# THE EFFECT OF THE SUBSTRATE TWO-DIMENSIONAL TEMPERATURE DISTRIBUTION ON THE TEC PERFORMANCE

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**Abstract**

Temperature distribution on a thermoelectric cooler (TEC) cold surface is of high practical value, as the size of the cooled object may not coincide with the dimensions of the TEC cold side and it is necessary to make the object temperature closer to the average cold substrate temperature. It is also very important to take into account the temperature distribution on the intermediate substrates of multistage TECs both in mathematical simulation and design modeling.

An approach to finding the approximate two-dimensional temperature distribution for the case of a heat source located on the surface has been developed in papers [1,2]. The results of quite a detailed model of the heat spread in the TEC substrates was considered for certain specific tasks in paper [3] but this model infinite series solution is hard to apply to independent problems. In this paper the method [1,2] is applied to calculations of the temperature 2D-profiles of the TEC substrates. The application of the above-mentioned method for performance improvement of TEC systems is discussed. The analytical form of the solution is open for a wide application.

### Two-Dimensional Temperature Distribution on the Cold Substrate of a Single-Stage TEC

Consider a problem of the temperature distribution on the cold substrate surface of a single-stage TEC. Assume it consists of N pellets. A heat source is localized on the TEC cold side.

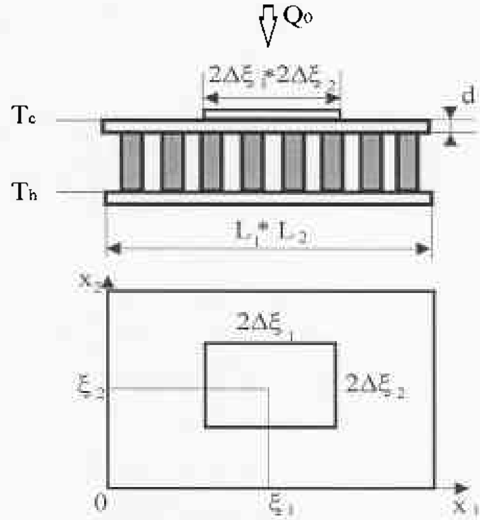



Figure 1. *Schematic view of a rectangular heat source on a single-stage TEC cold substrate surface*

Suppose the TEC substrate is an $L_1$ x $L_2$ rectangular, and the heat source is a $2\Delta\xi_1$ x $2\Delta\xi_2$ one. The heat source centre coordinates are $\xi_1$, $\xi_2$. The heat source load to be pumped by the TEC equals $Q_0$. The hot surface temperature is a fixed value $T_h$, and the cold surface temperature is a two-dimensional function $T_c(x_1,x_2)$. Hereinafter, not to take into account discreteness of pellets on the substrate surface, we do not restrict each pellet cooled (heated) area to the pellet cross-section $s_0$, but regard it as the full substrate area per a pellet – $L_1L_2/N$. That is, we assume a quasi-continuous pellets distribution on the substrate surface. Within this approach the calculated temperature two-dimensional field differs from the real one in lacking a slight periodicity (its period equals the distance between pellets). Then the pellets 2D-distribution density is equal to $N/L_1L_2$. Ignoring thermoelectric parameters dependence on temperature, we consider the Seebeck coefficient $\alpha$, thermal conductivity $\kappa$ and electrical resistivity $\rho$ to be constant values. When the pellet is exposed to the electrical current I, the heat flux $q_{pellet}$ [3] is pumped to the pellet cold end:

$$q_{pellet} = -\alpha I T_c + \frac{1}{2} I^2 R + k(T_h - T_c), \qquad (1)$$

where the first term on the right side of equation (1) expresses the Peltier heat extracted by the pellet from the substrate, the second term is the part of the Joule heat, arriving at the substrate from the pellet, and the third term describes the heat flux coming from the hot substrate by the pellet thermal conductance. Here $\alpha$ – the Seebeck coefficient, $R=\rho l/s_0$ – pellet electrical resistance, $k=\kappa s_0/l$ – pellet thermal conductance, $l$ – pellet length. Let d denote the substrate thickness and $\lambda$ stand for the substrate thermal conductivity. Then the heat conductance equation can be written as follows:

$$\lambda d\left(\frac{\partial^2 T_c}{\partial x_1^2}\right) + \lambda d\left(\frac{\partial^2 T_c}{\partial x_2^2}\right) - \frac{N(\alpha I + k)T_c}{L_1 L_2} + \\ + \frac{N\left(\frac{1}{2} I^2 R + k T_h\right)}{L_1 L_2} + \frac{Q_0 1\{u\}}{4\Delta\xi_1 \Delta\xi_2} = 0 \qquad (2)$$

where we write the symbol $1\{u\}$ for the function equal 1 within the area of the heat source $Q_0$ and 0 within the rest of the surface.

Suppose the heat is only absorbed from the cold substrate by the pellets and there are no lateral heat fluxes:

$$\left.\frac{\partial T_c}{\partial x_i}\right|_{x_i=0,\, x_i=L_i} = 0, \quad i = 1,2. \tag{3}$$

If turning the current I into the reduced current $j=Il/s_0$ and denoting the pellets filling coefficient $K_f$:

$$K_f = \frac{N s_0}{L_1 L_2}, \tag{4}$$

we define:

$$b^2 = \frac{(\alpha j + \kappa) K_f}{\lambda l d}, \tag{5}$$

$$A = \frac{K_f\left(\frac{1}{2} j^2 \rho + \kappa T_h\right)}{\lambda l d}, \tag{6}$$

$$C = \frac{Q_0}{4 \Delta\xi_1 \Delta\xi_2 \lambda d}. \tag{7}$$

Making in Eq. (2) the substitution of variables:

$$T_c = \theta + \frac{A}{b^2}, \tag{8}$$

we obtain the following equation:

$$\frac{\partial^2 \theta}{\partial x_1^2} + \frac{\partial^2 \theta}{\partial x_2^2} - b^2 \theta + C\{u\} = 0 \tag{9}$$

with boundary conditions:

$$\left.\frac{\partial \theta}{\partial \bar{x}_i}\right|_{\bar{x}_i=0,\, \bar{x}_i=1} = 0, \qquad i = 1,2. \tag{10}$$

The approximate solution of this problem is known and given in papers [1,2]:

$$\theta = \frac{C}{b^2} \phi_1 \phi_2. \tag{11}$$

In dimensionless coordinates

$$\bar{x}_i = \frac{x_i}{L_i}, \bar{\xi}_i = \frac{\xi_i}{L_i}, \Delta\bar{\xi}_i = \frac{\Delta\xi_i}{L_i}, \qquad i = 1,2. \tag{12}$$

the functions $\phi_i$ look as follows:

$$\phi_i = \begin{cases} K_i \mathrm{ch}(p_i \bar{x}_i), & x_i \in [0; \bar{\xi}_i - \Delta\bar{\xi}_i] \\ K_i \mathrm{ch}(p_i \bar{x}_i) - \mathrm{ch}(p_i(\bar{x}_i - \bar{\xi}_i + \Delta\bar{\xi}_i)) + 1, & x_i \in [\bar{\xi}_i - \Delta\bar{\xi}_i, \bar{\xi}_i + \Delta\bar{\xi}_i] \\ K_i \mathrm{ch}(p_i \bar{x}_i) - \mathrm{ch}(p_i(\bar{x}_i - \bar{\xi}_i + \Delta\bar{\xi}_i)) + \\ + \mathrm{ch}(p_i(\bar{x}_i - \bar{\xi}_i - \Delta\bar{\xi}_i)), & x_i \in [\bar{\xi}_i + \Delta\bar{\xi}_i, 1] \end{cases}, \tag{13}$$

$$K_i = \frac{2\mathrm{sh}(p_i \Delta\bar{\xi}_i)\mathrm{ch}(p_i(1-\bar{\xi}_i))}{\mathrm{sh} p_i}, \quad i = 1,2$$

where

$$p_i = \frac{L_i}{L_{i+(-1)^{i-1}}} \sqrt{B_{i+(-1)^{i-1}} \left[1{,}5 - \left(\frac{\mathrm{sh}\left(2\left(\sqrt{B_{i+(-1)^{i-1}}}\right)\right)}{2\left(\sqrt{B_{i+(-1)^{i-1}}}\right)} + 1\right)^{-1}\right]}, \tag{14}$$

$$B_i = b^2 L_i^2 \tag{15}$$

Therefore, the temperature distribution for the case in Fig. 1 is yielded by the expression:

$$T_c = \frac{Q_0 l}{S_q K_f (\alpha j + \kappa)} \phi_1 \phi_2 + \frac{\frac{1}{2} j^2 \rho + \kappa T_h}{\alpha j + \kappa} \tag{16}$$

where $S_q$ – the area covered by the heat source.

**Two-Dimensional Temperature Distribution on the Intermediate Substrate of a Two-Stage TEC**

Consider a two-stage TEC. Let the first (upper) stage (cascade) cover the area $2\Delta\xi_1 x 2\Delta\xi_2$ and consist of $N_1$ pellets. The heat load delivered onto the first stage is $q_0$. Let the second stage cover the area $L_1$ x $L_2$ and have $N_2$ pellets. Not to be concerned about the type of electrical connection we suppose power is supplied to the cascades independently, so that we can be free to choose the pellets geometry of each cascade. Thus we denote the pellets cross-section $s_i$ and the pellets height $l_i$, where indices i=1,2 correspond to the stage number. The values of the thermoelectric parameters are also taken different per stage and further are distinguished by the stage index. The reduced current values for the two stages can also differ and we denote them $j_i$, i = 1,2. Our objective is to obtain the temperature distribution $T_c(x,y)$ on the substrate between the first and the second cascade.

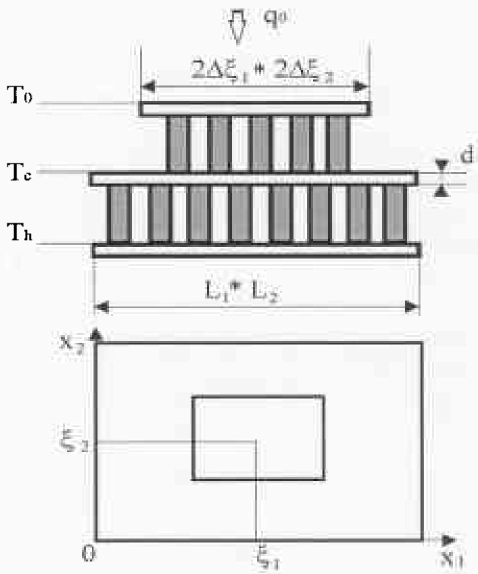



Figure 2. *Schematic view of geometry and temperatures on the cascades of a two-stage TEC*

Unfortunately, the solution [1] was obtained for the uniform heat load on the cooled substrate. Therefore, the solution is approximate and assumes that the heat flux from the first stage to the second one is evenly spread over their contact area. We also suggest the heat load on the upper stage be uniform as well and the pellets of both cascades be arranged quasi-continuously, as we supposed in the single-stage problem. Let us write $Q_0$ for the heat flux from the first stage. It can be expressed as follows:

$$Q_0 = \frac{N_1 s_1}{l_1}\left[\alpha_1 j_1 \overline{T}_c + \frac{1}{2} j_1^2 \rho_1 - \kappa_1\left(\overline{T}_c - T_0\right)\right]. \qquad (17)$$

Here $\overline{T}_c$ is the average temperature of the contact area between the two stages:

$$\overline{T}_c = \frac{1}{4\Delta\xi_1\Delta\xi_2}\int_{\xi_1-\Delta\xi_1}^{\xi_1+\Delta\xi_1}\int_{\xi_2-\Delta\xi_2}^{\xi_2+\Delta\xi_2} T_c\, d\xi_1 d\xi_2 \,. \qquad (18)$$

Analogous heat rate equations for the first stage cold substrate allows eliminating the temperature $T_0$ from Eq. (17):

$$Q_0 = \frac{N_1 s_1}{l_1}\left[\frac{\alpha_1^2 j_1^2 \overline{T}_c + \frac{1}{2} j_1^2 \rho_1\left(\alpha_1 j_1 + 2\kappa_1\right) + \kappa_1 q_0 \frac{l_1}{N_1 s_1}}{\alpha_1 j_1 + \kappa_1}\right] \qquad (19)$$

Therefore, we come to the equation similar to Eq. (2). Its solution is given by the following expression:

$$T_c = \frac{c}{\beta^2}\phi_1\phi_2 + \frac{a}{\beta^2}, \qquad (20)$$

where the functions $\phi_1$ and $\phi_2$ are determined by Eqs. (12) – (15), and the other terms are given below:

$$c = \frac{K_{f1}}{\lambda d l_1}\left[\frac{\alpha_1^2 j_1^2 \overline{T}_c + \frac{1}{2} j_1^2 \rho_1\left(\alpha_1 j_1 + 2\kappa_1\right) + \kappa_1 q_0 \frac{l_1}{N_1 s_1}}{\alpha_1 j_1 + \kappa_1}\right], \qquad (21)$$

$$a = \frac{K_{f2}\left(\frac{1}{2} j_2^2 \rho_2 + \kappa_2 T_h\right)}{\lambda d l_2} \qquad (22)$$

$$\beta^2 = \frac{K_{f2}\left(\alpha_2 j_2 + \kappa_2\right)}{\lambda d l_2} \qquad (23)$$

As a result of these transformations the value $\overline{T}_c$ remains unknown. It can be found by a multi-iteration procedure. For a zero approximation we take $\overline{T}_c$ as a solution of linear equations of the heat balance on the TEC substrate in the 1D-approach:

$$\overline{T}_c = \frac{4\Delta\xi_1\Delta\xi_2}{L_1 L_2}\frac{c}{\beta^2} + \frac{a}{\beta^2} \qquad (24)$$

The solution is

$$\overline{T}_c = \frac{\frac{N_2 s_2}{l_2}\left(\frac{1}{2} j_2^2 \rho_2 + \kappa_2 T_h\right)\left(\alpha_1 j_1 + \kappa_1\right) - \frac{N_1 s_1}{l_1}\frac{1}{2}\alpha_1 j_1^3 \rho_1 + q_0 \kappa_1}{\frac{N_2 s_2}{l_2}\left(\alpha_2 j_2 + \kappa_2\right)\left(\alpha_1 j_1 + \kappa_1\right) - \frac{N_1 s_1}{l_1}\alpha_1^2 J_1^2} \qquad (25)$$

After the temperature distribution is found in the first iteration, we can carry out the integration over the thermal contact area and calculate $\overline{T}_c$. As the solution of the heat conduction equation is expressed in the analytical form, the corresponding integrals are easily calculated. If denoting

$$\varphi_i = \frac{\int_{\bar{\xi}_i-\Delta\bar{\xi}_i}^{\bar{\xi}_i+\Delta\bar{\xi}_i}\phi_i(x)dx}{2\Delta\bar{\xi}_i} = \frac{\left[2K_i\left(\text{ch}p_i\bar{\xi}_i\,\text{sh}p_i\Delta\bar{\xi}_i\right) - \text{sh}2p_i\Delta\bar{\xi}_i\right]}{2p_i\Delta\bar{\xi}_i} + 1, \qquad (26)$$

i=1,2,

the expression for $\overline{T}_c$ can be written as follows:

$$\overline{T}_c = \frac{c}{\beta^2}\varphi_1\varphi_2 + \frac{a}{\beta^2} \qquad (27)$$

With the help of Eq. (27) we can find the value $\overline{T}_c$, calculate a new value of *c* from Eq. (21) and, with it, find a new $\overline{T}_c$, and etc. The procedure described above converges quickly and only a few iterations are required.

Due to the temperature losses the average temperature of the thermal contact area $\overline{T}_c$ is different from the average temperature of the whole intermediate substrate $\overline{T}_{cl}$:

$$\overline{T}_{cl} = \frac{c}{\beta^2}\zeta_1\zeta_2 + \frac{a}{\beta^2}, \qquad (28)$$

where we write $\zeta_i$ for the following:

$$\zeta_i = \int_0^1 \phi_i(x)dx = 2\Delta\xi_i\,, \quad i=1,2. \qquad (29)$$

Eq. (29), taken into account Eq. (28), coincides with Eq. (24), i.e. the temperature (24), used in the first iteration, is exactly the average over the substrate area. If the difference between $\overline{T}_c$ and $\overline{T}_{cl}$ is slight, one iteration may be enough.

**Numeric Calculations Results**

The above formulae allow calculating the temperature distribution over the substrates of a single-stage and multistage TEC. In fact, to perform this calculation it is sufficient to be capable of finding the temperature distribution on the cold substrate of a single-stage TEC (see (16)), as evaluating the operational heat load on a TEC stage is a standard task of a TEC mathematical simulation.

Eq. (28) may be applied not only to a two-stage TEC, but also to a second stage of a multicascade TEC. For this purpose one has to know, at least approximately, the temperature of the second stage hot substrate. Once the heat rejected by the previous cascades is found, it is possible to calculate the temperature distribution on any stage cold substrate of a multicascade TEC with the help of Eq. (28).

In practice it is often more important to obtain the average temperature $\overline{T}$ of the substrate and the average temperature $\overline{T}_q$ of the contact area under the heat load rather than the temperature two-dimensional field on the substrate surface. As an appropriate criterion of the distributional uniformity we take the difference $\Delta\overline{T} = \overline{T}_q - \overline{T}$. The analytical form of the heat conductance equation allows finding it easily. Thus we come to the following equation for a single-stage TEC:

$$\Delta\overline{T}_1 = \frac{C}{b^2}\left(\varphi_1\varphi_2 - 4\Delta\xi_1\Delta\xi_2\right) \qquad (30)$$

and for a two-stage TEC we have a similarly structured formula:

$$\Delta\overline{T}_2 = \frac{c}{\beta^2}\left(\varphi_1\varphi_2 - 4\Delta\xi_1\Delta\xi_2\right) \qquad (31)$$

In Table 1 we give the results of $\Delta\overline{T}$ calculation for various kinds of heat sources localized on the cold side of the standard 127-couple 40x40 $mm^2$ TEC (1.15x1.4x1.4 $mm^3$ pellets) for different materials of the cold substrate. The hot substrate temperature is taken 300 K. The reduced electric current is $j$=20 A/cm (I=3.4A). The heat source is placed in the centre of the cold substrate ($\xi_1$=20mm, $\xi_2$=20mm).

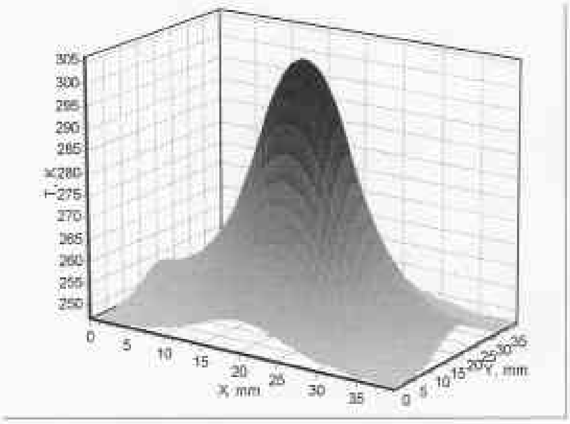
The data in Table 1 indicate that the localized heat load with the heat density 10 W/$cm^2$ is poorly spread over the substrate of $Al_2O_3$. In this case even the AlN ceramics is not sufficient. Only a 2 mm thick copper substrate allows reducing temperature losses to the extent of the calculation errors (on the order of 1 K).

Table 1. *Temperature difference $\Delta\overline{T}$ between the temperature averaged over the heat source area and that averaged over the whole substrate for the standard 127-couple 40x40 mm² TEC and various heat sources*

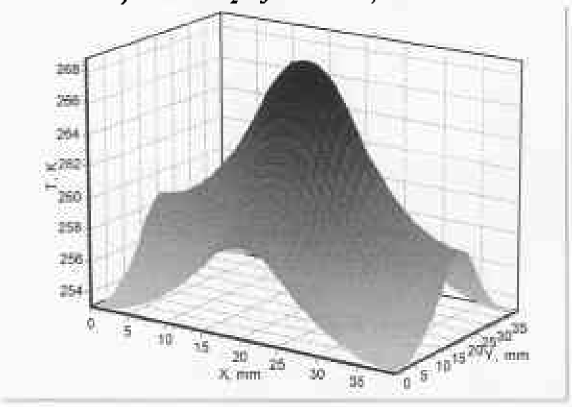
| # | Heat Load, W | Contact Area $2\Delta\xi_1 x 2\Delta\xi_2$, $mm^2$ | Substrate Thickness, mm | Substrate Material | Ceramics Thermal Conductivity, W/mK | $\Delta\overline{T}$, K |
|---|---|---|---|---|---|---|
| 1 | 10 | 10x10 | 1 | $Al_2O_3$ | 30 | 37.6 |
| 2 | 10 | 10x10 | 1 | AlN | 170 | 7.6 |
| 3 | 10 | 20x20 | 1 | $Al_2O_3$ | 30 | 12.6 |
| 4 | 10 | 20x20 | 1 | AlN | 170 | 3.1 |
| 5 | 10 | 30x30 | 1 | $Al_2O_3$ | 30 | 2.9 |
| 6 | 10 | 10x10 | 1 | Cu | 400 | 2.9 |
| 7 | 10 | 10x10 | 2 | Cu | 400 | 1.3 |

Fig. 3 illustrates the temperature distribution fields for cases 1, 2, 6.

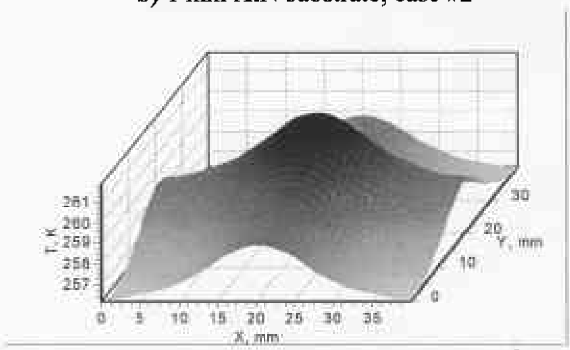
In Fig. 4 we summarize the above results by the picture of 2D-fields sections of cases 1 (a), 2 (b), 6 (c) of Table 1. The sections cut the topographic forms across the centre along the abscissa axis. The shaded area denotes the dimensions of the heat source (10 mm).




**a)** 1 mm $Al_2O_3$ substrate; case #1




**b)** 1 mm **AlN** substrate; case #2




**c)** 1 mm **Cu** substrate; heat density 10 W/$cm^2$ (#6)

Figure 3. *2D-temperature fields for cases 1 (a), 2 (b), 6 (c) of Table 1*

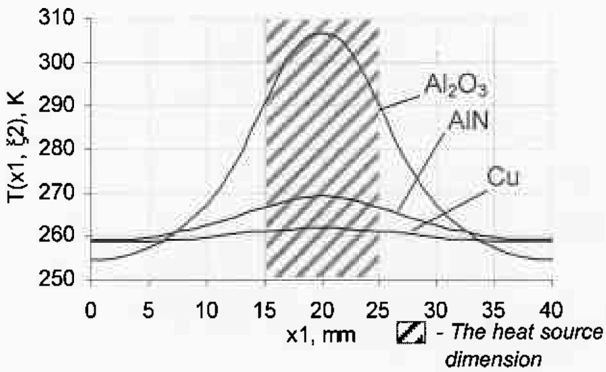



Figure 4. 2D-temperature fields central sections for cases 1, 2, 6 of Table 1

Another advantage of the method developed here is that it is not restricted to centre-symmetrical problems. Fig. 5 gives an example of the 2D-temperature field for situation # 2 of Table 1 in case the rectangular heat source 20x5 mm$^2$ is shifted from the centre ($\xi_1$=20 mm, $\xi_2$=10 mm).

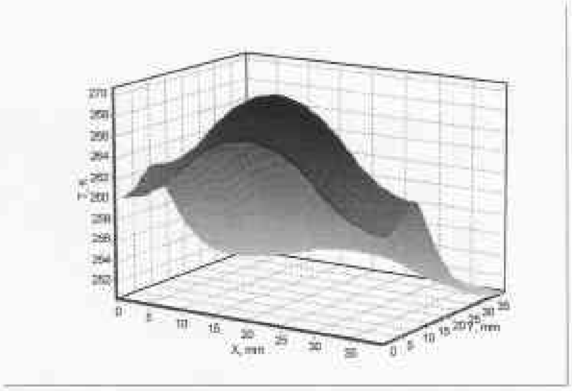


Figure 5. *Example of 2D-temperature field for non-central localization of the rectangular heat source; heat source 10 W, area $2\Delta\xi_1 x 2\Delta\xi_2$=20x5 mm$^2$. Centre $\xi_1$=20mm, $\xi_2$=10mm; substrate - case #2 of Table 1. $\overline{T}$=10.4 K.*

Now we investigate the problem of two-dimensional temperature losses on the intermediate substrate in two-stage TECs. Here we study how these losses are effected with the intercascade thermal resistance, varying the following aspects: 1) the ratio $N_2/N_1$; 2) intermediate substrate thickness and/or material. As earlier, the TEC hot side temperature equals 300K. We assume that the TEC heat load on the top stage is zero: $q_0$=0.

Let us examine the first aspect. We consider two TECs groups differing in heat density and dimensions. Each group consists of 3 TECs. The TECs bottom stage sizes expanded and the ratio $N_2/N_1$ growing, the criterion $\Delta\overline{T}$ behavior is studied.

The TECs are based on the $Al_2O_3$ ceramics (thermal conductivity is 30 W/mK).

Table 2 gives the results for two-stage TECs of group 1. The top cascade area and its pellets number $N_1$ are kept constant. The bottom stage pellets number N2 is varied. The pellets height is 1.5 mm, their cross-section is 0.6x0.6 mm$^2$. The ceramics is 0.5 mm thick. The electric current is 0.8 A (~$0.8I_{max}$).

Table 2. *The parameters and the* difference $\Delta\overline{T}$ *for the intermediate substrate of two-cascade TECs (group 1)*

| TEC # | $2\Delta\xi_1$x x$2\Delta\xi_2$, mm$^2$ | $L_1$x$L_2$, mm | $N_1$ | $N_2$ | $N_2/N_1$ | $Q_0$, W | Heat density, W/cm$^2$ | $\Delta\overline{T}$, K |
|---|---|---|---|---|---|---|---|---|
| 1 | 4x4 | 8x8 | 16 | 62 | 3.9 | 0.44 | 2.75 | 1.5 |
| 2 | 4x4 | 10x10 | 16 | 98 | 6.1 | 0.43 | 2.69 | 2.2 |
| 3 | 4x4 | 12x12 | 16 | 142 | 8.9 | 0.42 | 2.63 | 2.8 |

We see that miniature TECs provide comparatively small temperature losses on the intermediate substrate. The criterion $\Delta\overline{T}$ tends to grow with the ratio $N_2/N_1$.

The second case is much more thermally strenuous. Table 3 presents data similar to those of Table 2 for the two-stage TECs of group 2. The pellets height is 1.0 mm, their cross-section is 1.0x1.0 mm$^2$. The ceramics is 1.0 mm thick. The electric current is 3.0 A (~$0.8I_{max}$).

Table 3. *The parameters and the criterion $\Delta\overline{T}$ for the intermediate substrate of two-cascade TECs (group 2)*

| TEC # | $2\Delta\xi_1$x x$2\Delta\xi_2$, mm$^2$ | $L_1$x$L_2$, mm | $N_1$ | $N_2$ | $N_2/N_1$ | $Q_0$, W | Heat density, W/cm$^2$ | $\Delta\overline{T}$, K |
|---|---|---|---|---|---|---|---|---|
| 1 | 9x9 | 15x18 | 36 | 114 | 3.2 | 3.77 | 4.57 | 5.4 |
| 2 | 9x9 | 18x21 | 36 | 162 | 4.5 | 3.69 | 4.56 | 7.2 |
| 3 | 9x9 | 21x24 | 36 | 218 | 6.1 | 3.66 | 4.52 | 8.7 |

Table 3 shows that even for quite a moderate value of the ratio $N_2/N_1$ (#1) the criterion $\Delta\overline{T}$ is rather high. The situation is only worse for cases # 2 and 3. Fig. 6 gives the comparative picture of $\Delta\overline{T}$ versus the ratio $N_2/N_1$ for the two groups.

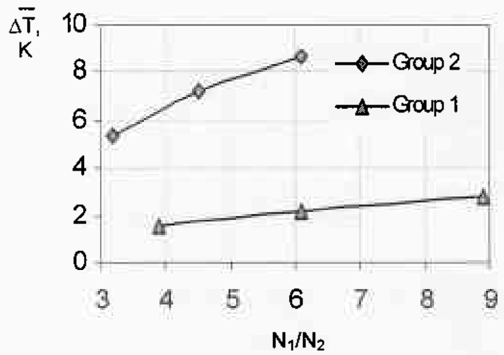



Figure 6. *The criterion $\Delta\overline{T}$ vs the ratio $N_2/N_1$ for TECs of groups 1 and 2.*

The intermediate substrates of the group 2 TECs are not substantially thick or their material thermal conductivity is not enough. As a result, the thermal resistance between the pellet on the edge of the substrate and the heat load becomes too high and the peripheral pellets in the bottom stage are not involved in cooling, unreasonably consuming additional power.

Thus, let us consider the second aspect. We take TEC # 1 of group 2 for the same electric current as in Table 2 and, varying the thickness and material of the intermediate substrate, study the criterion $\Delta\overline{T}$. The results obtained are given in Table 4.

Table 4. *The parameters and the criterion $\Delta\overline{T}$ of the two-cascade TEC (group 2, #1) based on intermediate substrates of different thermal resistance*

| **Intermediate Substrate Ceramics Material** | **Ceramics Thermal Conductivity, W/mK** | **Substrate Thickness, mm** | **$Q_0$, W** | **$\Delta\overline{T}$, K** |
|---|---|---|---|---|
| $Al_2O_3$ | 30 | 0.5 | 3.91 | 9.7 |
| $Al_2O_3$ | 30 | 1.0 | 3.77 | 5.4 |
| $Al_2O_3$ | 30 | 2.0 | 3.69 | 2.8 |
| AlN | 170 | 0.5 | 3.66 | 1.9 |
| AlN | 170 | 1.0 | 2.63 | 0.9 |
| BeO | 260 | 0.5 | 2.63 | 1.2 |

We see that for the case #1 of Table 3 alumina intermediate substrate can only be acceptable if it is thicker than 2mm. The similar result has been pointed out in paper[3]. Aluminum Nitride ceramics can reduce temperature losses to the level of calculation accuracy if the substrate is 0.5-1.0 mm thick; Beryllium Oxide does for a 0.5 mm thick substrate.

**Conclusions**

In TEC design and manufacturing a theoretical modeling should be of great reliability. Nowadays it is no problem to carry out *one-dimensional* computation and evaluate both specification and operation TEC parameters.

This paper proves that sometimes one-dimensional assessments are not sufficient. It occurs when a TEC is required to perform intense heat pumping in case of a forced geometrical compromise between concentration and dispersion of the heat flow. Then a detailed two-dimensional problem is to be solved and the criterion $\Delta\overline{T}$ should be taken into account.

The paper offers a thermoelectricist an open-to-use convenient analytical method for finding such 2D solutions.

As illustrations of the method efficiency, the analyses of two-dimensional temperature losses in single-stage TECs with localized heat sources and in two-stage TECs on the intermediate substrates are given.

The method is recommended for application-design optimization of TECs.

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