac{1}{2}(6e - 6 - 3b) = 3e - 3 - \frac{3}{2}b$$

$$3a + 5b = 15e - 30$$

$$9e - 9 - \frac{9}{2}b + 5b = 15e - 30$$

$$\frac{b}{2} = 6e - 21$$

$$b = 12e - 42$$

$$g = (60 - 15e)x^2 + 12e - 42$$

$$a = 3e - 3 - \frac{3}{2}(12e - 42)$$

$$= 3e - 3 - 18e + 63$$

$$= \underline{60 - 15e}$$