CALCULATION OF PROCESS OF BILATERAL FREEZING AND THERMOELECTRIC ICE-MAKER PRODUCTIVITY

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Abstract

The well-known evaluation procedures of ice freezing process on a cooled wall with a stationary value of temperature cannot be used in calculations of thermoelectric ice-makers with cellular ice-forms. One of the principal reasons is the oscillation of temperature within forms, both during freezing, and during thawing. The new approach to calculation of load characteristics of thermoelectric modules, treated as elements with known nominal characteristics, allows to combine these equations with calculations of freezing of ice in cells. This special approach is presented by authors as a calculation procedure of bilateral freezing of thin-walled pieces of ice in cells (for example, hemispherical and other proper shapes) with the use of cooled insertions that are disposed in cells while being integrated by design into the so-called upper ice-form. The procedure allows to evaluate a temperature of both forms at the end of freezing process, time of both freezing and thawing of ice and ice machine production output per hour including mass losses of ice during thawing. The procedure also presents a possibility for optimization of a thermoelectric ice-maker's both construction and cycle of production, which can be realised in selection of a particular relation in sizes of the lower and upper form and relation of cooling output of their refrigerating units. Outcome of calculations for both experimental and serial models of thermoelectric ice-makers demonstrate good concurrence to experimental data.

Introduction

Known procedures of calculation of process of ice formation upon a cooled wall of a stationary value of temperature [1,2] cannot be used at calculations of thermoelectric ice generators with the cellular shape (further - TEIG). One of the main reasons is the variability of shape temperature at all phases of the process of ice production. This feature is characteristic for thermoelectric ice-makers with thermal defrosting of ice from walls by reversing a current of feed of a thermoelectric refrigerator set. On the basis of the new approach of calculation of a thermoelectric set as object with known load characteristics enunciated in articles [3,4], in a paper [5], the example of calculation of TEIG productivity with unilateral freezing of ice on a flat wall and in cells of an ice form was submitted. This procedure and results of calculations made with it's help, are accepted by us as the base variant for the comparative analysis.

Unilateral ice freezing of different shapes

The freezing of a layer of ice by width δ_l can be viewed as a linear problem in a boundless bath, with height **h**, which is small as contrasted to equivalent diameter $\mathbf{d_{ekw}}$. These requirements can be presented as: $\mathbf{h/d_{ekw} \to 0}$. Then, the effect of lateral areas of a bath on a dynamics of ice growth can be neglected. Let's take the advantage of the formula (1) and form a definition of time τ_z of freezing of ice on a flat wall:

$$\tau_z = \frac{r \cdot \rho_l \cdot (\delta_l)^2}{2 \cdot \lambda_l \cdot \Delta T} \qquad (1)$$

where:

$$\Delta T = \frac{((T_0 - T_{fp}) + (T_0 - T_k))}{2} \qquad (2)$$

r - specific melting heat, ρ_l , λ_l , δ_l, - density, thermal conduction and ice depth, accordingly, $\mathbf{T_0}$ - solidification temperature, $\mathbf{T_{fp}}$, $\mathbf{T_k}$ - initial and terminal temperature of a wall of an ice form

Here, as well as in all subsequent formulas for the freezing and the defrosting time, the definition of the average arithmetical temperature of a wall during a time interval between the beginning and the end of the process is used. The linear character of change of the temperature in time is acknowledged by numerous experimental data obtained on a TEIG model with different ice forms. Generalizing upon major volume of experimental data, the temperature is assumed to be: $\mathbf{T_{fp}}$ = (-1 ...-3) °C.

Accordingly, for freezing on an interior and an exterior surface of the cylinder:

$$\tau_z = \frac{-r \cdot \rho_l}{2 \cdot \Delta t} \cdot \left[\frac{r_x^2}{\lambda_l} \cdot \ln\left(\frac{r_w}{r_x}\right) - \frac{1}{2 \cdot \lambda_l} \cdot \left(r_w^2 - r_x^2\right) \right] \qquad (3)$$

$$\tau_z = \frac{r \cdot \rho_l}{2 \cdot \Delta t} \cdot \left[\frac{r_x^2}{\lambda_l} \cdot \ln\left(\frac{r_x}{r_z}\right) - \frac{1}{2 \cdot \lambda_l} \cdot \left(r_x^2 - r_z^2\right) \right] \qquad (4)$$

where: r_z – radius of an interior cell , r_w – radius of an exterior cell , r_x – relative radius of a layer of ice

The time of complete freezing of a ball with a radius R:

$$\tau_{zamr} = \frac{\rho_l \cdot r \cdot (R)^2}{6 \cdot \lambda_l \cdot \Delta T} \qquad (5)$$

If the heat exchange between a free surface of water in a hemispherical form and an ambient air was neglected, the formula (5) could be used also for calculation of freezing of a hemisphere.

In case, when into an ice form, the water with temperature $\mathbf{T_w}$ is flooded, that is higher than 0°C, it is necessary to take the time of cooling of water up to a solidification temperature into account. Then, in formulas 1,3,4,5, we use a difference of enthalpies Δh instead of **r**:

$$\Delta h = r + c\,(T_w - T_0) \qquad (6)$$

For the increased precision of calculation, V.A. Bobkov [1] recommends to take the heat of supercooling of ice into account, which participation in an overall enthalpy usually does not exceed 2%. As contrasted to the freezing of ice on a flat wall or on pipes, where the heat capacity of a wall can be neglected, the cellular aluminium shapes of a TEIG have a major thermal lag. It is necessary to have this effect in mind, while calculating the time of defrosting. Hence, to maintain the logical uniformity of a calculation, the heat capacity of an ice form should be also regarded in overall enthalpy Δh and thus, it is necessary to enumerate this quantity onto a heat capacity of water. Applying the above changes, the formula (6) can be re-written as follows:

$$\Delta h = r_0 + c_w (T_w - T_{kr}) + A\, c_l\, (T_{kr} - T_k) + k_f c_f\, [(T_{kr} + 2) - T_k] \qquad (7)$$

where: A - coefficient depending on the shape of a mesh (for a flat wall A=1/2, for a hemisphere- A=5/6), T_k - terminal temperature of the shape, (T_{kr} + 2) - initial temperature of the shape, which, basing on experimental data, is accepted as a stationary value equal to temperature at the end of defrosting process, k_f - coefficient of metal consumption or otherwise scaling ratio; T_{kr} - solidification temperature,

$$k_f = \delta_f c_f / \delta_{ekw}\, c_l \qquad (8)$$

δ_{ekw} - equivalent ice depth, δ_f - mean width of an ice form, c_w, c_l, c_f - specific mass heat capacity of water, ice and ice form accordingly

The analysis of the conducted calculations displays, that a participation of the component $k_f c_f$ [(T_{kr} +2)-T_k] in overall quantity Δh oscillates in a scope from 0,2 % up to 1,0 %, depending on a construction of the shape.

The terminal temperature of the shape T_k is an unknown quantity in formulas (2) and (7). To define it, we shall take advantage of an equation of a heat flow continuity through ice - shape wall border in the moment of termination of a process of crystallization. The ice form with x_0 of cells is cooled by the use of *x* thermoelectric modules. Now a unit cell of volume V_0 with a surface F_k shall be distinguished in the shape, to which corresponds a known part of an entire cold productivity of a set (i.e. all modules). Then, it is possible to write down:

$$F_k\,(T_0 - T_k)\,\lambda_l / \delta_{ekw} = x/x_0\, Q_0 \qquad (9)$$

where: Q_0 - cold productivity of the module. It is possible to take not inflows of heat through isolation into account, seeing how small they are. In case of unilateral freezing, it is accepted to assume x/x_0 = 1.

Utilizing equations as shown in [3,4], the dependence (10) that defines the thermoelectric module's cold productivity can be written down as (11):

$$Q_0 = c_\Delta\, \Delta T_{max} [\,1 - (1 - I/I_{opt})^{2+c_i}\,] - c_t (T_W - T_g) - c_\Delta (T_g - T_Z) \qquad (10)$$

$$Q_0 = Q_0^* - c_\Delta (T_g - T_Z) \qquad (11)$$

where: Q_0^* - peak cold productivity of the module at a current of feed intensity **I** and temperature of the hot part of the module, T_g, T_z - temperature of the hot and cold part of the module accordingly, c_Δ - load coefficient. For a given module type quantities Q_0^* and c_Δ are fixed and specified by a producer in the technical data of a product.

Having specified $K = F_k \lambda_l / \delta_{ekw}$ and accepting $T_z = T_k$, after substitution of (11) into (9) we obtain:

$$K\,(T_0 - T_k) = Q_0^* - c_\Delta\,(T_g - T_k) \qquad (12)$$

Now (12) is solved, regarding T_k:

$$T_k = \frac{K T_0 + c_\Delta T_g - Q_0^*}{c_\Delta + K} \qquad (13)$$

More precise variant of calculation takes the thermal resistance of layers of heat-conducting paste on work sides of the module into account. Then, instead of temperatures T_g and T_z, corrected temperatures T_g^* and T_z^* are used. So, $T_g < T_g^*$ and $T_z > T_z^*$.

Having defined T_k, it is possible to spot the mean temperature of the shape during freezing, by the use of the formula (2).

Bilateral freezing in a TEIG with hemispherical cells

It is accepted as settlement, that during calculations, the second (or any subsequent) cycle of ice production is investigated, where the portion of water is flooded into the previously refrigerated ice form, as shown in the formula (7). The physical model of bilateral freezing differs from the model of unilateral freezing by the fact, that the layer of ice is considered thin enough to make the concept of equivalent width unusable. The total width of a piece of ice (without the width loss at defrosting) is represented by a subtraction of radiuses of a cell and insertion (fig. 1).

$$\delta = R - r \qquad (14)$$

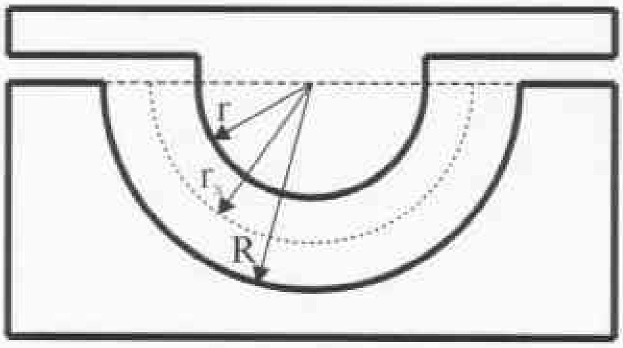

Fig. 1. The plan of a hemispherical cell of an ice-maker with bilateral freezing: R - radius of a cell, r - radius of an insertion, r_x - relative radius of a layer of ice.

The defining of dependence of the time of production of ice and productivity of an ice-maker from radius of a cooled insertion is an interesting question. In the beginning, it is stated that half of the width δ is freezed upon the lower part of the shape, i.e. the cell, and second half - upon the upper part of the shape, i.e. the insertion. In the calculations that followed, the scope of change of an insertion radius was accepted as 2 up to 20 mm. It corresponds to the change of the width of each half-layer from 1 up to 10 mm. It is also stated, that cold productivities of sets of the upper and lower shape are equal, and their sum makes for the cold productivity of base variant, i.e. as in case of the unilateral freezing. In computational dependences, this sectionalization is submitted in the number $x_0/2$ of modules, freezing each shape.

Calculations are begun by defining the parameter **K** for each radius of an insertion:

$$K_i = (F_k)_i \lambda_l / (\delta_l)_i \qquad (15)$$

Further, terminating temperatures of the shapes $(T_k)_{gf}$ and $(T_k)_{df}$ are determined:

$$(T_k)_i = \frac{K_i T_0 + (x_0/x) c_\Delta T_g - (x_o/x) Q_0^*}{(x_0/x) c_\Delta + K_i} \qquad (16)$$

and mean differences of temperatures $(\Delta T_s)_i$ are determined after substitution $(T_k)_i$ in the formula (2). Thus, it is should be remembered, that along with the change of the insertion's radius, coefficient k_f varies also. While calculating parameters of an insertion, the change of it's width can be taken into account in the simplified view, for example under the linear law from minimal up to the maximal value. As long as the participation of the component $(k_f)_i\, c_f\, [(T_{kr} + 2) - (T_k)_i]$ in overall difference of enthalpies does not exceed 1-2 %, such simplification practically does not influence the precision of calculations.

Time of freezing of each half-layer is determined by Lykov`s dependences [2] for spherical layers. For a layer iced outside (the cell):

$$\tau_z(r_x) = \frac{\rho_l \Delta h_i (r_x - R)^2 (2r_x + R)}{6\lambda_l (\Delta T_i) R} \qquad (17)$$

For a layer iced from within (the insertion):

$$\tau_z(r_x) = \frac{\rho_l \Delta h_i (r_x - r_i)^2 (2r_x + r_i)}{6\lambda_l (\Delta T_i) r_i} \qquad (18)$$

The moment of meeting of moving towards each other crystallization fronts is considered the moment of the termination of the freezing process. For this purpose, the combined equations (17) and (18) are solved in coordinates τ-r_x with the MATHCAD program. For increased precision of the calculation, the problem can be solved by a method of successive approximations, taking width differences of half-layers into account in the second run-through and adjusting the corresponding quantities in formulas (15), (17), (18).

In the following step, we determine the time of defrosting. The greatest impact upon this time is made by a thermal rating of a set and the temperature of a shape in the beginning of a defrosting process, that was spotted in the previous step as the terminal temperature of the shape during the freezing.

The required thermal rating of a set (or a separate module) Q_g consists of the cold productivity in the appropriate mode of operation and the electrical power **W**, consumed by a set in a phase of defrosting, what is followed in the equation of the thermal balance of the thermoelectric module (19):

$$Q_g = (Q_0)_{odmr} + W \qquad (19)$$

The cold productivity of the module in a mode of defrosting is defined according to the formula (10) with such a difference, that it is necessary to accept the temperature of the flowing water on an inlet into the heat exchanger as a value of T_z, and as a value T_g – the average arithmetical temperature of the shape during defrosting. At the beginning of the process of a separation of ice from walls of an ice form, the temperature of it's wall equals 2°C Accordingly, the temperature sensor of a termination of a defrosting process is usually attuned to this temperature. In view of this fact:

$$T_{g(odmr)} = \tfrac{1}{2}(T_{fk} + T_{kr} + 2) \qquad (20)$$

Power, consumed by the module in a mode of defrosting:

$$W = (I)_{odmr}^{\ 2} R \qquad (21)$$

where **R** - electric resistance of the module. For the simplification of calculation, the quantity $(I)_{odmr}$ can be accepted as a stationary value and equals the current of feed in a mode of freezing. In practical application, once after switching into a mode of defrosting, the current of the module increases approximately by 25%, but then it is promptly returned to the tentative value. The comparison of the settlement and the experimental data displays, that the simplification in calculations does not lead to increase of the calculated time of defrosting, as the procedure shown does not take the reverse effect of series of other factors into account, for example, masses of auxiliary elements of a construction of an ice form.

The following assumption can be made, that because of a low thermal conductivity of ice during defrosting, all it's mass doesn't have enough time to get warmed above the cryoscopic temperature and only the warmed-up part of the mass **Δm** gets thawed.

Ultimately, defining formula of the time of defrosting on each shape:

$$\tau_{odmr}(i) = \frac{(F_k)_i \{(\delta)_{odmr} \rho_l [r_0 + c_l (T_{kr} - T_{k_i})] + \delta_{f_i} \rho_f c_f (T_{kr} - T_{k_i} + 2)\}}{x / x_0 Q_{odmr}} \qquad (22)$$

where: ρ_l and ρ_f - density of ice and a material of an ice form.

The analysis of calculations results displays, that the time amount of defrosting of ice on an ice form is greater than time of defrosting on an insertion, independently of the size of an insertion. As the insertion is used for retrieving the ice from a cell, for exact operation of an ice-maker it is required at first to liberate the ice from the lower shape. From then, it follows that under definition of an overall time of production cycle of ice in the formula (23), times of defrosting are added up:

$$\tau_c = \tau_z + (\tau_{odmr})_{gf} + (\tau_{odmr})_{df} \qquad (23)$$

Productivity of an ice-maker:

$$G = (m_w - \Delta m)/\tau_c \qquad (24)$$

where: m_w - mass of water, poured into the shape, $m = V_k \rho_w$, **Δm** - losses of ice mass during defrosting;

$$\Delta m = (\delta)_{odmr} \rho_l F_k \qquad (25)$$

From experience of TEIG operation, it follows, that the width of a layer of ice $(\delta)_{odmr}$ makes 0,5 … 0,7 mm and depends on a surface roughness of the shape and force needed to separate a piece of ice from a shape surface.

The brief analysis of calculations results

Results of evaluations for an ice-maker LNT-0,5, presented in the book [4], are submitted in a fig. 2. The maximum of functions of absolute and relative productivity takes it's place close to the radius of an insertion of 18 mm, that corresponds to the ice depth of 4 mm. At the further diminution of an ice depth, despite of the diminution of time of freezing, the productivity of an ice-maker drops, because of the dominant effect of ice losses at defrosting.

Installing the hemispherical cooled insertion by the radius of 18 mm into the cell results in the diminution of the cell's volume by 45 %. At the same time, in the paper [7], it is agreed, that the optimum size (having a requirement G = max in mind) of uncooled insertions makes 8-12 % of volume of the cell.

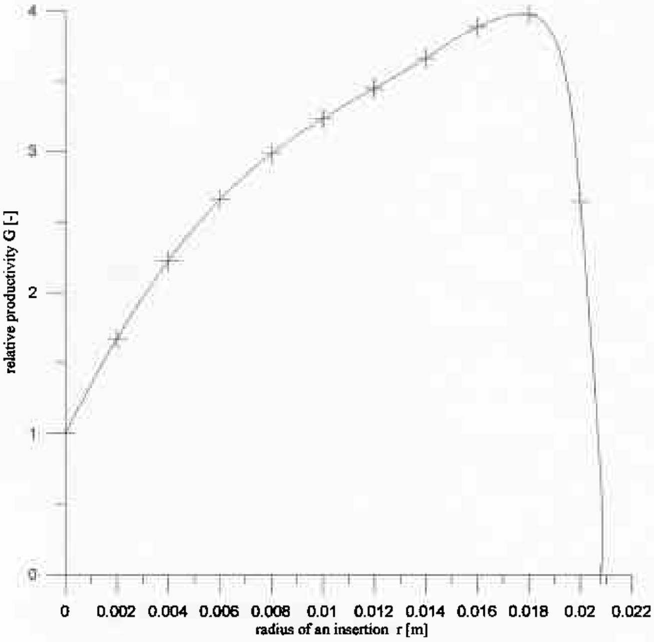

Fig. 2. Dependence of the relative productivity of a thermoelectric ice-maker with hemispherical cells from the radius of an insertion, where G_b - productivity of the base variant without an insertion.

Results of conducted calculations demonstrate the satisfactory concurrence to the experimental data obtained on the prototype of the LNT-0,5 ice-maker.

Conclusions

The offered procedure of calculation of a TEIG shows it's advantage in the combined view of different modes of the thermoelectric module operation, both in processes of the freezing and the defrosting of ice in a cell. The sectional requirement is especially actual in case of the bilateral freezing, when it is necessary to take differences in modes of operation of the upper and lower refrigerator sets into account. The bilateral freezing presents extra opportunities of the TEIG parameters optimization, involving the selection of a cell and insertion shape, the relative size of an insertion, the relation of sets productivity, making for the optimum algorithm of execution of separate operations of process of ice production.

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