Informatyka stosowana - ALGEBRA - Kartkówka 2

/30pkt

Zad.1

$$\left[\begin{array}{cccc|c} p & 5 & 0 & 10 & 20 \\ 0 & 0 & p+5 & p+8 & 50 \\ p & p & 0 & 10 & 2p+10 \\ 2p & p+5 & 10p+50 & 19p & 2p+539 \end{array}\right] \xrightarrow[w_4-2w_1]{w_3-w_1} \left[\begin{array}{cccc|c} p & 5 & 0 & 10 & 20 \\ 0 & 0 & p+5 & p+8 & 50 \\ 0 & p-5 & 0 & 0 & 2p-10 \\ 0 & p-5 & 10p+50 & 19p-20 & 2p+499 \end{array}\right] \xrightarrow[w_4-w_3]{}$$

$$\left[\begin{array}{cccc|c} p & 5 & 0 & 10 & 20 \\ 0 & 0 & p+5 & p+8 & 50 \\ 0 & p-5 & 0 & 0 & 2p-10 \\ 0 & 0 & 10p+50 & 19p-20 & 509 \end{array}\right] \xrightarrow[\substack{\text{kolejności} \\ w_2 \leftrightarrow w_3}]{\text{zmiana}} \left[\begin{array}{cccc|c} p & 5 & 0 & 10 & 20 \\ 0 & p-5 & 0 & 0 & 2p-10 \\ 0 & 0 & p+5 & p+8 & 50 \\ 0 & 0 & 10p+50 & 19p-20 & 509 \end{array}\right] \xrightarrow[w_4-10w_3]{} \left[\begin{array}{cccc|c} p & 5 & 0 & 10 & 20 \\ 0 & p-5 & 0 & 0 & 2p-10 \\ 0 & 0 & p+5 & p+8 & 50 \\ 0 & 0 & 0 & 9p-100 & 9 \end{array}\right]$$

a) WNIOSKI

① $p\in\mathbb{R}\setminus\{0,5,-5,\frac{100}{8}\}$ $\quad r(U)=r(A)=4=n$ układ oznaczony, 1 rozw.

② $p=0$

$$\left[\begin{array}{cccc|c} 0 & 5 & 0 & 10 & 20 \\ 0 & -5 & 0 & 0 & -10 \\ 0 & 0 & 5 & 8 & 50 \\ 0 & 0 & 0 & -100 & 9 \end{array}\right] \xrightarrow[w_1+w_2]{} \left[\begin{array}{cccc|c} 0 & 0 & 0 & 10 & 10 \\ 0 & -5 & 0 & 0 & -10 \\ 0 & 0 & 5 & 8 & 50 \\ 0 & 0 & 0 & -100 & 9 \end{array}\right]$$

układ sprzeczny brak rozwiązań

③ $p=5$

$$\left[\begin{array}{cccc|c} 5 & 5 & 0 & 10 & 20 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 10 & 13 & 50 \\ 0 & 0 & 0 & -65 & 9 \end{array}\right]$$

układ nieoznaczony / 1 parametr (nieskończenie wiele rozw.)

④ $p=-5$

$$\left[\begin{array}{cccc|c} -5 & 5 & 0 & 10 & 20 \\ 0 & -10 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3 & 50 \\ 0 & 0 & 0 & -145 & 9 \end{array}\right]$$

układ sprzeczny brak rozw.

⑤ $p=\frac{100}{9}$
ukł. sprzeczny, brak rozwiązań bo ostatnie równanie sprzeczne

---

Zad.2

$A,B,C,D\in M_4(\mathbb{R})$ $\quad$ nie obliczać $\quad M=\frac{1}{2}A^TB^{-1}C^{-1}$

$\det D=8$

$\det(CB)=\frac{1}{4}$

A - powstaje z D: $\quad k_1\leftrightarrow k_3 \wedge \sqrt{2}\cdot w_3$, w_4-w_2 $\Rightarrow \det A=-\sqrt{2}\cdot\det D=-8\sqrt{2}$

$$\det M=\det\left(\tfrac{1}{2}A^T\right)\cdot\det(B^{-1})\cdot\det(C^{-1})=\frac{\left(\frac{1}{2}\right)^4\cdot\det A}{\det B\cdot\det C}=\frac{-8\sqrt{2}}{2^4\cdot\det(CB)}=\frac{-8\sqrt{2}}{16\cdot\frac{1}{4}}=\frac{-8\sqrt{2}}{4}=\underline{\underline{-2\sqrt{2}}}$$

Zad.3

$B^9(10I-X)^{-1}-B^8A^T=0$ $\qquad X=?$ $\qquad A,B\in M_4(\mathbb{R})$

$$B=[b_{ij}]=\begin{cases} 0 & i\neq j \\ \frac{i+j+1}{i} & i=1 \end{cases} \qquad B=\begin{bmatrix} 3 & 0 & 0 & 0 \\ 0 & 5/2 & 0 & 0 \\ 0 & 0 & 7/3 & 0 \\ 0 & 0 & 0 & 9/4 \end{bmatrix}$$

diagonalna

$\det B=3\cdot\frac{5}{2}\cdot\frac{7}{3}\cdot\frac{9}{4}\neq 0\Rightarrow\exists B^{-1}$!

$$A=\begin{bmatrix} 2 & 0 & 0 & 4 \\ 0 & 0 & 0 & 1 \\ 0 & 2 & 0 & 0 \\ -1 & 0 & 1 & 0 \end{bmatrix} \qquad \det A=1\cdot(-1)^5\begin{vmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ -1 & 0 & 1 \end{vmatrix}=-(4+0)=-4\neq 0\Rightarrow\exists A^{-1}\ !$$

$B^{-9}\ /\ B^9(10I-X)^{-1}=B^8A^T$

$(10I-X)^{-1}=B^{-1}A^T$

$10I-X=(B^{-1}A^T)^{-1}$

$X=10I-(B^{-1}A^T)^{-1}$

$X=10I-(A^T)^{-1}\cdot B$

$(A^T)^{-1}=(A^{-1})^T=?$

$$[A|I]=\left[\begin{array}{cccc|cccc} 2&0&0&4&1&0&0&0 \\ 0&0&0&1&0&1&0&0 \\ 0&2&0&0&0&0&1&0 \\ -1&0&1&0&0&0&0&1 \end{array}\right]\rightarrow\left[\begin{array}{cccc|cccc} -1&0&1&0&0&0&0&1 \\ 0&2&0&0&0&0&1&0 \\ 2&0&0&4&1&0&0&0 \\ 0&0&0&1&0&1&0&0 \end{array}\right]\begin{array}{l} /\cdot -1 \\ /:2 \\ \\ \\ \end{array}\xrightarrow[w_3+2w_1]{}$$

$$\rightarrow\left[\begin{array}{cccc|cccc} 1&0&-1&0&0&0&0&-1 \\ 0&1&0&0&0&0&1/2&0 \\ 0&0&2&4&1&0&0&2 \\ 0&0&0&1&0&1&0&0 \end{array}\right]/:2\rightarrow\left[\begin{array}{cccc|cccc} 1&0&-1&0&0&0&0&-1 \\ 0&1&0&0&0&0&1/2&0 \\ 0&0&1&2&1/2&0&0&1 \\ 0&0&0&1&0&1&0&0 \end{array}\right]\rightarrow\left[\begin{array}{cccc|cccc} 1&0&-1&0&0&0&0&-1 \\ 0&1&0&0&0&0&1/2&0 \\ 0&0&1&0&1/2&-2&0&1 \\ 0&0&0&1&0&1&0&0 \end{array}\right]$$

Zad.3 - c.d

$$\xrightarrow[w_1+w_3]{} \left[\begin{array}{c|cccc} & 1/2 & -2 & 0 & 0 \\ I_4 & 0 & 0 & 1/2 & 0 \\ & \frac{1}{2} & -2 & 0 & 1 \\ & 0 & 1 & 0 & 0 \end{array}\right]$$

$$\underbrace{\begin{matrix} 1/2 & -2 & 0 & 0 \\ 0 & 0 & 1/2 & 0 \\ \frac{1}{2} & -2 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{matrix}}_{A^{-1}}$$

$$X = 10I - (A^{-1})^T \cdot B$$

$$\begin{array}{cccc|cccc} & & & & 3 & 0 & 0 & 0 \\ & & & & 0 & 5/2 & 0 & 0 \\ & & & & 0 & 0 & 7/3 & 0 \\ & & & & 0 & 0 & 0 & 9/4 \\ \hline \frac{1}{2} & 0 & \frac{1}{2} & 0 & \frac{3}{2} & 0 & 7/6 & 0 \\ -2 & 0 & -2 & 1 & -6 & 0 & -14/3 & 9/4 \\ 0 & 1/2 & 0 & 0 & 0 & 5/4 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 7/3 & 0 \end{array}$$

$$X = \begin{bmatrix} 10-3/2 & 0 & -7/6 & 0 \\ +6 & 10 & +14/3 & -9/4 \\ 0 & -5/4 & 10 & 0 \\ 0 & 0 & -7/3 & 10 \end{bmatrix}$$

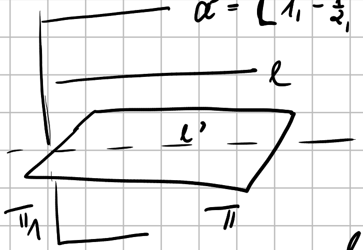
Zad. 4

$l: x-3 = 10-2y = 2z+12$ $\qquad \pi: 2x+3y-z+1=0$

$l: \frac{x-3}{1} = \frac{y-5}{-\frac{1}{2}} = \frac{z-(-6)}{\frac{1}{2}}$ $\qquad \vec{n} = [2, 3, -1] \perp \pi$

$P_0 = (3, 5, -6) \in l$

$\vec{a} = [1, -\frac{1}{2}, \frac{1}{2}] \parallel l$



π_1 t.że $l \subset \pi_1 \wedge \pi_1 \perp \pi$

$\Rightarrow \vec{n} \parallel \pi_1 \wedge \vec{a} \parallel \pi_1$

$\vec{b} := \vec{n} \times \vec{a} \perp \pi_1$ $\qquad \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & -1 \\ 1 & -\frac{1}{2} & \frac{1}{2} \end{vmatrix} = [1, -2, -4]$

$l \subset \pi_1 \Rightarrow P_0 \in \pi_1$

$\pi_1: 1(x-3) - 2(y-5) - 4(z+6) = 0$

$\pi_1 \quad x - 2y - 4z - 17 = 0$

$l': \begin{cases} 2x+3y-z+1=0 \\ x-2y-4z-17=0 \end{cases}$ $\longrightarrow$ $\vec{a} \parallel l \Rightarrow \vec{b} = -2\vec{a} = [-2, 1, -1] \parallel l'$

$\vec{a} \parallel l'$

$l': \begin{cases} x = 7 - 2t \\ y = -5 + t \\ z = 0 - t \end{cases} ; t \in \mathbb{R}$

$Q \in l' \rightarrow z = 0$, $\begin{cases} 2x + 3y = -1 \\ x - 2y = 17 \end{cases} \rightarrow x = 2y + 17$

$4y + 34 + 3y = -1$

$7y = -35 \quad y = -5$

$x = 7$

$\underline{Q = (7, -5, 0)}$