IS - 1 rok - Analiza mat. - Kartkówka 1

Gr 2

Zad. 1 Zbadaj (nie)parzystość funkcji f lub g

$$f(x)=\frac{e^{x^2+\sqrt{3}}}{\sqrt[3]{x}\cdot(x^4+\pi)}+\frac{tg\,x}{\ln|2x|}\sin^{345}\frac{x}{678}\cdot arctg\frac{x}{\pi}$$

Zał. $\sqrt[3]{x}\neq 0 \Rightarrow x\neq 0$

$x\neq\frac{\pi}{2}+k\pi,\ k\in\mathbb{Z}$

$|2x|>0 \Rightarrow x\neq 0$

$\ln|2x|\neq 0 \Rightarrow x\neq\pm\frac{1}{2}$

$D_f=\mathbb{R}\setminus\left\{-\frac{1}{2},\frac{1}{2},\frac{\pi}{2}+k\pi\ |\ k\in\mathbb{Z}\right\}$

$\forall x\in\mathbb{R}\quad x\in D_f\Rightarrow -x\in D_f$

$\sqrt[3]{(-x)}=-\sqrt[3]{x}$

$tg(-x)=-tg\,x$

$\ln|2(-x)|=\ln|-2x|=\ln|2x|$

$\sin\frac{-x}{678}=-\sin\frac{x}{678}$

$arctg\left(-\frac{x}{\pi}\right)=-arctg\frac{x}{\pi}$

$$f(-x)=\frac{e^{x^2+\sqrt{3}}}{-\sqrt[3]{x}\,(x^4+\pi)}+\frac{-tg\,x}{\ln|2x|}(-1)\cdot\sin^{345}\frac{x}{678}\cdot(-1)arctg\frac{x}{\pi}$$

$=-f(x)$ f - nieparzysta

Zad. 2 Oblicz

- $tg\,(arctg\sqrt{3})=\sqrt{3}$
- $arcctg\left(ctg\frac{37}{6}\pi\right)=arcctg\left(ctg\left(6\pi+\frac{\pi}{6}\right)\right)=$
  $=arcctg\left(ctg\frac{\pi}{6}\right)=\frac{\pi}{6}$
- $\arccos\left(-\frac{\sqrt{2}}{2}\right)=\pi-\arccos\frac{\sqrt{2}}{2}=\pi-\frac{\pi}{4}=\frac{3}{4}\pi$
- $\arcsin(\sin(-2\sqrt{2}))=-\arcsin(\sin 2\sqrt{2})=-\arcsin(\sin(\underbrace{\pi-2\sqrt{2}}_{\in(-\frac{\pi}{2},\frac{\pi}{2})}))$ 3,14 2·1,41
  $=-(\pi-2\sqrt{2})=2\sqrt{2}-\pi$

Zad. 3 Wyznacz dziedzinę funkcji

$$f(x)=\sqrt{\frac{x^2+1}{x^3+2x-3}}-\frac{\log_{\frac{2x}{1+x^2}}(3-x)+222}{\sqrt[6]{\sqrt{3}\,ctg\left(\frac{\pi}{3}-\pi x\right)-3}}$$

Zał. • $x^3+2x-3>0$

$$\begin{array}{c|c|c|c|c} & 1 & 0 & 2 & -3\\ \hline 1 & 1 & 1 & 3 & 0\end{array}$$

$(x-1)(x^2+x+3)>0$ $\quad\Delta=1-12<0$

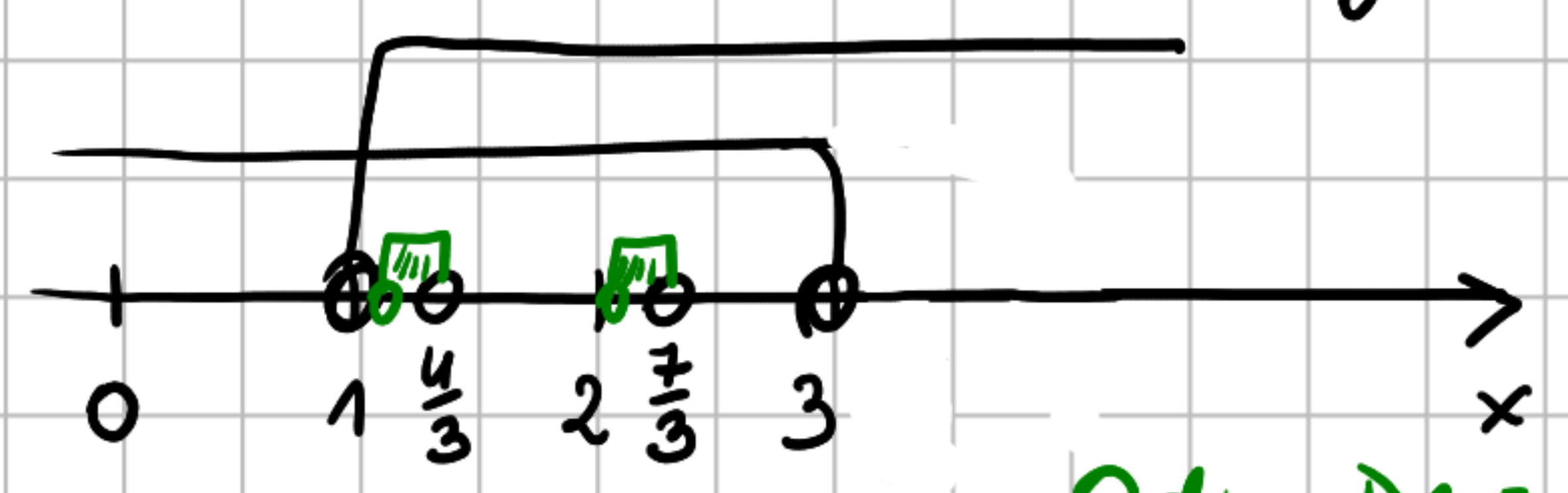
$\Rightarrow x-1>0,\ \underline{\underline{x>1}}$

- $3-x>0,\ \underline{\underline{x<3}}$
- $\frac{2x}{1+x^2}>0\Rightarrow\underline{\underline{x>0}}$
- $\frac{2x}{1+x^2}\neq 1,\ x^2+1\neq 2x$
  $x^2-2x+1=(x-1)^2\neq 0$
  $\underline{\underline{x\neq 1}}$
- $\frac{\pi}{3}-\pi x\neq k\pi,\ k\in\mathbb{Z}\ ,\ x\neq\frac{1}{3}-k,\ k\in\mathbb{Z}$
- $\sqrt{3}\,ctg\left(\frac{\pi}{3}-\pi x\right)-3>0$
  $ctg\left(\frac{\pi}{3}-\pi x\right)>\frac{3}{\sqrt{3}}=\sqrt{3}=ctg\frac{\pi}{6}$

$0+l\pi<\frac{\pi}{3}-\pi x<\frac{\pi}{6}+l\pi\ /:\pi$

$-\frac{1}{3}+l<-x<\frac{1}{6}-\frac{1}{3}+l\quad l\in\mathbb{Z}$

$\frac{1}{6}-l<x<\frac{1}{3}-l\ ,\ l\in\mathbb{Z}$



$l=-1\ \left(\frac{7}{6},\frac{4}{3}\right)$

$l=-2\ \left(\frac{13}{6},\frac{7}{3}\right)$

Odp $D_f=\left(\frac{7}{6},\frac{4}{3}\right)\cup\left(\frac{13}{6},\frac{7}{3}\right)$

Gr 3

$$g(x)=-\frac{1}{x}\cdot\sqrt[3]{arctg\,x}\cdot\log_2\left(x^{12}+2x^6+1\right)+\frac{5^{\sin^6 x}}{x^5-x^3}\arcsin\frac{x}{100}$$

Zał. $x\neq 0$

$x^{12}+2x^6+1=(x^6+1)^2>0\Rightarrow x^6+1\neq 0\Rightarrow x\in\mathbb{R}$

$x^5-x^3\neq 0\ ,\ x^3(x^2-1)\neq 0\Rightarrow x\neq 0, x\neq 1, x\neq -1$

$\left|\frac{x}{100}\right|\leq 1\Rightarrow x\in[-100,100]$

$D_g=[-100,100]\setminus\{-1,0,1\}$

$\forall x\in\mathbb{R}\quad x\in D_g\Rightarrow -x\in D_g$

$arctg(-x)=-arctg\,x$

$\arcsin\left(-\frac{x}{100}\right)=-\arcsin\frac{x}{100}$

$(-x)^5-(-x)^3=-x^5+x^3$

$\sin^6(-x)=(-\sin x)^6=\sin^6 x$

$$g(-x)=\frac{-1}{-x}\sqrt[3]{-arctg\,x}\cdot\log_2\left((-x)^6+1\right)^2+\frac{5^{\sin^6 x}}{-(x^5-x^3)}\left(-\arcsin\frac{x}{100}\right)$$

$=g(x)$ g - parzysta

Zad. 2 Oblicz

- $arcctg(-\sqrt{3})=\pi-arcctg\sqrt{3}=\pi-\frac{\pi}{6}=\frac{5}{6}\pi$
- $tg\left(arctg\frac{\sqrt{3}}{3}\right)=\frac{\sqrt{3}}{3}$
- $\arccos\left(\cos\frac{113}{11}\pi\right)=\arccos\left(\cos\left(10\pi+\frac{3}{11}\pi\right)\right)=\arccos\left(\cos\frac{3}{11}\pi\right)=\frac{3\pi}{11}$
- $\arcsin\left(\sin\left(-e-\frac{\pi}{2}\right)\right)=-\arcsin\left(\sin\left(e+\frac{\pi}{2}\right)\right)=$
  $=-\arcsin\left(\sin\left(\pi-e-\frac{\pi}{2}\right)\right)=-\arcsin\left(\sin\left(\frac{\pi}{2}-e\right)\right)=e-\frac{\pi}{2}$ $\in\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$ 1,57 - 2,72

Zad. 3 Dziedzina

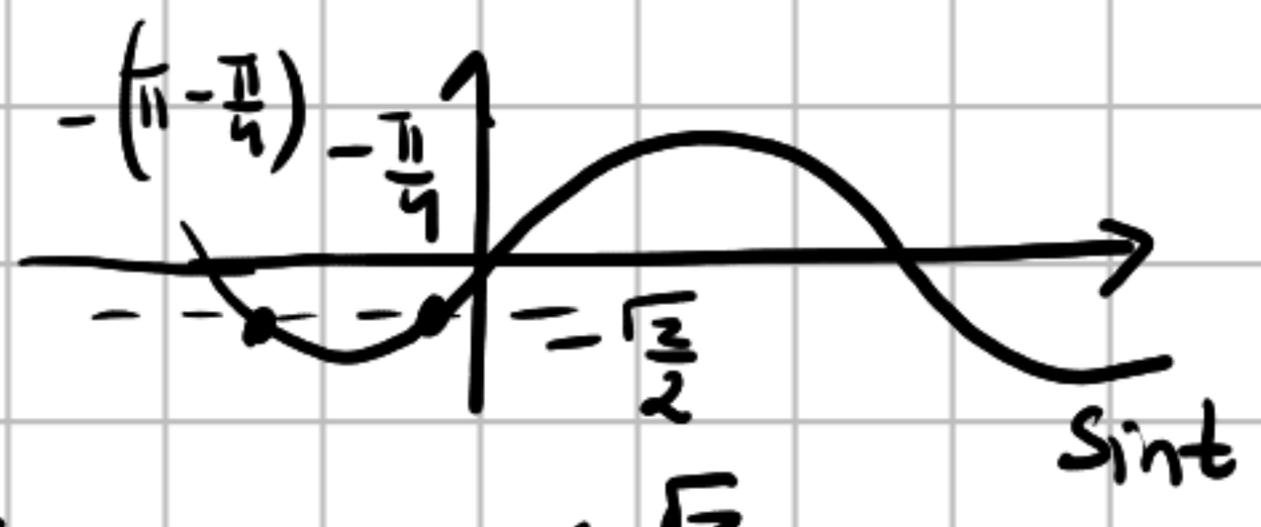
$$g(x)=\sqrt{\frac{x^3+2x^2-x-2}{x^4+1}}+\frac{\sqrt[8]{\arccos\frac{1}{2}-\arccos(\log_2 x)}}{\sqrt{-\sqrt{2}-2\sin\pi x}}$$

Zał. • $x^3+2x^2-x-2\geq 0$ $\quad x^2(x+2)-(x+2)=(x+2)(x^2-1)\geq 0$

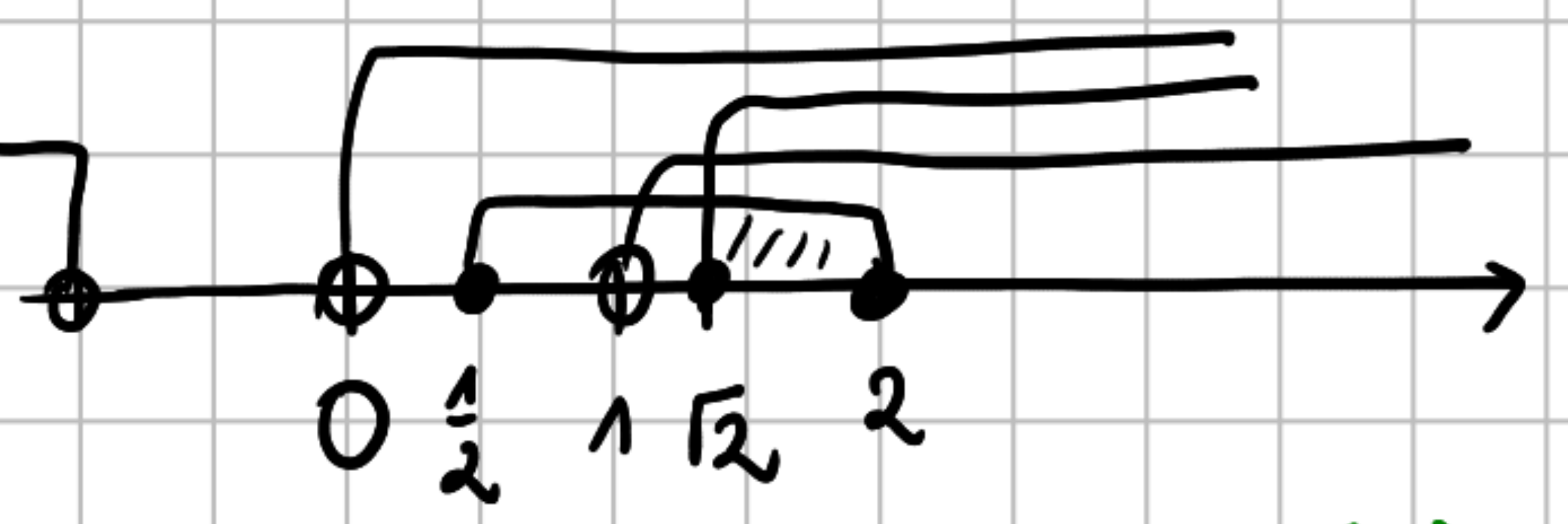
$x\in[-2,-1]\cup[1,\infty)$

- $\underline{\underline{x>0}}$
- $-1\leq\log_2 x\leq 1\ ,\ \log_2\frac{1}{2}\leq\log_2 x\leq\log_2 2\ ,\ \underline{\underline{x\in\left[\frac{1}{2},2\right]}}$
- $\arccos\frac{1}{2}-\arccos(\log_2 x)\geq 0\ ,$
  $\arccos\frac{1}{2}\geq\arccos(\log_2 x)$

$\log_2\sqrt{2}=\frac{1}{2}\leq\log_2 x\quad\Rightarrow\underline{\underline{\sqrt{2}\leq x}}$



- $-\sqrt{2}-2\sin\pi x>0\ ,\ 2\sin\pi x<-\sqrt{2}\ ,\ \sin\pi x<-\frac{\sqrt{2}}{2}$
  $-\frac{3}{4}\pi+2k\pi<\pi x<-\frac{\pi}{4}+2k\pi\ ,\ -\frac{3}{4}+2k<x<-\frac{1}{4}+2k\ ,\quad k\in\mathbb{Z}$



$k=1\quad\left(\frac{5}{4},\frac{7}{4}\right)$

$\sqrt{5}\approx 1{,}41=\frac{14}{10}=\frac{56}{40}$

$\frac{56}{40}>\frac{50}{40}=\frac{5}{4}$

$7/4<2=8/4$

Odp. $D_g=\left[\sqrt{2},\frac{7}{4}\right)$

Zad. 4 Oblicz granice

a)
$$\lim_{x\to\infty} \underbrace{x\cdot 7^x}_{\downarrow\ \infty}\cdot\underbrace{(\sqrt[x]{e}-1)}_{\downarrow\ 0} = \lim_{x\to\infty} \underbrace{\frac{e^{\frac{1}{x}}-1}{\frac{1}{x}}}_{\downarrow\ 1}\cdot \underbrace{7^x}_{\downarrow\ \infty} = \infty$$

b)
$$\lim_{n\to\infty} \underbrace{\sqrt[n^4]{10^{n^3}+11^{n^3}+\ldots+20^{n^3}}}_{a_n} = 1 \qquad \boxed{[\infty^0]}$$

$$\sqrt[n^4]{20^{n^3}} \le a_n \le \sqrt[n^4]{11\cdot 20^{n^3}} = \sqrt[n^4]{11}\cdot\sqrt[n]{20}$$

$$\sqrt[n^4]{20^{n^3}} = \sqrt[n]{20} \to 1$$

$$a_n \xrightarrow{\text{tw. o 3 ciągach}} \boxed{1}$$

$$\sqrt[n^4]{11}\to 1, \quad \sqrt[n]{20}\to 1, \quad \to 1$$

jako podciąg $\sqrt[n]{11}$

a)
$$\lim_{n\to\infty} \underbrace{n\cdot 2^n}_{\downarrow\ \infty}\cdot \underset{\downarrow\ 0}{\ln}\left(1-\frac{3}{n}\right) = \lim_{n\to\infty}(-3)\cdot \underset{\downarrow\ \infty}{2^n}\cdot \underset{\downarrow\ 1}{\ln}\underbrace{\left(1+\frac{-3}{n}\right)^{\frac{-n}{3}}}_{\downarrow\ e}$$
$$= -\infty$$

b)
$$\lim_{n\to\infty} \underbrace{\left(\frac{4}{\sqrt[4]{n^4+n+1}}+\frac{4}{\sqrt[4]{n^4+n+2}}+\ldots+\frac{4}{\sqrt[4]{n^4+n+n}}\right)}_{a_n} \qquad \boxed{\infty\cdot 0}$$

$$\frac{4\cdot n}{\sqrt[4]{n^4+2n}} \le a_n \le \frac{4n}{\sqrt[4]{n^4+n+1}} = \frac{4}{\sqrt[4]{1+\frac{1}{n^3}+\frac{1}{n^4}}} \to 4$$

$$\frac{4\cdot n}{\sqrt[4]{n^4+2n}} = \frac{4}{\sqrt[4]{1+\frac{2}{n^3}}} \to 4$$

$$a_n \xrightarrow{\text{tw. o 3 ciągach}} \boxed{4}$$

Zad. 5 Zbadaj ciągłość funkcji w całej dziedzinie

$$f(x) = \begin{cases} \sqrt[6]{5x}\cdot\cos\frac{4}{3x^2} & ; \ x<0 \\ 0 & ; \ x=0 \\ \sqrt{\frac{1}{x^2}-1}-\sqrt{\frac{1}{x^2}+1} & ; \ 0<x\le 1 \\ (7-6x)^{\frac{x^3}{x^4-1}} & ; \ 1<x<\frac{7}{6} \\ \frac{1}{14} & ; \ x=\frac{7}{6} \\ \frac{\sin(6x-7)}{36x^2-49} & ; \ x>\frac{7}{6} \end{cases}$$

$D_f = (0,\infty)$ W każdym z punktów ze zbioru $(0,\infty)\setminus\{1,\frac{7}{6}\}$ ciągła jako f. elementarna

- $f(0) = (0)$

$$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+}\left(\sqrt{\frac{1}{x^2}-1}-\sqrt{\frac{1}{x^2}+1}\right) \overset{[\infty-\infty]}{=}$$
$$= \lim_{x\to 0^+}\frac{\cancel{\frac{1}{x^2}}-1-\cancel{\frac{1}{x^2}}-1}{\sqrt{\frac{1}{x^2}-1}+\sqrt{\frac{1}{x^2}+1}} \overset{\left[\frac{-2}{+\infty}\right]}{=} 0$$

f ciągła prawostronnie w x=0

- $f(1) = \sqrt{\frac{1}{1}-1}-\sqrt{\frac{1}{1}+1} = 0-\sqrt{2} = -\sqrt{2}$

$$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-}\left(\sqrt{\frac{1}{x^2}-1}-\sqrt{\frac{1}{x^2}+1}\right) = -\sqrt{2}$$

$$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+}(7-6x)^{\frac{x^3}{x^4-1}} \overset{[1^\infty]}{=}$$
$$= \lim_{x\to 1^+}\Bigg(\underbrace{\left[1+(6-6x)\right]^{\frac{1}{6(1-x)}}}_{\downarrow\ e}\Bigg)^{\underbrace{\frac{6(1-x)x^3}{x^4-1}}} = e^{-3/2}$$

$$\frac{-6x^3(x\cancel{-1})}{(x\cancel{-1})(x^3+x^2+x+1)} \xrightarrow[x\to 1]{} \frac{-6}{4}$$

$$\begin{array}{c|c|c|c|c|c} & x^4 & x^3 & x^2 & x^1 & x^0 \\ & 1 & 0 & 0 & 0 & -1 \\ \hline 1 & 1 & 1 & 1 & 1 & 0 \end{array}$$

f nie jest ciągła w x=1
Jest lewostr. ciągła
p.n. I rodz
skok

- $f(\frac{7}{6}) = \frac{1}{14}$

$$\lim_{x\to \frac{7}{6}^-} f(x) = \lim_{x\to\frac{7}{6}^-}\underbrace{(7-6x)}_{\downarrow\ 0}^{\underbrace{\frac{x^3}{x^4-1}}_{\downarrow\ \text{stała}}} = 0$$

$$\lim_{x\to\frac{7}{6}^+} f(x) = \lim_{x\to\frac{7}{6}^+}\frac{\sin(6x-7)}{(6x-7)(6x+7)} = \frac{1}{14}$$

$$\frac{\sin(6x-7)}{6x-7}\to 1$$

f nie jest ciągła w x = 7/6
p.n. I rodz
skok
prawostr.
ciągła

$$f(x) = \begin{cases} e^3\cdot|1-3x|^{1/x} & ; \ x<0 \\ 1 & ; \ x=0 \\ \frac{1-\cos 2x}{x\sin x} & ; \ 0<x<1 \\ 1/2 & ; \ x=1 \\ \sqrt{e^{\frac{2}{x-1}}+e^{\frac{1}{x-1}}+e}-\sqrt{e^{\frac{2}{x-1}}+e+1} & ; \ 1<x<2 \\ \frac{1}{2} & ; \ x=2 \\ \frac{x-2}{2}\cdot\operatorname{arctg}\frac{1}{x-2} & ; \ x>2 \end{cases}$$

$D_f = \mathbb{R}$ W każdym z punktów ze zbioru $\mathbb{R}\setminus\{0,1,2\}$ f ciągła jako f. elementarna

- $f(0) = 1$

$$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} e^3\cdot|1-3x|^{\frac{1}{x}} = e^3\cdot\lim_{x\to 0^-}(1-3x)^{\frac{1}{x}} \overset{[1^\infty]}{=}$$
$$= e^3\cdot\lim_{x\to 0^-}\Big[\underbrace{(1-3x)^{\frac{1}{-3x}}}_{\downarrow\ e}\Big]^{\frac{-3x}{x}} = e^3\cdot e^{-3} = 1$$

$$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+}\frac{1-\cos 2x}{x\sin x} = \lim_{x\to 0^+}\frac{2\sin^2 x}{x\sin x} =$$
$$= \lim_{x\to 0^+}\frac{2\sin x}{x} = 2$$

$\Rightarrow$ f nie jest ciągła w x=0
jest lewostr. ciągła
p.n. I rodzaju, skok

- $f(1) = 1/2$

$$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-}\frac{1-\cos 2x}{x\sin x} = \frac{1-\cos 2}{\sin 1} = \frac{2\sin^2 1}{\sin 1} = 2\sin 1$$

$$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+}\left(\sqrt{e^{\frac{2}{x-1}}+e^{\frac{1}{x-1}}+e}-\sqrt{e^{\frac{2}{x-1}}+e+1}\right) =$$
$$\underset{[\infty-\infty]}{=} \lim_{x\to 1^+}\frac{e^{\frac{1}{x-1}}-1}{\sqrt{e^{\frac{2}{x-1}}+e^{\frac{1}{x-1}}+e}+\sqrt{e^{\frac{2}{x-1}}+e+1}} \overset{\left[\frac{\infty}{\infty}\right]}{=}$$
$$= \lim_{x\to 1^+}\frac{1-\underbrace{\frac{1}{e^{\frac{1}{x-1}}}}_{\to 0}}{\sqrt{1+\underbrace{\frac{1}{e^{\frac{1}{x-1}}}}_{\to 0}+\underbrace{\frac{e}{e^{\frac{2}{x-1}}}}_{\to 0}}+\sqrt{1+\underbrace{\frac{e+1}{e^{\frac{2}{x-1}}}}_{\to 0}}} = \frac{1}{2}$$

f nie jest ciągła w x=1, jest prawostr. ciągła
p.n. I rodzaju, skok

- $f(2) = \frac{1}{2}$

$$\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+}\frac{1}{2}\underset{\nearrow 0}{(x-2)}\,\underbrace{\operatorname{arctg}\frac{1}{x-2}}_{\text{og}} = 0$$

$$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-}\left(\sqrt{}-\sqrt{}\right) = \sqrt{e^2+e+e}-\sqrt{e^2+e+1}$$

w x=2
Nie jest
ciągła
ani
jednostr.
ciągła
p.n. I