Informatyka stosowana - AW - Kartkówka 1 - 2024r. | 20pkt

Zad. 1 (Pół)grupa (przemienna)?

$(\mathbb{R}, \circ)$ $\quad \forall a,b \in \mathbb{R} \quad a \circ b := \log_3(3^a + 3^b)$

- Działanie $\circ$ jest wewnętrzne $\quad a,b \in \mathbb{R} \Rightarrow \log_3(3^a+3^b) \in \mathbb{R}$
- oraz przemienne bowiem $3^a + 3^b = 3^b + 3^a$
- Łączność? $\quad a,b,c \in \mathbb{R}$ dowolne

$$L = (a\circ b)\circ c = \log_3(3^a+3^b)\circ c = \log_3\left(3^{\log_3(3^a+3^b)} + 3^c\right) = \log_3(3^a+3^b+3^c)$$
$$P = a\circ(b\circ c) = a \circ \log_3(3^b+3^c) = \log_3\left(3^a + 3^{\log_3(3^b+3^c)}\right) = \log_3(3^a+3^b+3^c)$$

$L = P$ działanie łączne

- el. neutralny? $\quad e \in \mathbb{R} \wedge a\circ e = a \iff \log_3(3^a+3^e) = a$

$$\log_3(3^a+3^e) = \log_3(3^a) \Rightarrow 3^e = 0 \text{ sprzeczne!}$$

Brak el. neutralnego

PÓŁGRUPA PRZEMIENNA

Zad. 2 Rozwiąż $\quad \dfrac{|4-3i|}{i}\cdot \bar{z}^2 = \dfrac{4+12i}{3-i}\cdot \dfrac{\left(\sin\frac{\pi}{6} - i\cos\frac{\pi}{3}\right)^7}{\left(\sin\frac{\pi}{3} - i\cos\frac{\pi}{3}\right)^5}\cdot z^5$

$|4-3i| = \sqrt{16+9} = 5$

$\frac{1}{i} = \frac{i}{i^2} = -i$

$\frac{4+12i}{3-i} = 4\cdot\frac{1+3i}{3-i} = 4\,\frac{(1+3i)(3+i)}{9+1} = \frac{4}{10}(\cancel{3}+i+9i-\cancel{3}) = \frac{4}{10}\cdot 10i = 4i$

$\sin\frac{\pi}{6} - i\cos\frac{\pi}{3} = \frac{1}{2} - \frac{1}{2}i = \frac{1}{2}(1-i) = \frac{\sqrt{2}}{2}\left(\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i\right) = \frac{1}{\sqrt{2}}e^{-\frac{\pi}{4}i}$

$\sin\frac{\pi}{3} - i\cos\frac{\pi}{3} = \frac{\sqrt{3}}{2} - \frac{1}{2}i = \cos\left(-\frac{\pi}{6}\right) + i\sin\left(-\frac{\pi}{6}\right) = e^{-\frac{\pi}{6}i}$

$z=0$ jest rozwiązaniem

zał. $z \neq 0$

$z = |z|\cdot e^{\varphi i}$

Wstawiając

$$\underbrace{-5}_{e^{\pi i}}\cancel{i}\cdot|z|^2 e^{-2\varphi i} = 4\cancel{i}\cdot\frac{\left(\frac{1}{\sqrt{2}}\right)^7 e^{-\frac{7}{4}\pi i}}{e^{-\frac{5}{6}\pi i}}\cdot|z|^5 e^{5i\varphi} \quad /:|z|^2$$

$$5e^{\pi i}e^{-2\varphi i} = \frac{\cancel{4}}{\cancel{8}_2\sqrt{2}}\cdot e^{-\pi i\left(\frac{7}{4}-\frac{5}{6}\right)}\cdot|z|^3 e^{5i\varphi} \quad /\cdot 2\sqrt{2} \quad /\cdot e^{2\varphi i}$$

$\frac{7}{4} - \frac{5}{6} = \frac{42-20}{24} = \frac{22}{24} = \frac{11}{12}$

$$10\sqrt{2}\, e^{\pi i} e^{\frac{11}{12}\pi i} = |z|^3 e^{7\varphi i} \Rightarrow |z|^3 = 10\sqrt{2} \;\wedge\; 7\varphi = \frac{23}{12}\pi + 2k\pi$$

$|z| = \sqrt[3]{10\sqrt{2}}$ $\qquad \varphi_k = \frac{23}{84}\pi + \frac{2}{7}k\pi$, $\; k\in\{0,1,\dots,6\}$

ROZW: $z = 0 \vee z_k = \sqrt[3]{10\sqrt{2}}\cdot e^{\varphi_k i}$

Zad. 3 Zaznacz na płaszczyźnie zespolonej $A = \{z\in\mathbb{C}: |3\bar{z}+2i| \geq 6 \wedge |z| < |z+2| \wedge$

- $|3\bar{z}+2i| \geq 6$

$|3(\bar{z}+\frac{2}{3}i)| \geq 6$

$|\bar{z}+\frac{2}{3}i| \geq 2$

$|\bar{z}+\frac{2}{3}i| = |\overline{\bar{z}+\frac{2}{3}i}| = |z-\frac{2}{3}i| \geq 2$

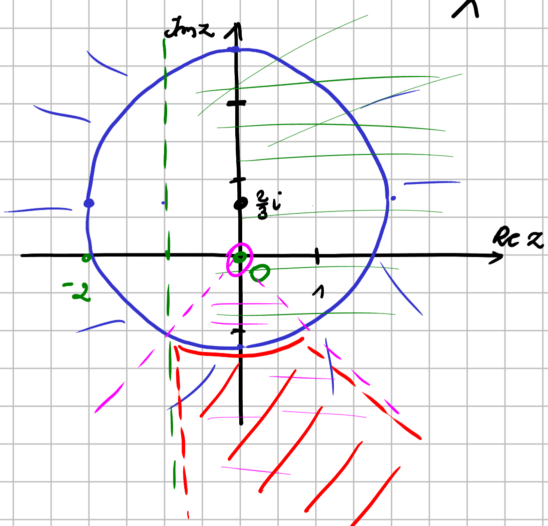
- $|z| < |z+2|$

$|z-0| < |z-(-2)|$

- $\frac{\pi}{4} < \arg(\bar{z}) < \frac{3}{4}\pi$

$\frac{\pi}{4} < 2\pi - \arg(z) < \frac{3}{4}\pi$

$\arg(z) < \frac{7}{4}\pi \;\wedge\; \arg z > \frac{5}{4}\pi$ $\}$



Część wspólna