Trygonometria

$$\sin^2 x = \frac{\operatorname{tg}^2 x}{1+\operatorname{tg}^2 x}, \quad \cos^2 x = \frac{1}{1+\operatorname{tg}^2 x}, \quad \sin x \cos x = \frac{\operatorname{tg} x}{1+\operatorname{tg}^2 x}$$

$$\sin x = \frac{2\operatorname{tg}(x/2)}{1+\operatorname{tg}^2(x/2)}, \quad \cos x = \frac{1-\operatorname{tg}^2(x/2)}{1+\operatorname{tg}^2(x/2)}, \quad \operatorname{tg} x = \frac{2\operatorname{tg}(x/2)}{1-\operatorname{tg}^2(x/2)}$$

$$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y, \quad \cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$$

$$\sin x + \sin y = 2\sin\frac{x+y}{2}\cos\frac{x-y}{2}, \quad \sin x - \sin y = 2\sin\frac{x-y}{2}\cos\frac{x+y}{2}$$

$$\cos x + \cos y = 2\cos\frac{x+y}{2}\cos\frac{x-y}{2}, \quad \cos x - \cos y = -2\sin\frac{x+y}{2}\sin\frac{x-y}{2}$$

$$\sin x \sin y = \frac{\cos(x-y)-\cos(x+y)}{2}$$

$$\cos x \cos y = \frac{\cos(x-y)+\cos(x+y)}{2}$$

$$\sin x \cos y = \frac{\sin(x-y)+\sin(x+y)}{2}$$

Całka nieoznaczona

$$\int \frac{1}{x^2+k^2}dx = \frac{1}{k}\operatorname{arctg}\frac{x}{k}+c, \quad \int \frac{1}{x^2-k^2}dx = \frac{1}{2k}\ln\left|\frac{x-k}{x+k}\right|+c$$

$$\int \frac{1}{\sqrt{x^2+k}}dx = \ln|x+\sqrt{x^2+k}|+c, \quad \int \frac{1}{\sqrt{k^2-x^2}}dx = \arcsin\frac{x}{k}+c$$

$$\int \frac{1}{(1+x^2)^n}dx = \frac{x}{(2n-2)(1+x^2)^{n-1}} + \frac{2n-3}{2n-2}\int \frac{1}{(1+x^2)^{n-1}}dx$$

Równania zupełne (czynnik całkujący dla równania $P(x,y)dx+Q(x,y)dy=0$)

$$\mu(x) = \exp\left(\int \varphi(x)\,dx\right), \text{ jeżeli } \frac{P'_y(x,y)-Q'_x(x,y)}{Q(x,y)} = \varphi(x)$$

$$\mu(y) = \exp\left(\int \varphi(y)\,dy\right), \text{ jeżeli } -\frac{P'_y(x,y)-Q'_x(x,y)}{P(x,y)} = \varphi(y)$$

$$\mu(x\pm y) = \exp\left(\int \varphi(t)\,dt\right)|_{t=x\pm y}, \text{ jeżeli } \frac{P'_y(x,y)-Q'_x(x,y)}{Q(x,y)\mp P(x,y)} = \varphi(x\pm y)$$

$$\mu(xy) = \exp\left(\int \varphi(t)\,dt\right)|_{t=xy}, \text{ jeżeli } \frac{P'_y(x,y)-Q'_x(x,y)}{yQ(x,y)-xP(x,y)} = \varphi(xy)$$

$$\mu(x/y) = \exp\left(\int \varphi(t)\,dt\right)|_{t=x/y}, \text{ jeżeli } \frac{y^2(P'_y(x,y)-Q'_x(x,y))}{xP(x,y)+yQ(x,y)} = \varphi(x/y)$$

Transformata Laplace'a: $\mathcal{L}\{f\}|_s = F(s), \ \mathcal{L}\{g\}|_s = G(s)$

$$\mathcal{L}\{t\to t^n\}|_s = \frac{n!}{s^{n+1}}, \quad \mathcal{L}\{t\to e^{at}\}|_s = \frac{1}{s-a}, \quad \mathcal{L}\{\sin\}|_s = \frac{1}{1+s^2}, \quad \mathcal{L}\{\cos\}|_s = \frac{s}{1+s^2}$$

$$\mathcal{L}\left\{f^{(n)}\right\}\Big|_s = s^nF(s) - s^{n-1}f(0) - s^{n-2}f'(0) - \ldots - sf^{(n-2)}(0) - f^{(n-1)}(0)$$

$$\mathcal{L}\{t\to t^nf(t)\}|_s = (-1)^n F^{(n)}(s)$$

$$\mathcal{L}\{t\to f(at)\}|_s = \frac{1}{a}F\left(\frac{s}{a}\right)$$

$$\mathcal{L}\{t\to e^{at}f(t)\}|_s = F(s-a)$$

$$\mathcal{L}\left\{t\to \int_0^t f(x)\,dx\right\}\Big|_s = \frac{F(s)}{s}$$

$$\mathcal{L}\{f*g\}|_s = F(s)\,G(s)\,, \text{ gdzie } (f*g)(t) = \int_0^t f(x)\,g(t-x)\,dx$$