ALGEBRA - Zadanie domowe nr 1

① $(\mathbb{Z}, 0)$, $x \circ y = x + (-1)^x y$ czy łączne? TAK

a) Niech $x, y, z \in \mathbb{Z}$ dowolne. $L = (x \circ y) \circ z = [x + (-1)^x y] \circ z = x + (-1)^x y + (-1)^{x + (-1)^x y} z$

$\overset{?}{=} P = x \circ (y \circ z) = x \circ [y + (-1)^y z] = x + (-1)^x [y + (-1)^y z] = x + (-1)^x y + (-1)^{x+y} z$

$L = P$ bowiem $(-1)^{x + (-1)^x y} = (-1)^{x - y + 2y} = (-1)^{x-y}$

b) $(\mathbb{R} \setminus \{-1\}, 0)$ czy (półgrupa)? $x \circ y = x + y + xy$

"0" wewnętrzne $x, y \in \mathbb{R} \setminus \{-1\}$ tj. $x \neq -1, y \neq -1 \Rightarrow x \circ y = x + y + xy = x(1+y) + y + 1 - 1 = (x+1)(y+1) - 1 \neq -1 \Rightarrow x \circ y \in \mathbb{R} \setminus \{-1\}$

przemienne $x \circ y = x + y + xy = y + x + yx$ dla dowolnych $x, y \neq -1$

łączne Niech $x, y, z \in \mathbb{R} \setminus \{-1\}$ dowolne $L = (x \circ y) \circ z = (x + y + xy) \circ z = x + y + xy + z + xz + yz + xyz$

$P = x \circ (y \circ z) = x \circ (y + z + yz) = x + y + z + yz + xy + xz + xyz$ $L = P$

el. neutralny? $x \circ e = e \circ x$ bo działanie przemienne

$x \circ e = x \iff x + e + xe = x \iff e + xe = e(1+x) = 0 \overset{x \neq -1}{\Rightarrow} e = 0 \in \mathbb{R} \setminus \{-1\}$ istnieje

el. symetryczny do $x \in \mathbb{R} \setminus \{-1\}$ $x \circ x' = x' \circ x$ bo działanie przemienne

$x \circ x' = e \iff x + x' + xx' = 0 \iff x'(1+x) = -x$, $x' = \frac{-x}{1+x}$ ← określone $\forall x \neq -1$

Ponadto $x' \in \mathbb{R} \setminus \{-1\}$ bo $\frac{-x}{1+x} = -1 \iff -x = -1 - x$, $0 = -1$ sprzeczne

Grupa abelowa

c) $(\mathbb{R} \setminus \{0\}, 0)$, $x \circ y = x \cdot |y|$ Czy (półgrupa)?

"0" wewnętrzne $x, y \neq 0 \Rightarrow x \circ y = x \cdot |y| \neq 0 \Rightarrow x \circ y \in \mathbb{R} \setminus \{0\}$

nie jest przemienne $2 \circ (-1) = 2 \cdot |-1| = 2 \neq (-1) \circ 2 = -1 \cdot |2| = -2$

łączne Niech $x, y, z \in \mathbb{R} \setminus \{0\}$ dowolne $L = (x \circ y) \circ z = (x \cdot |y|) \circ z = x \cdot |y| \cdot |z|$

$P = x \circ (y \circ z) = x \circ (y \cdot |z|) = x \cdot |y \cdot |z|| = x \cdot |y| \cdot |z|$ $L = P$

el. neutralny? $\begin{cases} x \circ e = x \\ e \circ x = x \end{cases} \iff \begin{cases} x \cdot |e| = x \\ e \cdot |x| = x \end{cases}$ ← dla $x > 0$ spełnia $e = 1$; $x < 0$ spełnia $e = -1$ BRAK el. neutralnego!

Półgrupa niepryemienna

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② $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 3 & 1 & 5 & 7 & 2 & 6 & 9 & 8 & 4 \end{pmatrix} \in S_9$

a) $\sigma^{-1} = ?$ $\sigma^{-1} = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 2 & 5 & 1 & 9 & 3 & 6 & 4 & 8 & 7 \end{pmatrix}$

b) Rozkład na cykle $\sigma = (1352)(479)$

c) Rozkład na transpozycje $\sigma = (12)(15)(13)(49)(47)$

$\sigma =$ ↙ 5 transpozycji

c) $\operatorname{sgn}(\sigma) = -1$

d) liczba inwersji:

$3\,(1)\,5\,7\,(2)\,6\,9\,8\,4$	2
$1\,5\,7\,2\,6\,9\,8\,4$	0
$5\,7\,(2)\,6\,9\,8\,(4)$	2
$7\,(2)\,(6)\,9\,8\,(4)$	3
$2\,6\,9\,8\,4$	0
$6\,9\,8\,(4)$	1
$9\,(8)\,(4)$	2
$8\,(4)$	1

11 inwersji

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④ a) $z^4 = (1+i)^8$ $z \neq 0$ $z = |z| e^{i\varphi}$

$z_1 = (1+i)^2 = 1 + 2i + i^2 = 2i$

$z_2 = z_1 \cdot i = -2$ $z_3 = z_2 \cdot i = -2i$ $z_4 = z_3 \cdot i = 2$

b) $z^4 = \frac{-18}{-1+i\sqrt{3}} = \frac{-18(-1-i\sqrt{3})}{1 - i^2 \cdot 3} = -\frac{9}{2}(-1-i\sqrt{3}) = 9\left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right) = 9\left(\cos\left(\pi - \frac{\pi}{3}\right) + i\sin\left(\pi - \frac{\pi}{3}\right)\right)$

$z \neq 0$ $z = |z| e^{i\varphi}$ $z^4 = |z|^4 e^{4i\varphi} = 9 \cdot e^{\frac{2}{3}\pi i}$ $|z|^4 = 9 \Rightarrow |z| = \sqrt{3}$

$4\varphi = \frac{2}{3}\pi + 2k\pi$, $\varphi_k = \frac{\pi}{6} + k\frac{\pi}{2}$; $k \in \{0,1,2,3\}$ $z_k = \sqrt{3} e^{\varphi_k i}$ $k \in \{0,1,2,3\}$

c) $z^4 = \frac{i-1}{\sin\frac{\pi}{6} + i\cos\frac{\pi}{6}} = \frac{\sqrt{2}\left(-\frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}\right)}{\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}} = \frac{\sqrt{2} e^{\frac{3}{4}\pi i}}{1 \cdot e^{\frac{\pi}{3}i}}$

$z \neq 0$ $z = |z| e^{i\varphi}$

$|z|^4 e^{4i\varphi} = \sqrt{2} e^{\left(\frac{3}{4} - \frac{1}{3}\right)\pi i} = \sqrt{2} e^{\frac{5}{12}\pi i}$

$|z|^4 = \sqrt{2} \Rightarrow |z| = \sqrt[8]{2}$

Odp. $z_k = \sqrt[8]{8} \cdot e^{\varphi_k i}$ $k \in \{0,1,2,3\}$ ←

$4\varphi = \frac{5}{12}\pi + 2k\pi$, $\varphi_k = \frac{5}{48}\pi + k\frac{\pi}{2}$ $k \in \{0,1,2,3\}$

d) $z^3 = \frac{(i-1)^7}{(\sqrt{3}-i)^5} = \frac{\left[\sqrt{2}\left(-\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i\right)\right]^7}{\left[2\left(\frac{\sqrt{3}}{2} - \frac{1}{2}i\right)\right]^5} = \frac{\left[\sqrt{2} e^{\frac{3}{4}\pi i}\right]^7}{\left[2 e^{-\frac{\pi}{6}i}\right]^5} = \frac{8\sqrt{2} e^{\frac{21}{4}\pi i}}{32 \cdot e^{-\frac{5}{6}\pi i}} = \frac{\sqrt{2}}{4} e^{\left(\frac{21}{4} + \frac{5}{6}\right)\pi i} = \frac{\sqrt{2}}{4} e^{\frac{73}{12}\pi i}$

$z \neq 0$ $z = |z| e^{i\varphi}$ $|z|^3 e^{3i\varphi} = \frac{\sqrt{2}}{4} e^{\frac{73}{12}\pi i} \Rightarrow |z|^3 = \frac{\sqrt{2}}{4} = \frac{1}{2\sqrt{2}} \Rightarrow z = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

$3\varphi = \frac{73}{12}\pi + 2k\pi$, $\varphi_k = \frac{73}{36}\pi + \frac{2}{3}k\pi$ $k \in \{0,1,2\}$

$z_k = \frac{\sqrt{2}}{2} e^{\varphi_k i}$ $k \in \{0,1,2\}$

c) $z^3 = (2-2i)^9$

$z_1 = (2-2i)^3 = 2^3 + 3 \cdot 2^2 (-2i) + 3 \cdot 2 (-2i)^2 + (-2i)^3 = 8 - 24i - 24 - 8i^3 = -16(1+i) =$ ($8i$)

$z_2 = z_1 e^{\frac{2}{3}\pi i} = -16(1+i)\left(-\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right) = -16(1+i)\left(-\frac{1}{2} + i\frac{\sqrt{3}}{2}\right) = 8(1+\sqrt{3} + i - \sqrt{3}i)$

$z_2 = z_1 e^{\frac{4}{3}\pi i} = -16(1+i)\left(-\cos\frac{\pi}{3} - i\sin\frac{\pi}{3}\right) = 16(1+i)\frac{1}{2}(1+\sqrt{3}) = 8(1-\sqrt{3} + i + \sqrt{3}i)$

ALGEBRA – Zadanie domowe nr 1

(4) f) $(iz+2)^4 = (z-4i)^4$ Zał. $z-4i \neq 0$, $z \neq 4i$ Gdy $z=4i$: $P=0$, $L=(4i+2)^4 \neq 0$

$\left(\frac{iz+2}{z-4i}\right)^4 = 1$ $w := \frac{iz+2}{z-4i}$ $w^4=1 \Rightarrow w \in \{1, i, -1, -i\}$

- $iz+2 = z-4i$, $(i-1)z = -2-4i$, $z = \frac{2+4i}{1-i} = \underline{-1+3i}$
- $iz+2 = i(z-4i)$, $2=4$, Sprzeczne
- $iz+2 = -z+4i$, $(i+1)z = -2+4i$, $z = \frac{-2+4i}{i+1} = \underline{1+3i}$
- $iz+2 = -i(z-4i)$, $2iz = -6$, $z = -\frac{3}{i} = \underline{3i}$

g) $9z^3 = (\bar{z})^5$

$z=0$ jest rozwiązaniem, zał. $z \neq 0$, $z = |z|e^{i\varphi}$

$9|z|^3 e^{3i\varphi} = |z|^5 e^{-5i\varphi}$ / : $|z|^3$, $\cdot e^{-3i\varphi}$ $z \neq 0$, $|z| \neq 0$

$9e^{\pi i} = |z|^2 e^{-8i\varphi}$

$|z|^3 = 9$, $|z| = 3$ $-8\varphi = \pi + 2k\pi$ $\varphi_k = \frac{-\pi}{16} - k\frac{\pi}{4}$

Odp. $z=0 \vee z = 3e^{\varphi_k i}$; $k \in \{0,1,\dots,7\}$ $k \in \{0,1,\dots,7\}$

h) $z^{11} = (\bar{z})^{11}$ $z=0$ jest rozw., zał. $z \neq 0$, $z = |z|e^{i\varphi}$ $\Rightarrow$ $|z|^{11} e^{11\varphi i} = |z|^{11} e^{-11\varphi i}$ / : $|z|$

$e^{22\varphi i} = 1$ $|z| \neq 0$ bo $z \neq 0$

$|z|$ dowolne $\wedge$ $22\varphi = 2k\pi$, $\varphi_k = \frac{\pi}{11}k$; $k \in \{0,1,\dots,21\}$ potrzeba

i) $z^3 - 3z^2 + 6z - 4 = 0$

$(z-1)(z^2-2z+4) = 0$

$\Delta = 4 - 4\cdot 1\cdot 4 = -12 = 12i^2$

$\sqrt{\Delta} = \{2\sqrt{3}i, -2\sqrt{3}i\}$

	1	-3	6	-4
(1)	1	-2	4	0

$z_0 = 1$

$z_1 = \frac{2-2\sqrt{3}i}{2} = 1-\sqrt{3}i$ $z_2 = 1+\sqrt{3}i$

j) $z^4 + (i-2)z^2 - 2i = 0$ $w = z^2 \in \mathbb{C}$

$w^2 + (i-2)w - 2i = 0$ $\Delta = (i-2)^2 - 4\cdot 1\cdot(-2i) = i^2 - 4i + 4 + 8i = 4i + 3 = 4 + 4i - 1 = 4 + 4i + i^2 = (2+i)^2$

$\sqrt{\Delta} = \{2+i, -2-i\}$ $w_1 = \frac{-i+2-i-2}{2} = -i$ $w_2 = \frac{-i+2+2+i}{2} = 2$

$z^2 = -i = e^{-\frac{\pi}{2}i}$ lub $z^2 = 2$

$z_1 = e^{-\frac{\pi}{4}i}$ $\vee$ $z_2 = e^{\frac{3}{4}\pi i}$ $z_3 = \sqrt{2}$, $z_4 = -\sqrt{2}$

$z_1 = \cos\frac{\pi}{4} - i\sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}(1-i)$ $\vee$ $z_2 = \frac{\sqrt{2}}{2}(i-1)$

k) $i^3 \bar{z} z^6 = \operatorname{Im}\left(e^{\frac{\pi}{2}i}\right) \cdot \frac{1+i}{1-i} \cdot \frac{(\sqrt{12}-2i)^7}{(-\sin\frac{\pi}{5} + i\cos\frac{\pi}{5})^4}$ $\Rightarrow$

$z \neq 0$, $z = |z|e^{i\varphi}$

$i^3 = -i = e^{-\frac{\pi}{2}i}$ $\operatorname{Im}(e^{\frac{\pi}{2}i}) = \sin\frac{\pi}{2} = 1$

$\frac{1+i}{1-i} = \frac{(1+i)^2}{1^2-i^2} = \frac{1}{2}(1+2i-1) = i = e^{\frac{\pi}{2}i}$

$\sqrt{12} - 2i = 2\sqrt{3} - 2i = 4\left(\frac{\sqrt{3}}{2} - \frac{1}{2}i\right) = 4e^{-\frac{\pi}{6}i}$

$-\sin\frac{\pi}{5} + i\cos\frac{\pi}{5} = \cos\left(\frac{\pi}{2}+\frac{\pi}{5}\right) + i\sin\left(\frac{\pi}{2}+\frac{\pi}{5}\right) = e^{\frac{7}{10}\pi i}$

$e^{-\frac{\pi}{2}i} \cdot |z|e^{-i\varphi} \cdot |z|^6 e^{6i\varphi} = 1 \cdot e^{\frac{\pi}{2}i} \cdot \frac{4^{15} e^{\frac{35}{6}\pi i}}{e^{\frac{14}{5}\pi i}}$

$|z|^7 e^{5i\varphi} = e^{\pi i\left(1 - \frac{15}{6} - \frac{14}{5}\right)i}$

$|z|^7 = 4^{15}$ $5\varphi = \frac{30-75-84}{30}\pi + 2k\pi = \frac{-129}{30}\pi + 2k\pi = -\frac{9}{30}\pi + 2k\pi = -\frac{3}{10}\pi + 2k\pi$

$z_k = \sqrt[7]{4^{15}} e^{\varphi_k i}$ $\varphi_k = -\frac{3}{50}\pi + \frac{2}{5}k\pi$ $k \in \{0,1,2,3,4\}$

l) $\frac{|4-3i|}{i}\bar{z}^2 = \frac{4+12i}{3-i} \cdot \frac{(\sin\frac{\pi}{6} - i\cos\frac{\pi}{3})^7}{(\sin\frac{\pi}{3} - i\cos\frac{\pi}{3})^5} \cdot z^5$

$z=0$ jest rozw., zał. $z \neq 0$, $z = |z|e^{i\varphi}$

$|4-3i| = 5$ $\sin\frac{\pi}{6} - i\cos\frac{\pi}{3} = \frac{1}{2}(1-i) = \frac{\sqrt{2}}{2}e^{-\frac{\pi}{4}i}$

$\frac{1}{i} = -i$ $\sin\frac{\pi}{3} - i\cos\frac{\pi}{3} = \frac{\sqrt{3}}{2} - \frac{1}{2}i = e^{-\frac{\pi}{6}i}$

$\frac{4+12i}{3-i} = 4\frac{1+3i}{3-i} = 4\frac{(1+3i)(3+i)}{9+1} = \frac{4}{10}\cdot 10i = 4i$

Korzystając

$-5i \cdot |z|^2 e^{-2\varphi i} = 4i \cdot \frac{\left(\frac{1}{\sqrt{2}}\right)^7 e^{-\frac{7}{4}\pi i}}{e^{-\frac{5}{6}\pi i}} \cdot |z|^5 \cdot e^{5i\varphi}$ / : $|z|^2$, $\cdot e^{2i\varphi}$ $|z| \neq 0$ bo $z \neq 0$

$5e^{\pi i} = 4\cdot\frac{1}{8\sqrt{2}} e^{\left(-\frac{7}{4}+\frac{5}{6}\right)\pi i} \cdot |z|^3 e^{5i\varphi}$

$10\sqrt{2}\, e^{\left(1+\frac{7}{4}-\frac{5}{6}\right)\pi i} = |z|^3 e^{5i\varphi}$

$|z|^3 = 10\sqrt{2}$ $\frac{\pi}{12} + 2k\pi = 5\varphi$

$|z| = \sqrt[3]{10\sqrt{2}}$ $\varphi_k = \frac{\pi}{60} + \frac{2}{5}k\pi$ $k \in \{0,1,2,3,4\}$

Odp. $z=0 \vee z = \sqrt[3]{10\sqrt{2}}\, e^{\varphi_k i}$, $k \in \{0,1,2,3,4\}$

$1 + \frac{7}{4} - \frac{5}{6} = \frac{12+21-10}{12} = \frac{1}{12}$

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(3) $\arg\left[(\sin\frac{\pi}{3} - i\cos\frac{\pi}{3})^{17}\right] = \arg\left[\left(\frac{\sqrt{3}}{2} - \frac{1}{2}i\right)^{17}\right] = \arg\left[\cos(-\frac{\pi}{6}) + i\sin(-\frac{\pi}{6})\right]^7 = \arg\left(e^{-\frac{7}{6}\pi i}\right)$

$= -\frac{7}{6}\pi + 2\pi = \frac{5}{6}\pi$

ALGEBRA - Zadanie domowe nr 1

5

a) $W(z) = z^4 - 2z^3 + 27z^2 - 2z + 26$ ← współczynniki rzeczywiste

$z_1 = i$ - pierwiastek W $\Rightarrow \bar{z_1} = -i$ to też pierwiastek $\Rightarrow W$ dzieli się przez $(z+i)(z-i) = z^2+1$

$$\begin{array}{r} z^2 - 2z + 26 \\ \hline (z^4 - 2z^3 + 27z^2 - 2z + 26) : (z^2+1) \\ z^4 \quad + z^2 \\ \hline -2z^3 + 26z^2 - 2z + 26 \\ -2z^3 \quad\quad - 2z \\ \hline 26z^2 \quad + 26 \\ 26z^2 \quad + 26 \\ \hline 0 \end{array}$$

$W(z) = (z+i)(z-i)(z^2 - 2z + 26)$

$\Delta = 4 - 4 \cdot 1 \cdot 26 = 4 - 104 = -100 = 100i^2$

$\sqrt{\Delta} = \{10i, -10i\}$

$z_1 = i$, $z_2 = -i$, $z_3 = \frac{2 - 10i}{2} = 1 - 5i$, $z_4 = 1 + 5i$

b) Faktoryzacja $W(z) = z^6 - 64$

$W(z) = (z^3)^2 - 8^2 = (z^3 - 8)(z^3 + 8) = (z-2)(z^2 + 2z + 4)(z+2)(z^2 - 2z + 4)$

$0 = z^2 + 2z + 4 = (z+1)^2 + 3 = (z+1)^2 - (-3) = (z+1)^2 - (3i^2) = (z+1)^2 - (\sqrt{3}i)^2 = (z + 1 - \sqrt{3}i)(z + 1 + \sqrt{3}i)$

$0 = z^2 - 2z + 4 = (z-1)^2 + 3 = (z-1)^2 - (-3) = (z-1)^2 - (3i^2) = (z-1)^2 - (\sqrt{3}i) = (z - 1 - \sqrt{3}i)(z - 1 + \sqrt{3}i)$

$W(z) = (z-2)(z+2)(z+1-\sqrt{3}i)(z+1+\sqrt{3}i)(z-1-\sqrt{3}i)(z-1+\sqrt{3}i)$

6

$D = \{z \in \mathbb{C} : \arg[(\bar{z} + i)^4] = \frac{3}{2}\pi\}$

argument główny tj. $z \in [0, 2\pi)$

$\arg(z \cdot w) = \arg(z) + \arg(w) + 2k\pi$

$\arg(\bar{w}) = 2\pi - \arg(w)$ gdy $\arg(w) \neq 0$

$\Rightarrow$

$\frac{3}{2}\pi = \arg[(\bar{z}+i)^4] = 4\arg(\bar{z}+i) + 2k\pi =$

$= 4[2\pi - \arg(\overline{\bar{z}+i})] + 2k\pi =$

$= 4[2\pi - \arg(z - i)] + 2k\pi =$

$= -\arg(z-i) + 8\pi + 2k\pi = -\arg(z-i) + 2k\pi$

$\Rightarrow \arg(z-i) = -\frac{3}{2}\pi + 2k\pi = (4+k)2\pi$

$k = 1 \quad \arg(z-i) = \frac{\pi}{2}$

$A = \{z \in \mathbb{C} : \operatorname{Im}(\sqrt{2} \cdot e^{\frac{\pi}{4}i}) \leq |\frac{1}{2}i \cdot \bar{z} - 2 + 6i| < |\sqrt{3} + i|^2\}$

$\sqrt{2}\, e^{\frac{\pi}{4}i} = \sqrt{2}(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}) = \sqrt{2}(\frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}) = 1 + i \qquad \operatorname{Im}(\sqrt{2} \cdot e^{\frac{\pi}{4}i}) = 1$

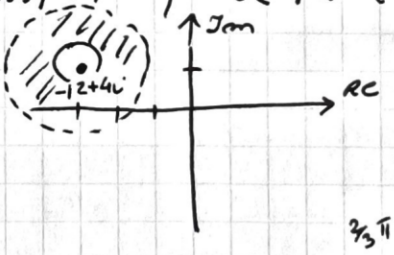
$|\sqrt{3} + i| = \sqrt{3+1} = 2$

tj. $1 \leq |\frac{1}{2}i\bar{z} - 2 + 6i| < 4$

$|\frac{1}{2}i\bar{z} - 2 + 6i| = |\frac{1}{2}i(\bar{z} - \frac{4}{i} + 12)| = |\frac{1}{2}| \cdot |i| \cdot |\bar{z} + 4i + 12| =$

$= \frac{1}{2} \cdot |\overline{\bar{z} + 12 + 4i}| = \frac{1}{2}|z + 12 - 4i| = \frac{1}{2}|z - (-12 + 4i)|$

$1 \leq \frac{1}{2}|z - (-12 + 4i)| < 4 \qquad 2 \leq |z - (-12 + 4i)| < 8$



$B = \{z \in \mathbb{C} : \operatorname{Im}(z^6) < 0\}$

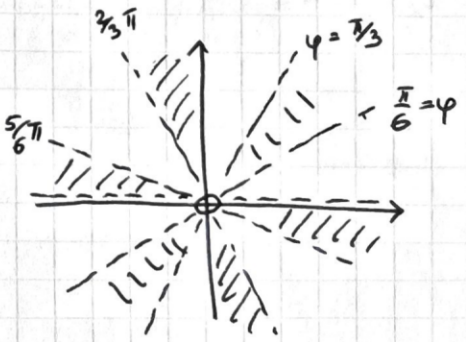
$z^6 = r^6(\cos 6\varphi + i\sin 6\varphi)$

$\operatorname{Im}(z^6) = \underbrace{r^6}_{>0} \sin 6\varphi < 0 \iff \sin 6\varphi < 0$

$\pi + 2k\pi < 6\varphi < 2\pi + 2k\pi$

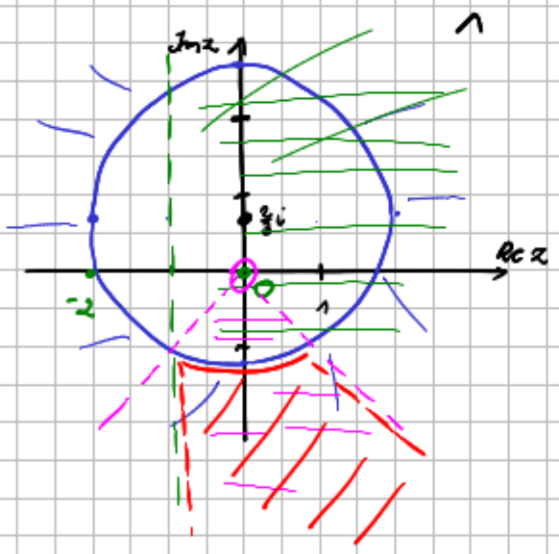
$\frac{\pi}{6} + k \cdot \frac{\pi}{3} < \varphi < \frac{\pi}{3} + k \cdot \frac{\pi}{3}$

$k \in \mathbb{Z}$



(6) Zaznacz na płaszczyźnie zespolonej $C=\{z\in\mathbb{C}: |3\bar{z}+2i|\geq 6 \wedge |z|<|z+2|\}$

- $|3\bar{z}+2i|\geq 6$
  $|3(\bar{z}+\frac{2}{3}i)|\geq 6$
  $|\bar{z}+\frac{2}{3}i|\geq 2$
  $|\bar{z}+\frac{2}{3}i| = |\overline{\bar{z}+\frac{2}{3}i}| = |z-\frac{2}{3}i|\geq 2$
- $|z|<|z+2|$
  $|z-0|<|z-(-2)|$
- $\frac{\pi}{4}<\arg(\bar{z})<\frac{3}{4}\pi$
  $\frac{\pi}{4}<2\pi-\arg(z)<\frac{3}{4}\pi$
  $\arg(z)<\frac{7}{4}\pi \wedge \arg z>\frac{5}{4}\pi$



Część wspólna

(7) I.

*	a	b	c
a	b	c	a
b	c	b	b
c	a	b	c

niewątpliwie → w tabeli tylko a, b, c
przemienność → symetria tabeli wzgl. przekątnej. PRAWDA

element neutralny $e=c\in\{a,b,c\}$

II. $\{z\in\mathbb{C}: z^{11}=1\} = \sqrt[11]{1}$ ← pierwiastki leżą na okręgu o promieniu 1 wierzchołki 11-kąta foremnego $z_1=z_0\cdot e^{\frac{2\pi}{11}i}$ itd...
PRAWDA – Grupa abelowa z mnożeniem

III. $Z=\{\begin{bmatrix}a & a\\ a & a\end{bmatrix} : a\in\mathbb{R}\setminus\{0\}\}$ mnożenie macierzy – wewnętrzne w Z
PRAWDA
$a\neq 0, b\neq 0$ $\begin{bmatrix}a & a\\ a & a\end{bmatrix}\begin{bmatrix}b & b\\ b & b\end{bmatrix} = \begin{bmatrix}2ab & 2ab\\ 2ab & 2ab\end{bmatrix}\in Z$ $a\neq 0, b\neq 0 \Rightarrow 2ab\neq 0$

ODPOWIEDŹ (A)

(8)
I. Prawda → to zasadnicze twierdzenie algebry
II. $W(z)$ ← wielomian o współcz. z $\mathbb{R}$, stopnia 7 → pierwiastki zespolone występują parami z_0 i $\bar{z}_0$
FAŁSZ
III. $e^{ix}\neq 0$ $\forall x\in\mathbb{R}$ prawda $e^{ix}=\cos x+i\sin x$ gdy $\cos x=0$ to $\sin x=\pm 1$

Odpowiedź: (D)