Zadanie domowe nr 8

ZAD. 1

$$\lim_{n\to\infty}\frac{1}{n}\ln\left[\left(1+\frac{1}{n}\right)\left(1+\frac{2}{n}\right)\cdots\left(1+\frac{n}{n}\right)\right]=\lim_{n\to\infty}\frac{1}{n}\cdot\left[\ln\left(1+\frac{1}{n}\right)+\ln\left(1+\frac{2}{n}\right)+\dots+\ln\left(1+\frac{n}{n}\right)\right]=$$

$$=\lim_{n\to\infty}\frac{1}{n}\sum_{k=1}^{n}\ln\left(1+\frac{k}{n}\right)=\int_0^1\ln(1+x)\,dx=x\ln(1+x)\Big|_0^1-\int_0^1\frac{x}{x+1}\,dx=\Big[x\ln(1+x)-x+\ln|1+x|\Big]_0^1=$$

$$\int\frac{x}{x+1}\,dx=\int\frac{x+1-1}{x+1}\,dx=\int dx-\int\frac{dx}{x+1}=x-\ln|x+1|+C$$

$$=\ln 2-1+\ln 2=2\ln 2-1=\ln 4-1$$

ZAD. 2

a) $$\int_0^{\pi}x\cdot \operatorname{sgn}(\cos x)\,dx=\int_0^{\frac{\pi}{2}}x\,dx-\int_{\frac{\pi}{2}}^{\pi}x\,dx=\left[\frac{x^2}{2}\right]_0^{\frac{\pi}{2}}-\left[\frac{x^2}{2}\right]_{\frac{\pi}{2}}^{\pi}=\left(\frac{\pi^2}{8}-0\right)-\left(\frac{\pi^2}{2}-\frac{\pi^2}{8}\right)=-\frac{\pi^2}{4}$$

$$\operatorname{sgn}(\cos x)=\begin{cases}1; & \cos x>0\\ 0; & \cos x=0\\ -1; & \cos x<0\end{cases}\Rightarrow x\operatorname{sgn}(\cos x)=\begin{cases}x; & x\in[0,\frac{\pi}{2})\\ 0; & x=\frac{\pi}{2}\\ -x; & x\in[\frac{\pi}{2},\pi]\end{cases}$$

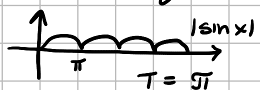
b) $$\int_{-1}^{0}x^2\cdot\sqrt{1-5x}\,dx=\left|\begin{matrix}1-5x=t \text{ malejąca}\\ x\in[-1,0]\Rightarrow t\in[6,1]\\ x=\frac{1-t}{5}\quad dx=-\frac{1}{5}dt\end{matrix}\right|=-\frac{1}{5}\int_6^1\left(\frac{1-t}{5}\right)^2\cdot\sqrt{t}\,dt=\frac{-1}{125}\int_6^1(1-2t+t^2)\sqrt{t}\,dt=\frac{1}{125}\int_1^6\left(\sqrt{t}-2t^{3/2}+t^{5/2}\right)dt$$

$1-5x\geq 0$, $x\leq\frac{1}{5}$ OK

$$=\frac{1}{125}\cdot\left[\frac{2}{3}t^{3/2}-\frac{4}{5}t^{5/2}+\frac{2}{7}t^{7/2}\right]_1^6=\dots$$

c) $$\int_0^{100\pi}\sqrt{1-\cos 2x}\,dx=\int_0^{100\pi}\sqrt{2\sin^2 x}\,dx=\sqrt{2}\int_0^{100\pi}|\sin x|\,dx=\sqrt{2}\cdot 100\cdot\int_0^{\pi}|\sin x|\,dx=100\sqrt{2}\int_0^{\pi}\sin x\,dx=100\sqrt{2}(-\cos x)_0^{\pi}=$$

$\cos 2\alpha=\cos^2\alpha-\sin^2\alpha=1-2\sin^2\alpha$



$T=\pi$

$$=100\sqrt{2}(1+1)=200\sqrt{2}$$

d) $$\int_{-\sqrt{2}}^{\sqrt{2}}\frac{2x^{17}+3x^6-10x^5-7x^3-12x^2+x+1}{x^2+2}\,dx=\int_{-\sqrt{2}}^{\sqrt{2}}\underbrace{\frac{2x^{17}-10x^5-7x^3+x}{x^2+2}}_{\text{nieparzysta}}\,dx+\int_{-\sqrt{2}}^{\sqrt{2}}\underbrace{\frac{3x^6-12x^2+1}{x^2+2}}_{\text{parzysta}}\,dx=$$

$$=2\int_0^{\sqrt{2}}\frac{3x^2(x^4-4)+1}{x^2+2}\,dx=6\int_0^{\sqrt{2}}x^2(x^2-2)\,dx+2\int_0^{\sqrt{2}}\frac{dx}{x^2+2}=6\left[\frac{x^5}{5}-\frac{2}{3}x^3\right]_0^{\sqrt{2}}+\left[\sqrt{2}\operatorname{arctg}\frac{x}{\sqrt{2}}\right]_0^{\sqrt{2}}=6\left[\frac{4\sqrt{2}}{5}-\frac{4\sqrt{2}}{3}\right]+$$

$$\int\frac{dx}{x^2+2}=\frac{1}{2}\int\frac{dx}{1+\left(\frac{x}{\sqrt{2}}\right)^2}=\left|\begin{matrix}\frac{x}{\sqrt{2}}=t\\ x=\sqrt{2}t\\ dx=\sqrt{2}\,dt\end{matrix}\right|=\frac{1}{2}\int\frac{\sqrt{2}\,dt}{1+t^2}=\frac{\sqrt{2}}{2}\operatorname{arctg}t+C$$

$$+\sqrt{2}(\operatorname{arctg}1-0)=24\sqrt{2}\left(\frac{1}{5}-\frac{1}{3}\right)+\sqrt{2}\cdot\frac{\pi}{4}=\sqrt{2}\left(\frac{\pi}{4}-\frac{16}{5}\right)$$

e) $$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\sqrt{\cos x-\cos^3 x}\,dx=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\sqrt{\cos x\cdot(1-\cos^2 x)}\,dx=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\underbrace{|\sin x|\cdot\sqrt{\cos x}}_{\text{parzysta}}\,dx=2\int_0^{\frac{\pi}{2}}|\sin x|\sqrt{\cos x}\,dx=2\int_0^{\frac{\pi}{2}}\sin x\cdot\sqrt{\cos x}\,dx=$$

$$=\left|\begin{matrix}\cos x=t & x\in[0,\frac{\pi}{2}]\\ -\sin x\,dx=dt & t\in[1,0]\end{matrix}\right|=-2\int_1^0\sqrt{t}\,dt=2\int_0^1\sqrt{t}\,dt=\frac{4}{3}t^{3/2}\Big|_0^1=\frac{4}{3}$$

f) $$\int_0^{\infty}\frac{x}{\sqrt[3]{x^2-1}}\,dx=\int_0^{1}\frac{x\,dx}{\sqrt[3]{x^2-1}}+\int_1^{\infty}\frac{x\,dx}{\sqrt[3]{x^2-1}}=\int_0^{1}\frac{x\,dx}{\sqrt[3]{x^2-1}}+\int_1^{2}\frac{x\,dx}{\sqrt[3]{x^2-1}}+\int_2^{\infty}\frac{x\,dx}{\sqrt[3]{x^2-1}}=$$

$x^2-1\neq 0\quad x^2\neq 1,\ x\neq\pm 1$

$$=\lim_{\alpha\to 1^-}\int_0^{\alpha}\frac{x\,dx}{\sqrt[3]{x^2-1}}+\lim_{\beta\to 1^+}\int_{\beta}^{2}\frac{x\,dx}{\sqrt[3]{x^2-1}}+\lim_{\gamma\to+\infty}\int_2^{\gamma}\frac{x\,dx}{\sqrt[3]{x^2-1}}=\dots$$

$$\int\frac{x\,dx}{\sqrt[3]{x^2-1}}=\left|\begin{matrix}x^2-1=t\\ 2x\,dx=dt\\ x\,dx=\frac{1}{2}dt\end{matrix}\right|=\frac{1}{2}\int\frac{dt}{\sqrt[3]{t}}=\frac{1}{2}\int t^{-1/3}dt=\frac{1}{2}\cdot\frac{3}{2}t^{2/3}+C=\frac{3}{4}\cdot\sqrt[3]{t^2}+C=\frac{3}{4}\cdot\sqrt[3]{(x^2-1)^2}+C$$

$$\dots=\lim_{\alpha\to 1^-}\left[\frac{3}{4}\cdot\sqrt[3]{(x^2-1)^2}\right]_0^{\alpha}+\lim_{\beta\to 1^+}\left[\frac{3}{4}\cdot\sqrt[3]{(x^2-1)^2}\right]_{\beta}^{2}+\lim_{\gamma\to\infty}\left[\frac{3}{4}\cdot\sqrt[3]{(x^2-1)^2}\right]_2^{\gamma}=$$

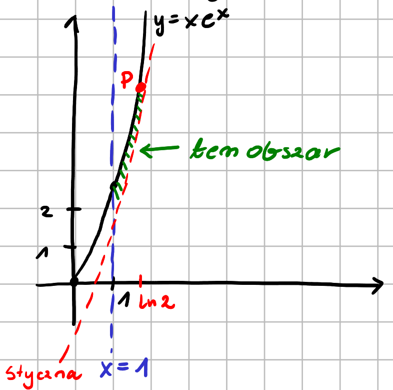
$$=\lim_{\alpha\to 1^-}\left[\frac{3}{4}\cdot\underbrace{\sqrt[3]{(\alpha^2-1)^2}}_{0}-\frac{3}{4}\right]+\lim_{\beta\to 1^+}\left[\cancel{\frac{3}{4}\cdot\sqrt[3]{9}}-\frac{3}{4}\cdot\underbrace{\sqrt[3]{(\beta^2-1)^2}}_{0}\right]+\lim_{\gamma\to\infty}\left[\frac{3}{4}\cdot\sqrt[3]{(\gamma^2-1)^2}-\cancel{\frac{3}{4}\cdot\sqrt[3]{9}}\right]=$$

$$=-\frac{3}{4}+\lim_{\gamma\to\infty}\frac{3}{4}\cdot\sqrt[3]{(\gamma^2-1)^2}=+\infty\quad \text{całka rozbieżna do }+\infty$$

Zadanie domowe nr 8

ZAD. 3 Pole obszaru

a) $x=1$, $y=xe^x$ i styczna do $y=xe^x$ w $P=(\ln 2, \ln 4)$



$x=\ln 2 \Rightarrow y=\ln 2\cdot e^{\ln 2}=2\ln 2=\ln 4$

OK P leży na krzywej

Równanie stycznej: $y=Ax+B$

$A=(xe^x)'\big|_{x=\ln 2}=e^x+xe^x\big|_{x=\ln 2}=2+\ln 4$

$B=?$ $\ln 4=(2+\ln 4)\cdot\ln 2+B \Rightarrow B=\cancel{\ln 4-2\ln 2}-\ln 4\cdot\ln 2=-\ln 2\cdot\ln 4$

styczna $y=(2+\ln 4)x-\ln 2\cdot\ln 4$

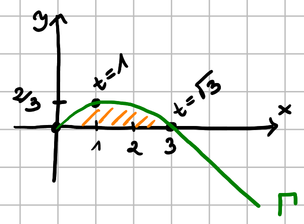
Pole obszaru $P=\int_1^{\ln 2}\left[xe^x-(2+\ln 4)x+\ln 2\cdot\ln 4\right]dx$

$\int \underset{f\ g'}{xe^x}\,dx=\underset{f\ g}{xe^x}-\int \underset{f'\ g}{1\cdot e^x}\,dx=xe^x-e^x+C=(x-1)e^x+C$

$P=(x-1)e^x-(1+\ln 2)x^2+\ln 2\cdot\ln 4\, x\,\Big|_1^{\ln 2}=(\ln 2-1)\cdot 2-(1+\ln 2)\ln^2 2+\ln^2 2\cdot\overset{2\ln 2}{\ln 4}-0+1+\ln 2-\ln 2\cdot\overset{2\ln 2}{\ln 4}$

$=2\ln 2-2-\ln^2 2-\ln^3 2+2\ln^3 2+1+\ln 2-2\ln^2 2=\ln^3 2-3\ln^2 2+3\ln 2-1=(\ln 2-1)^3$

b) Γ: $x=t^2$, $y=t-\frac{1}{3}t^3$; $t\geq 0$



$x=t^3\geq 0$ w prawo $\rightarrow$

$y=t-\frac{1}{3}t^3$ $y'=1-t^2>0 \iff |t|<1 \Rightarrow$ $t\in[0,1]$ $y(t)\nearrow$ w górę; $t>1$ $y(t)\searrow$ w dół

$t=0$ $P_0=(0,0)$

$t=1$ $P_1=(1, 2/3)$

Dla jakiego t krzywa przecina oś Ox ? $y=0 \iff t(1-\frac{t^2}{3})=0$

$t=0 \vee t^2=3 \underset{t\geq 0}{\Rightarrow} t=\sqrt{3}$

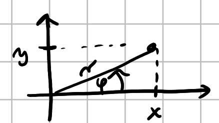
$t=\sqrt{3}$ $P_2=(3,0)$

$P=\int_0^{\sqrt{3}}|y(t)|\cdot|x'(t)|\,dt=\int_0^{\sqrt{3}}\left|t-\frac{1}{3}t^3\right|\cdot|2t|\,dt=\int_0^{\sqrt{3}}\left(t-\frac{1}{3}t^3\right)\cdot(2t)\,dt=2\int_0^{\sqrt{3}}\left(t^2-\frac{1}{3}t^4\right)dt=$

$=2\cdot\left[\frac{t^3}{3}-\frac{t^5}{15}\right]_0^{\sqrt{3}}=\frac{2}{3}\left[t^3-\frac{t^5}{5}\right]_0^{\sqrt{3}}=\frac{2}{3}\left[3\sqrt{3}-\frac{1}{5}\cdot 9\sqrt{3}\right]=2\sqrt{3}\cdot\left(1-\frac{3}{5}\right)=\frac{4}{5}\sqrt{3}$

c) $r=\sqrt{3}\sin\varphi$, $r=1+\cos\varphi$

Γ_1: $r=\sqrt{3}\sin\varphi$



$\sin\varphi=\frac{y}{r}$

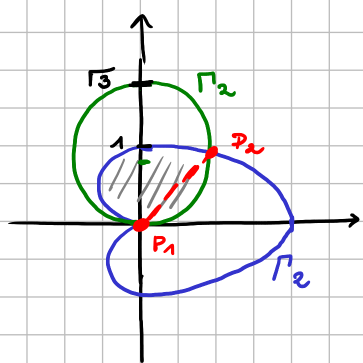
$r=\sqrt{x^2+y^2}$

$\sin\varphi=\frac{r}{\sqrt{3}}=\frac{y}{r} \Rightarrow y=\frac{r^2}{\sqrt{3}}$, $\sqrt{3}y=r^2$

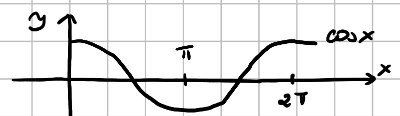
$x^2+y^2-\sqrt{3}y=0$

okrąg $x^2+(y-\frac{\sqrt{3}}{2})^2=\frac{3}{4}$

o środku $S=(0,\frac{\sqrt{3}}{2})$ i promieniu $R=\frac{3}{4}$



Γ_2: $r=1+\cos\varphi$



$\varphi=0$ $r=2$

$\varphi=\frac{\pi}{2}$ $r=1$

$\varphi=\pi$ $r=0$ kardioida

$P_1=?$ $P_2=?$

$\sqrt{3}\sin\varphi=1+\cos\varphi$ $\iff$ $\sqrt{3}\cdot 2\sin\frac{\varphi}{2}\cos\frac{\varphi}{2}=2\cos^2\frac{\varphi}{2}$ $/:2$ $\iff \cos\frac{\varphi}{2}\left[\cos\frac{\varphi}{2}-\sqrt{3}\sin\frac{\varphi}{2}\right]=0$

$\varphi\in[0,\pi]$

$\cos\frac{\varphi}{2}=0$ $\vee$ $\operatorname{ctg}\frac{\varphi}{2}=\sqrt{3}$

$\frac{\varphi}{2}=\frac{\pi}{2}$ $\quad$ $\frac{\varphi}{2}=\frac{\pi}{6}$

$\varphi=\pi$ $\quad$ $\varphi=\frac{\pi}{3}$

$P_1=(0,0)$

Pole $P=\frac{1}{2}\int_0^{\pi/3}3\sin^2\varphi\,d\varphi+\frac{1}{2}\int_{\pi/3}^{\pi}(1+\cos\varphi)^2d\varphi=\frac{3}{2}\int_0^{\pi/3}\frac{1-\cos 2\varphi}{2}\,d\varphi+\frac{1}{2}\int_{\pi/3}^{\pi}(1+2\cos\varphi+\underset{}{\overset{\frac{1+\cos 2\varphi}{2}}{\cos^2\varphi}})\,d\varphi=$

$=\frac{3}{4}\left[\varphi-\frac{1}{2}\sin 2\varphi\right]_0^{\pi/3}+\frac{1}{2}\left[\varphi+2\sin\varphi+\frac{\varphi}{2}+\frac{1}{4}\sin 2\varphi\right]_{\frac{\pi}{3}}^{\pi}=\ldots=\frac{3}{4}(\pi-\sqrt{3})$

Zadanie domowe nr 8

ZAD. 4 Długość łuku krzywej

$\Gamma: \quad x(t) = 5\cos t\,(1+\cos t)\,; \quad y(t) = 5\sin t\,(1+\cos t)\,, \quad t\in[0,2\pi]$

$$L = |\Gamma| = \int_0^{2\pi} \sqrt{(x'(t))^2 + (y'(t))^2}\,dt$$

$\sin 2\alpha = 2\sin\alpha\cos\alpha$
$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha =$
$= 2\cos^2\alpha - 1$

$x'(t) = -5\sin t(1+\cos t) + 5\cos t\cdot(-\sin t) = -5\sin t - 10\sin t\cos t = -5\sin t - 5\sin 2t$

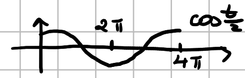
$y'(t) = 5\cos t(1+\cos t) + 5\sin t(-\sin t) = 5\cos t + 5\cos^2 t - 5\sin^2 t = 5\cos t + 5\cos 2t$

$(x')^2+(y')^2 = 25\sin^2 t + 50\sin t\sin 2t + 25\sin^2 2t + 25\cos^2 t + 50\cos t\cos 2t + 25\cos^2 2t = 50 + 50[\sin t\sin 2t + \cos t\cos 2t]$

$= 50+50[2\sin^2 t\cos t + \cos^3 t - \cos t\sin^2 t] = 50 + 50\cos t\cdot[\underbrace{2\sin^2 t + \cos^2 t - \sin^2 t}_{1}] = 50 + 50\cos t = 50(1+\cos t)$

$L = \int_0^{2\pi}\sqrt{50(1+\cos t)}\,dt = 5\sqrt2\int_0^{2\pi}\sqrt{1+\cos t}\,dt = 5\sqrt2\int_0^{2\pi}\sqrt{2\cos^2\frac t2}\,dt = 10\int_0^{2\pi}\left|\cos\frac t2\right|dt =$

okres $f(t) = \cos\frac t2$
wynosi $T = 4\pi$

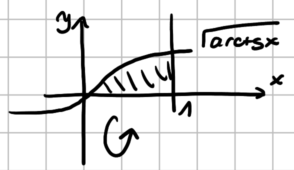


$= 10\int_0^{\pi}\cos\frac t2\,dt - 10\int_{\pi}^{2\pi}\cos\frac t2\,dt =$

$= \left[20\sin\frac t2\right]_0^{\pi} - \left[20\sin\frac t2\right]_{\pi}^{2\pi} = [20\cdot1 - 20\cdot0] - [20\cdot0 - 20\cdot1] = 20+20 = \underline{\underline{40}}$

ZAD. 5 Objętość bryły obrotowej: $y = \sqrt{\operatorname{arcctg} x}\ ;\ y = 0\ ;\ x = 1$ obrót wokół Ox

$$V = \pi\int_a^b y^2\,dx$$



$V = \pi\int_0^1 \operatorname{arcctg} x\,dx = \pi\left[x\operatorname{arcctg}x\Big|_0^1 - \int_0^1\frac{x}{1+x^2}\,dx\right] =$

$= \pi\left[x\operatorname{arcctg}x - \frac12\ln(1+x^2)\right]_0^1 = \pi\left[\frac{\pi}{4} - \frac12\ln2\right]$

ZAD. 6

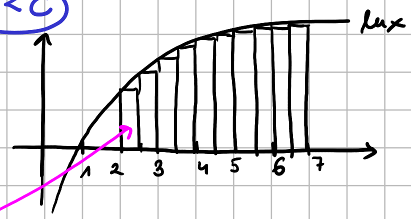
$A = \lim_{h\to0}\frac{\ln(2+h)-\ln2}{h}$ $\quad B = \ln\frac26$ $\quad C = 0$ $\quad D = \ln3+\ln4+\ln5+\ln6$

$E = \ln2+\ln3+\ln4+\ln5$ $\quad F = \sum_{k=0}^{7}\frac{\ln\left(2+\frac k2\right)}{2}$ $\quad G = \int_2^6\ln x\,dx$

$A = (\ln x)'|_{x=2} = \frac12$ $\quad A > C$

$B = \ln\frac13 < \ln1 = 0 = C$ $\quad B < C$

$F = \sum_{k=0}^{7}\frac12\cdot\ln\left(2+\frac k2\right) \le \int_2^6\ln x\,dx = G$ $\quad F < G$



$G = \int_2^6\ln x\,dx =$
$= [x\ln x - x]_2^6 =$
$= 6\ln6 - 6 - 2\ln2 + 2$
$= 6\ln6 - \ln4 - 4$

$\|$

$\frac12(\underbrace{\ln2 + \ln\tfrac52}_{\vee\ 2\ln2} + \underbrace{\ln3 + \ln\tfrac72}_{\vee\ 2\ln3} + \underbrace{\ln4 + \ln\tfrac92}_{\vee\ 2\ln4} + \underbrace{\ln5 + \ln\tfrac{11}{2}}_{\vee\ 2\ln5})$

$\vee$

$E = \ln2+\ln3+\ln4+\ln5$ $\quad E < F$

$D = \ln3+\ln4+\ln5+\ln6 > \int_2^6\ln x\,dx = G$

$G < D$

$\ln 2 > 1 \Rightarrow E > A$

$B < C < A < E < F < G < D$

ZAD. 7 a) $\left(\frac12\operatorname{arctg}\frac{2x}{1-x^2}\right)' = \frac12\cdot\frac{1}{1+\frac{4x^2}{(1-x^2)^2}}\cdot\frac{2(1-x^2)-2x(-2x)}{(1-x^2)^2} = \frac12\,\frac{\cancel{(1-x^2)^2}}{(1-x^2)^2+4x^2}\cdot\frac{2-2x^2+4x^2}{\cancel{(1-x^2)^2}} =$

$= \frac{1}{\cancel{2}}\cdot\frac{\cancel{2}+\cancel{2}x^2}{1-2x^2+x^4+4x^2} = \frac{1+x^2}{1+2x^2+x^4} = \frac{1+x^2}{(1+x^2)^2} = \frac{1}{1+x^2}$

b) $f(x) = \frac{1}{1+x^2}$ $\quad F(x) = \frac12\cdot\operatorname{arctg}\frac{2x}{1-x^2}$ to pierwotna f na PEWNYM przedziale

Nie jest prawdą, że $\forall x\in[0,\sqrt3]\quad F'(x) = f(x)$

Równość ta nie zachodzi dla $x = 1$ $\quad D_f = \mathbb{R}$ $\quad D_F = \mathbb{R}\setminus\{-1,1\}$

Stosując tw. Newtona-Leibniza nie można posłużyć się F.

Zadanie domowe nr 8

ZAD. 8 $f \in C^\infty(\mathbb{R})$ $\quad f(1)=f(2)=2 \quad f'(1)=6 \quad f''(1)=3$

$f'(2)=5 \quad f''(2)=5$

$$\int_1^2 \underbrace{f(x)}_{u}\underbrace{f''(x)}_{v'}\,dx \underset{\substack{\text{przez}\\ \text{części}}}{=} \underbrace{f(x)}_{u}\cdot \underbrace{f'(x)}_{v}\Big|_1^2 - \int_1^2 \underbrace{[f'(x)]^2}_{u'\,v}\,dx = f(2)f'(2)-f(1)f'(1) - \int_1^2 [f'(x)]^2\,dx =$$

$$= 2 - \underbrace{\int_1^2 \underbrace{[f'(x)]^2}_{\geq 0}\,dx}_{\geq 0} \leq 2 \quad \text{cbdo}$$

ZAD. 9

$$\lim_{x\to 1}\frac{\int_1^x t^t\,dt}{x^{x+1}-1} \overset{[\frac{0}{0}]}{=} \lim_{x\to 1}\frac{\int_1^x t^t\,dt}{e^{(x+1)\ln x}-1} \underset{H}{=} \lim_{x\to 1}\frac{x^x}{e^{(x+1)\ln x}\cdot\left(\ln x+\frac{x+1}{x}\right)} = \lim_{x\to 1}\frac{x^x}{x^{x+1}\left(\ln x+1+\frac{1}{x}\right)}$$

$$\left(\int_1^x t^t\,dt\right)'_x = x^x \quad \text{Zasadnicze tw. rachunku całkowego}$$

$$= \lim_{x\to 1}\frac{1}{\underbrace{x}_{0}\underbrace{\ln x}_{0}+x+1} = \frac{1}{2}$$

ZAD. 10 $I=\int_0^\infty\left(\frac{ax^3}{x^4+2}-\frac{1}{8x+5}\right)dx$ $\quad$ Dla jakich $a\in\mathbb{R}$ jest zbieżna?

$8x+5\neq 0\ ;\ x\neq -\frac{5}{8}\notin[0,\infty)$ OK

$$I=\lim_{T\to\infty}\int_0^T\left(\frac{ax^3}{x^4+2}-\frac{1}{8x+5}\right)dx$$

$$\int\frac{ax^3}{x^4+2}\,dx-\int\frac{dx}{8x+5} = \frac{a}{4}\cdot\int\frac{4x^3}{x^4+2}\,dx-\int\frac{dx}{8x+5} = \frac{a}{4}\ln(x^4+2)-\frac{1}{8}\ln|8x+5|+C = \frac{2a}{8}\ln(x^4+2)-\frac{1}{8}\ln|8x+5|+C$$

$$= \frac{1}{8}\ln\left|\frac{(x^4+2)^{2a}}{8x+5}\right|+C$$

wartość

$$I=\lim_{T\to\infty}\frac{1}{8}\left(\ln\left|\frac{(T^4+2)^{2a}}{8T+5}\right|-\ln\frac{2^{2a}}{5}\right)$$

dla a $2a=\frac{1}{4}$: $\lim_{T\to\infty}\ln\frac{\sqrt[4]{T^4+2}}{8T+5}=\ln\frac{1}{8}$ $\quad|\quad$ $2a<\frac{1}{4}$: $-\infty$ $\quad|\quad$ $2a>\frac{1}{4}$: $+\infty$

Odp. $a=\frac{1}{8}$

$$I=\frac{1}{8}\left(\ln\frac{1}{8}-\ln\frac{\sqrt[4]{2}}{5}\right)$$