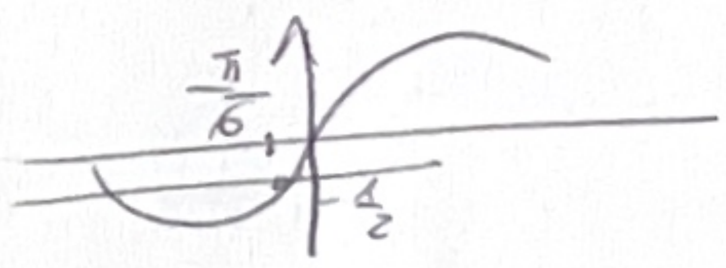
ANALIZA Kartkówka 1 – gr. 2 28. X. 2025r.

Zad.1
a) $tg(-arctg 5) = -tg(arctg 5) = -5$

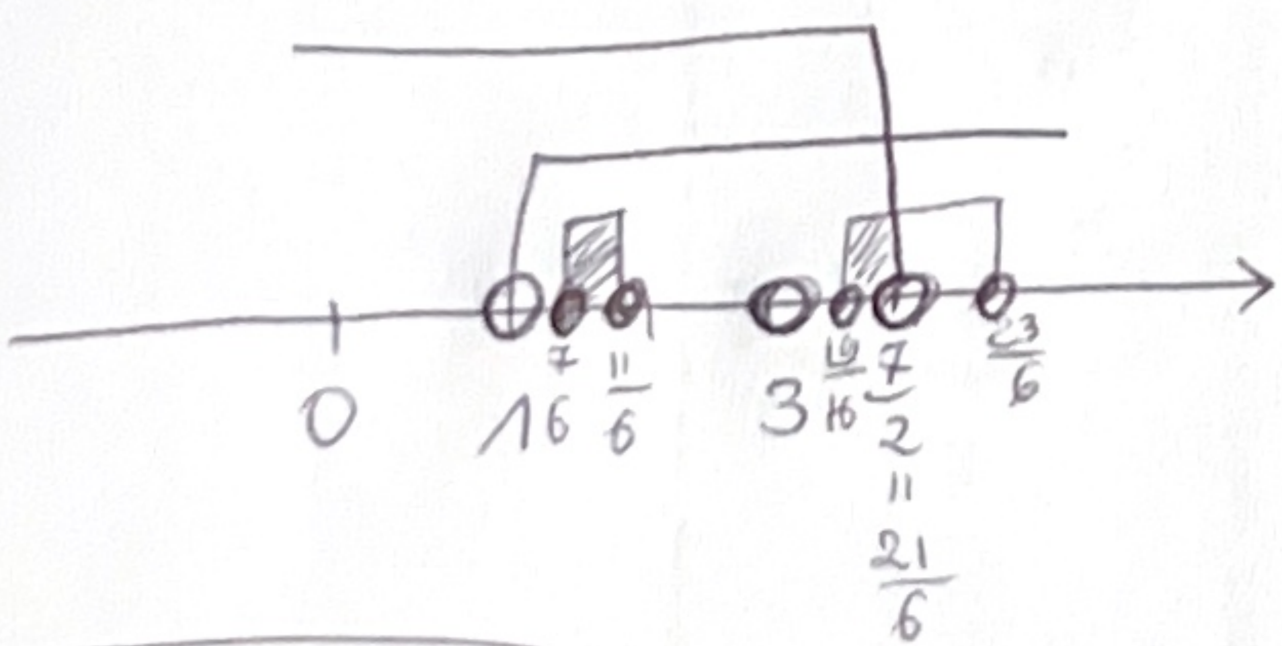
b) $\arccos\left(\sin\left(-\frac{11}{4}\pi\right)\right) = \arccos\left(\sin\frac{5}{4}\pi\right) = \arccos\left(\sin\left(\frac{\pi}{2}+\frac{3\pi}{4}\right)\right) =$
$= \arccos\left(\cos\frac{3}{4}\pi\right) = \frac{3}{4}\pi$

Zad.2
$$f(x) = \frac{\ln(\operatorname{arcctg}(x-4))}{\sin(2\operatorname{arctg}(x-3))} \cdot \frac{\arccos\frac{2x^2-8}{6x^2}}{\sqrt{x-1}} + \frac{\log\left(\frac{7}{2}-x\right)}{\sqrt{|\sin\pi x| - \sin\pi x - 1}}$$

- $\operatorname{arcctg}(x-4) > 0$ zawsze
- $\sin(2\operatorname{arctg}(x-3)) \neq 0$, $2\operatorname{arctg}(x-3) \neq k\pi$
  $\operatorname{arctg}(x-3) \neq k\frac{\pi}{2}$ $k \in \mathbb{Z}$
  $\frac{\pi}{2} < \operatorname{arctg}(x-3) < \frac{\pi}{2} \Rightarrow \operatorname{arctg}(x-3) \neq 0 \Rightarrow x-3 \neq 0$, $\underline{x \neq 3}$
- $-1 \leq \frac{2x^2-8}{6x^2} \leq 1$, $-1 \leq \frac{x^2-4}{3x^2} \leq 1$, $-3x^2 \leq x^2-4 \leq 3x^2$
- $x \neq 0$
  $4 \leq 4x^2 \wedge -4 \leq 2x^2$
  $1 \leq x^2$ zawsze
  $\boxed{1 \leq |x|}$
  $x \geq 1 \vee x \leq -1$
- $x-1 > 0$, $\underline{x > 1}$
- $\frac{7}{2} - x > 0$, $x < \frac{7}{2}$



- $|\sin\pi x| - \sin\pi x - 1 > 0$
  $|\sin\pi x| > \sin\pi x + 1$
  $\sin\pi x > 0 \Rightarrow \sin\pi x > \sin\pi x + 1$ sprzeczne
  $\sin\pi x < 0 \wedge -\sin\pi x > \sin\pi x + 1$
  $-\pi + 2k\pi < \pi x < 2k\pi \wedge 2\sin\pi x < -1$
  $\sin\pi x < -\frac{1}{2}$
  zawarte
  $-\frac{5}{6}\pi + 2k\pi < \pi x < -\frac{\pi}{6} + 2k\pi$
  $-\frac{5}{6} + 2k < x < -\frac{1}{6} + 2k$ $k \in \mathbb{Z}$



$k=1$ $\left(\frac{7}{6}, \frac{11}{6}\right)$
$k=2$ $\left(\frac{19}{6}, \frac{23}{6}\right)$

Odp $\left(\frac{7}{6}, \frac{11}{6}\right) \cup \left(\frac{19}{6}, \frac{7}{2}\right)$

Zad.3
a) $\lim_{n\to\infty} (n+1)[\ln(n+1) - \ln(n+2)] = \lim_{n\to\infty} (n+1)\ln\frac{n+1}{n+2} = \lim_{n\to\infty} \ln\left[\left(1+\frac{-1}{n+2}\right)^{\frac{n+2}{-1}}\right]^{\frac{n+1}{-(n+2)}}$
$= \ln e^{-1} = -1$

b) $\lim_{n\to\infty} (-1)^n \sqrt{n}\left(\sqrt{n+1} - \sqrt{n}\right) = \lim_{n\to\infty} (-1)^n\sqrt{n}\,\frac{n+1-n}{\sqrt{n+1}+\sqrt{n}} = \lim_{n\to\infty} (-1)^n \frac{\sqrt{n}}{\sqrt{n+1}+\sqrt{n}}$

NIE ISTNIEJE

$n = 2k$ $\lim_{k\to\infty} \frac{\sqrt{2k}}{\sqrt{2k+1}+\sqrt{2k}} = \frac{1}{2}$ $n = 2k+1$ $\lim_{k\to\infty} (-1)\frac{\sqrt{2k+1}}{\sqrt{2k+2}+\sqrt{2k+1}}$
$= -\frac{1}{2}$

c) $\lim_{x\to 0}\left[\frac{1-2^{-5x}}{5^x-1} + \operatorname{arcctg}\left(-\frac{1}{x}\right)\cdot\operatorname{arctg}\frac{x}{5}\right]$

$= \lim_{x\to 0}\left[-1\cdot\underbrace{\frac{2^{-5x}-1}{-5x}}_{\to\ln 2}\cdot\underbrace{\frac{\cancel{x}}{5^x-1}}_{\to\frac{1}{\ln 5}}\cdot(-5) + \overbrace{\operatorname{arcctg}\left(-\frac{1}{x}\right)}^{\text{ogr}}\cdot\operatorname{arctg}\frac{x}{5}\right] = 5\frac{\ln 2}{\ln 5}$

Zad. 4

$$f(x)=\begin{cases} 2x+\dfrac{\operatorname{arctg}x}{3x-\frac{6}{x+1}} & ; x\in\mathbb{R}\setminus\{-2,-1,1\} \\ -2 & ; x\in\{-2,-1,1\}\end{cases}$$

W każdym z przedziałów $(-\infty,-2)$, $(-2,-1)$, $(-1,1)$, $(1,\infty)$

f ciągła jako f. elementarna

$$3x-\frac{6}{x+1}=0$$
$$\frac{3x(x+1)-6}{x+1}=0$$
$$3x^2+3x-6=0$$
$$x^2+x-2=0$$
$$(x+2)(x-1)=0$$
$$x=-2 \vee x=1$$

• $f(-2)=-2$

$$\lim_{x\to-2^+} f(x)=\lim_{x\to-2^+}\left[2x+\frac{\overset{-1}{(x+1)}\cdot \overset{\alpha<0}{\operatorname{arctg}x}}{\underset{-6}{3(x-1)}\underset{0^+}{(x+2)}}\right]=-\infty$$

$$\lim_{x\to-2^-} f(x)=+\infty$$

f nie jest ciągła ani jednostr. ciągła w $x=-2$

• $f(-1)=-2$

$$\lim_{x\to-1} f(x)=\lim_{x\to-1}\left[\underset{-2}{2x}+\frac{\overset{0}{(x+1)}\cdot \overset{-\frac{\pi}{4}}{\operatorname{arctg}x}}{\underset{3}{3}\underset{-2}{(x-1)}\underset{1}{(x+2)}}\right]=-2+0=-2$$

f ciągła w $x=-1$

• $f(1)=-2$

$$\lim_{x\to1^+} f(x)=\lim_{x\to1^+}\left[2x+\frac{\overset{2}{(x+1)}\overset{\frac{\pi}{2}}{\operatorname{arctg}x}}{\underset{3}{3}\underset{0^+}{(x-1)}\underset{2}{(x+2)}}\right]=+\infty$$

$$\lim_{x\to1^-} f(x)=-\infty$$

f nie jest ciągła ani jednostronnie ciągła w $x=1$