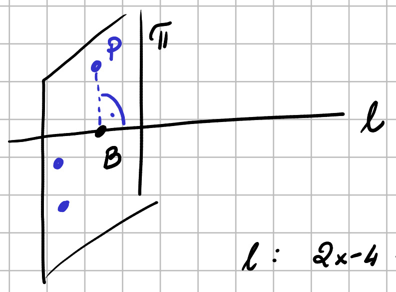
ALGEBRA - Zadanie domowe nr 3
Geometria analityczna

Zad. 1 Punkty, których rzutem prostokątnym na $l: 2x-4 = 2-y = 2z+4$ jest $B=(3,0,-1)$.



$B \in l$, istotnie

$2\cdot 3 - 4 = 2 \qquad 2-0=2 \qquad 2\cdot(-1)+4=2$

$\Pi \perp l$, $B \in \Pi$ $\qquad \forall P \in \Pi$ rzut P na l to B

$l: 2x-4 = 2-y = 2z+4$

$l: \frac{x-2}{\frac{1}{2}} = \frac{y-2}{-1} = \frac{z-(-2)}{\frac{1}{2}} \qquad \vec{a}=[1,-2,1] \parallel l \Rightarrow \vec{a} \perp \Pi$

$\vec{a} \perp \Pi \wedge B \in \Pi \qquad \Pi: 1\cdot(x-3) - 2(y-0) + 1\cdot(z+1) = 0$

$\Pi: x - 2y + z - 2 = 0$

Zad. 2

a) $\Pi: \begin{cases} x = t \\ y = t-s \\ z = -1+2t+s \end{cases} ; t,s \in \mathbb{R}$ $\qquad$ Znaleźć równanie ogólne Π.

$t=0, s=0 \Rightarrow P_0=(0,0,-1) \in \Pi \qquad \vec{a}=[1,1,2] \parallel \Pi \qquad \vec{b}=[0,-1,1] \parallel \Pi$

$\vec{n} := \vec{a} \times \vec{b} \qquad \vec{n} \perp \Pi \qquad \vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 2 \\ 0 & -1 & 1 \end{vmatrix} = [3,-1,-1] \qquad \Pi: 3(x-0) - 1\cdot(y-0) - 1(z+1) = 0$

$\Pi: 3x - y - z - 1 = 0$

b) $\Pi: 3x-y-z-1=0$ $\qquad$ Znaleźć parametryzację Π.

Niech $y=\alpha$, $z=\beta$. Wówczas $3x-\alpha-\beta-1=0$, $x=\frac{1}{3}(\alpha+\beta+1)$

$\Pi: \begin{cases} x = \frac{1}{3}(\alpha+\beta+1) \\ y = \alpha \\ z = \beta \end{cases} ; \alpha,\beta \in \mathbb{R} \leftarrow$ Inna parametryzacja niż w podpunkcie a)

Zad. 3 $\quad l_1: x+4 = 2y+2 = z \qquad l_2: \begin{cases} x = -2s \\ y = -13+6s \\ z = 0 \end{cases} ; s \in \mathbb{R}$

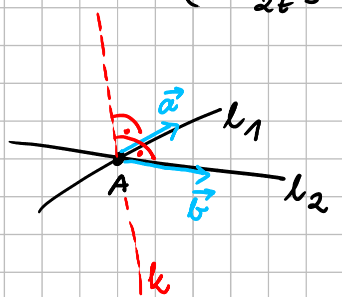
$\{A\} = l_1 \cap l_2$

$k = ?$ t. że $k \perp l_1 \wedge k \perp l_2 \wedge A \in k$

$l_1: \frac{x-(-4)}{1} = \frac{y-(-1)}{\frac{1}{2}} = \frac{z-0}{1} \qquad \vec{a} = [2,1,2] \parallel l_1 \qquad l_1: \begin{cases} x = -4+2t \\ y = -1+t \\ z = 2t \end{cases} ; t \in \mathbb{R}$

$\vec{b} = [-2,6,0] \parallel l_2$

$A = ? \quad \begin{cases} -4+2t = -2s \\ -1+t = -13+6s \\ 2t = 0 \rightarrow t=0 \end{cases} \quad \begin{cases} -4=-2s \rightarrow s=2 \\ -1=-13+6s \end{cases}$ $\Downarrow$ spełnia $\qquad A = (-4,-1,0)$



$\vec{n} := \vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & 2 \\ -2 & 6 & 0 \end{vmatrix} = [-12,-4,14] \parallel k$

$A \in k \Rightarrow k: \begin{cases} x = -4-12t \\ y = -1-4t \\ z = 14t \end{cases} ; t \in \mathbb{R}$

Algebra – Zad. domowe nr. 3 – Geometria analityczna

Zad. 4

$l_1: \dfrac{x-9}{8} = \dfrac{y-5}{3} = \dfrac{z-2}{1}$ $\qquad l_2: \dfrac{3x+9}{4} = y+1 = \dfrac{24-3z}{7}$

$l_2: \dfrac{x+3}{\frac{4}{3}} = \dfrac{y+1}{1} = \dfrac{z-8}{-7/3}$

a) $l_1 \cap l_2 = \{P_0\}$ $\quad P_0 = ?$

$$l_1: \begin{cases} x = 9+8t \\ y = 5+3t \\ z = 2+t \end{cases} \quad t \in \mathbb{R} \qquad l_2: \begin{cases} x = -3+\frac{4}{3}s \\ y = -1+s \\ z = 8-\frac{7}{3}s \end{cases}$$

$$\begin{cases} 9+8t = -3+\frac{4}{3}s \\ 5+3t = -1+s \\ 2+t = 8-\frac{7}{3}s \end{cases}$$

$s = 6+3t$

$2+t = 8-\frac{7}{3}(6+3t) =$

$= 8-14-7t$

$8t = -8 \quad , t = -1$

$s = 3$

$9+8\cdot(-1) = 1 = -3+\frac{4}{3}\cdot 3$ ok.

$P_0 = (1, 2, 1)$

b) $\pi = ?$ $\quad l_1, l_2 \subset \pi$

$\vec{a} = [8,3,1] \parallel l_1, \quad \vec{b} = [\frac{4}{3}, 1, -\frac{7}{3}] \parallel l_2$

równ. param.

$$\pi: \begin{cases} x = 9 + 8t + \frac{4}{3}s \\ y = 5 + 3t + s \\ z = 2 + t - \frac{7}{3}s \end{cases}$$

$\vec{n} = \vec{a} \times \vec{b} \perp \pi$

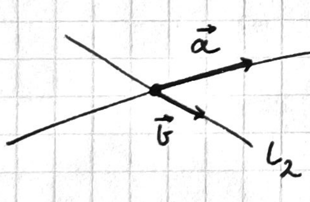
$$\vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 8 & 3 & 1 \\ \frac{4}{3} & 1 & -\frac{7}{3} \end{vmatrix} = [-8, 20, 4]$$

$\pi: \quad -8(x-9)+20(y-5)+4(z-2)=0$

$-2(x-9)+5(y-5)+z-2=0$

$-2x+5y+z-9=0$

c) l_4, l_3 dwusieczne kątów między l_1 i l_2



$\vec{v}_3 = \dfrac{\vec{a}}{|\vec{a}|} + \dfrac{\vec{b}}{|\vec{b}|} \qquad \vec{v}_4 = \dfrac{\vec{a}}{|\vec{a}|} - \dfrac{\vec{b}}{|\vec{b}|}$

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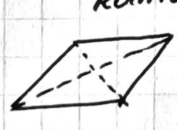


dwusieczne kątów to przekątne

$|\vec{a}| = \sqrt{64+9+1} = \sqrt{74}$

$|\vec{b}| = \sqrt{\frac{16}{9}+1+\frac{49}{9}} = \frac{1}{3}\sqrt{16+9+49}$

$= \dfrac{\sqrt{74}}{3}$

$\vec{v}_3 = \dfrac{1}{\sqrt{74}}\left([8,3,1]+3[\frac{4}{3}, 1, -7]\right) =$

$= \dfrac{1}{\sqrt{74}} \cdot [12, 6, -20]$

$\vec{v}_4 = \dfrac{1}{\sqrt{74}}[(8,3,1)-(4,3,-21)] = \dfrac{1}{\sqrt{74}}[4,0,22]$

$P_0 \in l_3, \ P_0 \in l_4$

$$l_3: \begin{cases} x = 1 + 12t \\ y = 2 + 6t \\ z = 1 - 20t \end{cases} \quad t \in \mathbb{R} \qquad l_4: \begin{cases} x = 1 + 4s \\ y = 2 \\ z = 1 + 22s \end{cases} \quad s \in \mathbb{R}$$

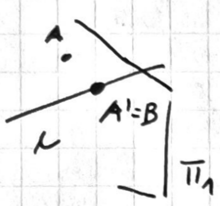
Zad. 5 $P = P_{\Delta ABC} = ?$ $\quad A=(1,2,3)$ $\quad l:\ 3x-3 = 6-y = 2z-8$

$\Pi:\ -3x+2y-2z-29=0$

B – rzut prostokątny A na l

C – p. symetryczny do A wzgl. Π

$$l:\ \frac{x-1}{\frac{1}{3}} = \frac{y-6}{-1} = \frac{z-4}{\frac{1}{2}} \qquad \vec{v} = [\tfrac{1}{3}, -1, \tfrac{1}{2}] \parallel l \qquad l_1: \begin{cases} x = 1 + \frac{1}{3}t \\ y = 6 - t \\ z = 4 + \frac{1}{2}t \end{cases}$$



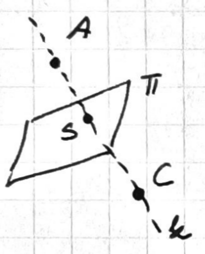
$\Pi_1 \ni A$ $\quad \vec{v} \perp \Pi_1$ $\quad \Pi_1:\ \frac{1}{3}(x-1) - (y-2) + \frac{1}{2}(z-3) = 0$

$\Pi_1 \perp l$

$\{B\} = \Pi_1 \cap l$

$$2(x-1) - 6(y-2) + 3(z-3) = 0$$
$$2\cdot\tfrac{1}{3}t - 6(4-t) + 3(1+\tfrac{1}{2}t) = 0$$
$$\tfrac{2}{3}t - 24 + 6t + 3 + \tfrac{3}{2}t = 0$$
$$\frac{4+36+9}{6}t = 21 \qquad t = \cancel{21}^{3}\cdot\frac{6}{\cancel{49}_{7}} = \frac{18}{7}$$

$$B = \left(1+\frac{6}{7}, \frac{42-18}{7}, 4+\frac{9}{7}\right) = \left(\frac{13}{7}, \frac{24}{7}, \frac{37}{7}\right)$$



$k \perp \Pi$

$A \in k$

$\vec{m} = [-3,2,-2] \perp \Pi$

$\vec{m} \parallel k$

$$k: \begin{cases} x = 1 - 3t \\ y = 2 + 2t \\ z = 3 - 2t \end{cases} ;\ t \in \mathbb{R}$$

$\{S\} = \Pi \cap k$

$$-3(1-3t) + 2(2+2t) - 2(3-2t) - 29 = 0$$
$$17t = 34$$
$$t = 2$$
$$S = (-5, 6, -1)$$

$C = T_{\vec{AS}}(S)$ $\quad \vec{AS} = [-6, 4, -4]$ $\quad C = (-5-6, 6+4, -1-4)$

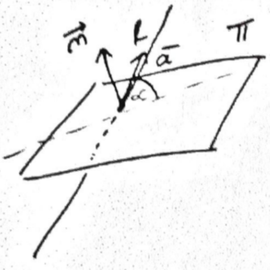
$C = (-11, 10, -5)$

$P_\Delta = \frac{1}{2}|\vec{AB} \times \vec{AC}|$ $\quad \vec{AB} = [\frac{6}{7}, \frac{10}{7}, \frac{16}{7}]$ $\quad \vec{AC} = [-12, 8, -8]$

$$\vec{AB} \times \vec{AC} = \frac{1}{7}\begin{vmatrix} i & j & k \\ 6 & 10 & 16 \\ -12 & 8 & -8 \end{vmatrix} = \frac{1}{7}[-208, -144, 168]$$

$$P_\Delta = \frac{1}{2}\cdot\sqrt{\frac{208^2+144^2+168^2}{7^2}} = \frac{1}{14}\sqrt{208^2+144^2+168^2}$$

Zad. 6 Oblicz miarę kąta między płaszczyzną $\Pi:\ x-z=0$ a prostą $l: \begin{cases} x = 3+2t \\ y = 1 \\ z = -2-3t \end{cases} ;\ t \in \mathbb{R}.$



$\vec{m} = (1,0,-1) \perp \Pi$

$\vec{a} = (2,0,-3) \parallel l$

$\alpha = \sphericalangle(\Pi, l) = \frac{\pi}{2} - \overset{\beta}{\overset{\parallel}{\sphericalangle(\vec{m},\vec{a})}}$ $\qquad \alpha + \beta = \pi/2$

$$\cos\beta = \cos(\tfrac{\pi}{2} - \alpha) = \sin\alpha = \frac{|\vec{m}\circ\vec{a}|}{|\vec{m}|\cdot|\vec{a}|} = \frac{|2+0+3|}{\sqrt{2}\cdot\sqrt{4+9}} = \frac{5}{\sqrt{26}}$$

$$\alpha = \arcsin\frac{5}{\sqrt{26}}$$

Zad 6

$k: \begin{cases} x = t \\ y = 1 - t \\ z = a \cdot t \end{cases} ; t \in \mathbb{R}$ $\quad l: \begin{cases} x = -s \\ y = 1 + (1+a)s \\ z = 2 - 2as \end{cases} ; s \in \mathbb{R}$

$\vec{a} = [1, -1, a]$

$\vec{b} = [-1, 1+a, -2a)$

$A = (0, 1, 0) \quad B = (0, 1, 2)$

$\vec{AB} = [0, 0, 2]$

$[\vec{a}, \vec{b}, \vec{AB}] = 2a$

$\begin{bmatrix} 1 & -1 & a \\ -1 & 1+a & -2a \\ 0 & 0 & 2 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & -1 & a \\ 0 & a & -a \\ 0 & 0 & 2 \end{bmatrix}$

$a \neq 0 \Rightarrow$ rząd 3

P a) $a = ?$ by $k \parallel l$ $\quad -\vec{a} = [-1, 1, -a]$

$a = 0 \quad [-1, 1, 0]$

$[-1, 1, 0]$

P b) k i l skośne $\Rightarrow a = 0$ $\quad \cancel{2a \neq 0} \quad 2a = 0$ skośne

P c) $\forall a < 2 \quad k \cap l \neq \emptyset$