

### COMPUTATIONAL INTELLIGENCE AND KNOWLEDGE ENGINEERING

## Associative Graph Data Structures AGDS with an Efficient Access via AVB+trees



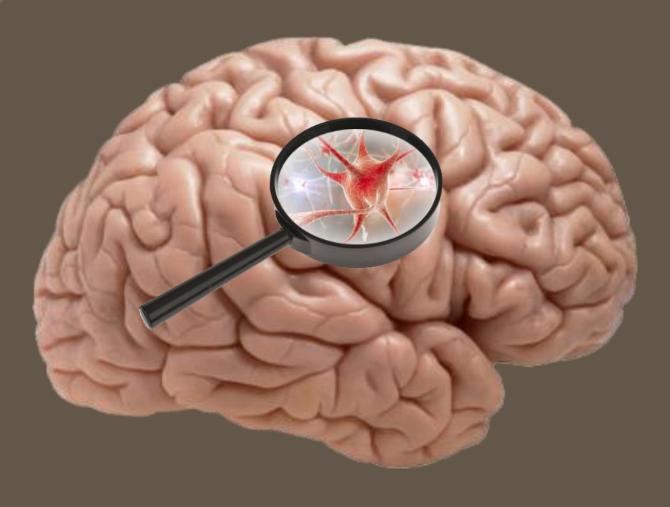


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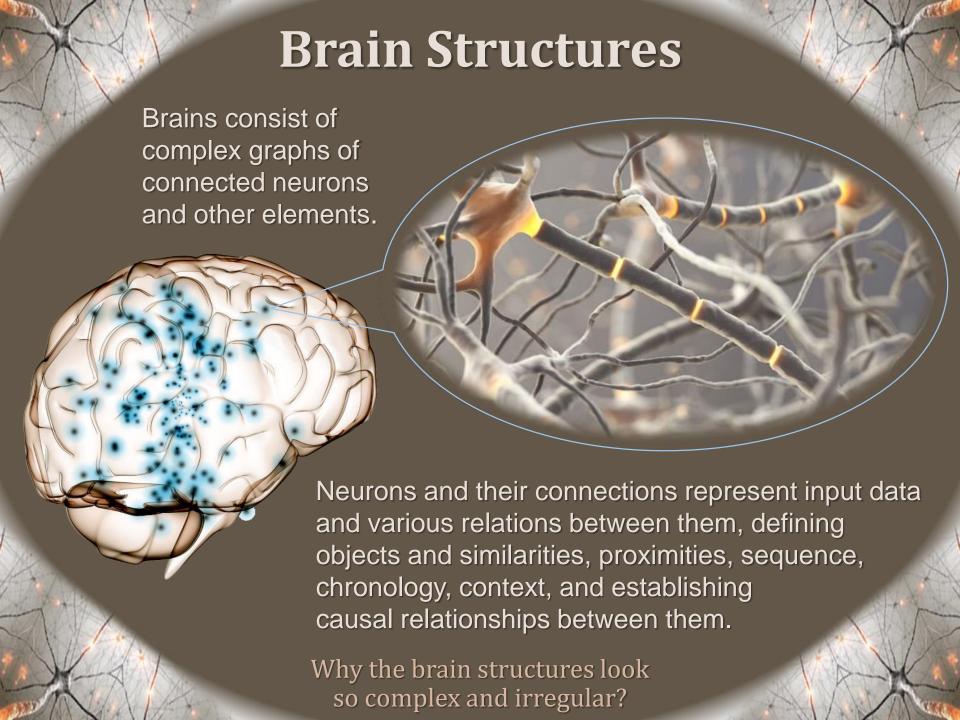
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#### How do they really work?

How we can use brain-like structures to make computations more efficient and intelligent?



### Data Tables

In computer science, we mostly use tables to store, organize and manage data,

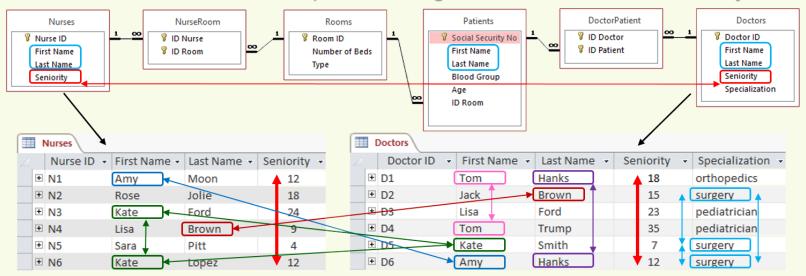
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<b>OBJECTS</b>	LENGTH	WIDTH	LENGTH	WIDTH	LABEL
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02	6.3	3.3	4.7	1.6	Versicolor
О3	6.0 ▲	2.7	5.1	1.6	Versicolor
04	6.7	3.0	5.0	1.7	Versicolor
<b>O</b> 5	6.0	2.2	5.0	1.5	Virginica
06	5.9	3.2	4.8	1.8	Versicolor
07	6.0 🕈	3.0	4.8	1.8	Virginica
08	5.7	2.5	5.0	2.0	Virginica
O9	6.5	3.2	5.1	2.0	Virginica

but common **relations** like identity, similarity, neighborhood, minima, maxima, number of duplicates **must be found**. The more data we have the bigger time loss we face!

Such relations are not enough!

#### **Relational Databases**

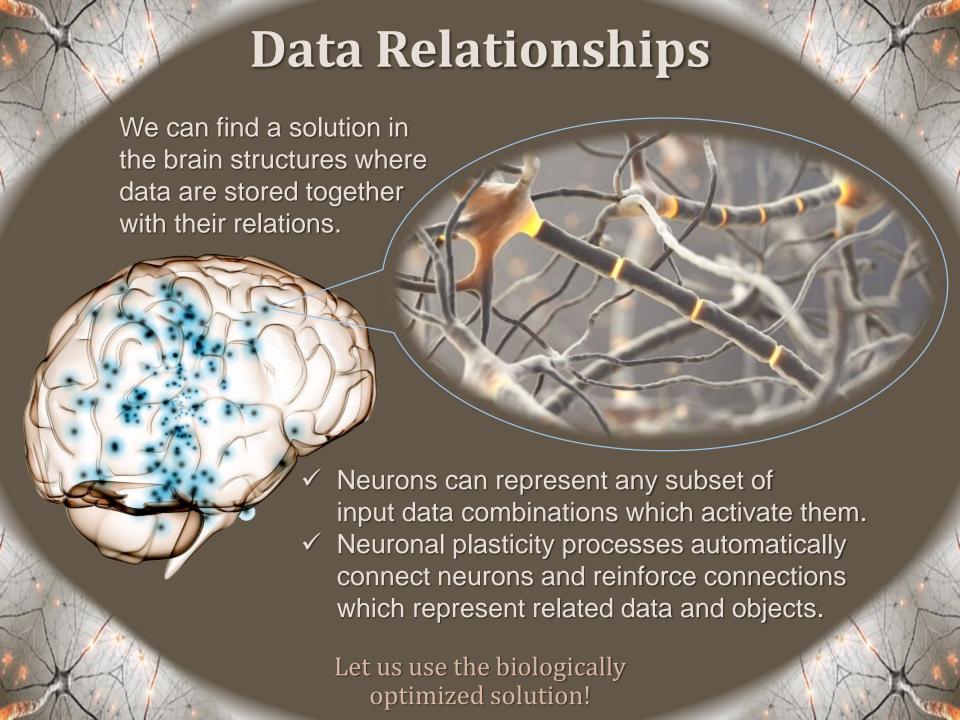
Relational databases relate stored data only horizontally, not vertically, so we still have to search for duplicates, neighbor or similar values and objects.



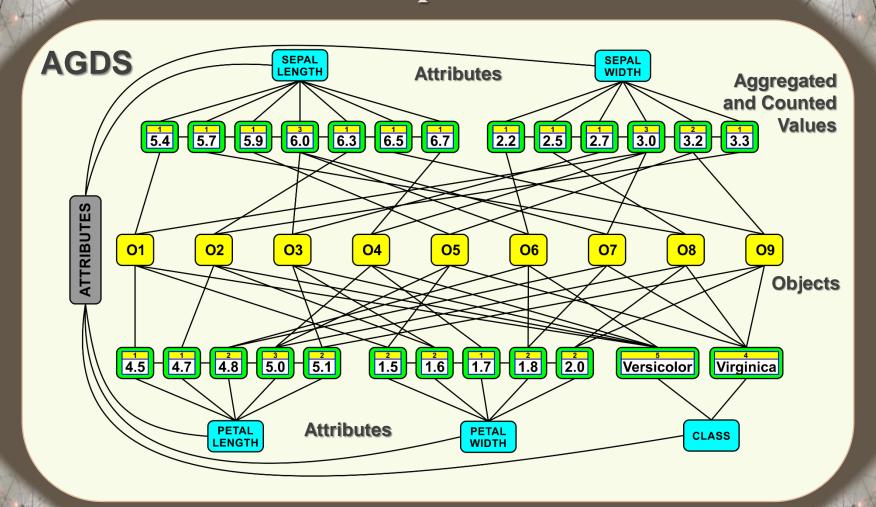
Even horizontally, data are not related perfectly and many duplicates of the same categories occur in various tables which are not related anyhow. In result, we need to lose a lot of computational time to search out necessary data relations to compute results or make conclusions.



Is it wise to lose the majority of the computational time for searching for data relations?!



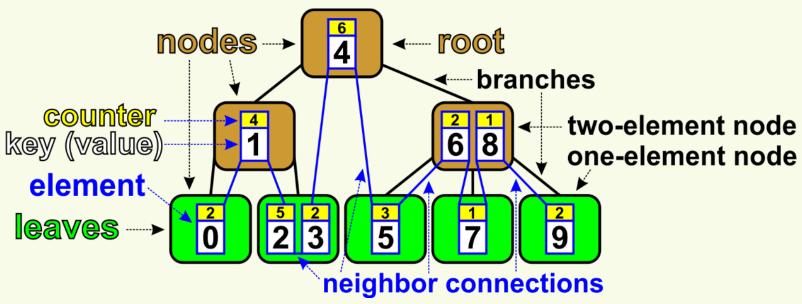
## AGDS Associative Graph Data Structure



Connections represent various relations between AGDS elements like similarity, proximity, neighborhood, definition etc.

## **AVB+Trees**Sorting Aggregated-Value B-Trees

An AVB+tree is a hybrid structure that represent sorted list of elements which are quickly accessed via self-balancing B-tree structure. Elements aggregate and count up all duplicates of represented values.



AVB+trees are typically much smaller in size and height than B-trees and B+trees thanks to the aggregations of duplicates and not using any extra internal nodes as signposts as used in B+trees.

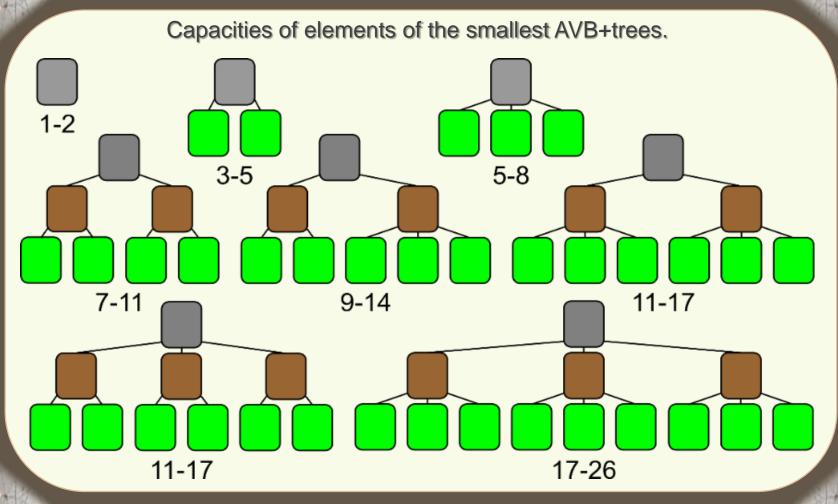
Internal states of APN neurons are updated only at the end of internal processes (IP) that are supervised by the Global Event Queue.

#### **Properties of AVB+trees**

- ✓ Each tree node can store one or two elements.
- ✓ Elements aggregate representations of duplicates and store counters of aggregated duplicates of values.
- ✓ Elements are connected in a sorted order, so it is possible to move between neighbor values very quickly.
- ✓ AVB+trees do not use extra nodes to organize access to the elements stored in leaves as B+trees.
- ✓ AVB+trees use all advantages of B-trees, B+trees, and AVB-trees removing their inconvenience.
- ✓ They implement common operations like Insert, Remove, Search, GetMin, GetMax, and can be used to compute Sums, Counts, Averages, Medians etc. quickly.
- ✓ They supply us with sorted lists of elements which are quickly accessible via this tree structure and thanks to the aggregations of duplicates that substantially reduce the number of elements storing values.

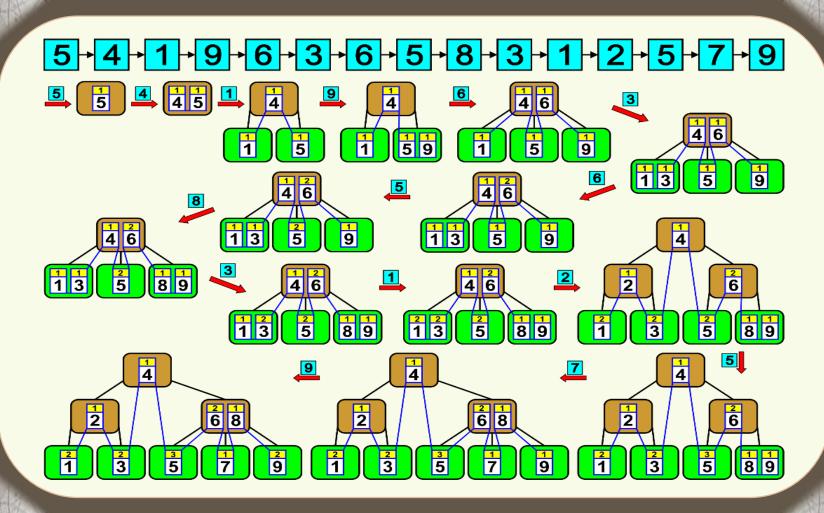
Efficient hybrid structure!

## **Capacity of AVB+Trees**



The same number of elements can be stored by various AVB-tree structures, e.g. 11 or 17 elements!

# Insert Operation on AVB+Trees



AVB+trees self-balance, self-sort and self-organize the structure during the insert operation!

### **Insert Operation**

#### The Insert operation on the AVB+tree is processed as follows:

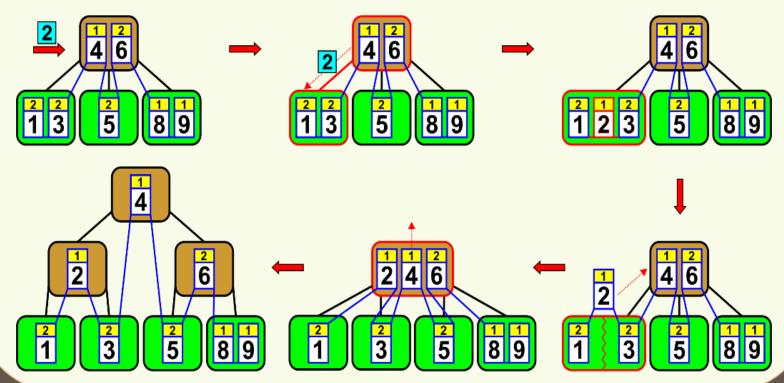
- 1. Start from the root and go recursively down along the branches to the descendants until the leaf is not achieved after the following rules:
- if one of the elements stored in the node already represents the inserted key, increment the counter of this element, and finish this operation;
- else go to the left child node if the inserted key is less than the key represented by the leftmost element in this node;
- else go to the right child node if the inserted key is greater than the key represented by the rightmost element in this node;
- else go to the middle child node.
- 2. When the leaf is achieved:
- and if the inserted key is already represented by one of the elements in this leaf, increment the counter of this element, and finish this operation;
- else create a new element to represent the inserted key and initialize its counter to one, next insert this new element to the other elements stored in this leaf in the increasing order, update the neighbor connections, and go to step 3.

### **Insert Operation**

- 3. If the number of all elements stored in this leaf is greater than two, divide this leaf into two leaves in the following way:
- let the divided leaf represent the leftmost element representing the least key in this node together with its counter;
- create a new leaf and let it represent the rightmost element representing the greatest key in this node together with its counter;
- and the middle element (representing the middle key together with its counter) and the pointer to the new leaf representing the rightmost element pass to the parent node if it exists, and go to step 4;
- if the parent node does not exist, create it (a new root of the AVB+tree) and let it represent this middle element (representing the middle key together with its counter), and create new branches to the divided leaf representing the leftmost element and to the leaf pointed by the passed pointer to the new leaf representing the rightmost element. Next, finish this operation.

## Rebalancing during Insert Operation

A self-balancing mechanism of an AVB+tree during the Insert operation when adding the value (key) "2" to the current structure which must be reconstructed because the node is overfilled and must be divided.



Self-balancing and self-sorting mechanism of the Insert Operation when a node is overfilled and must be divided!

## **Insert Operation**

- 4. Insert the passed element between the element(s) stored in this node in the key increasing order after the following rules:
- if the element has come from the left branch, insert it on the left side of the existing element(s) in this node;
- if the element has come from the right branch, insert it on the right side of the existing element(s) in this node;
- if the element has come from the middle branch, insert it between the existing element(s) in this node.
- 5. Create a new branch to the new node (or leaf) pointed by the passed pointer and insert this pointer to the child list of pointers immediately after the pointer representing the branch to the divided node (or leaf).

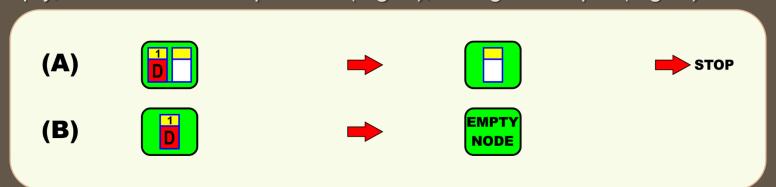
## **Insert Operation**

- 6. If the number of all elements stored in this node is greater than two, divide this node into two nodes in the following way:
- let the existing node represent the leftmost element representing the least key in this node together with its counter;
- create a new node and let it represent the rightmost element representing the greatest key in this node together with its counter;
- the middle element (representing the middle key together with its counter) and the pointer to the new node representing the rightmost element pass to the parent node if it exists; and go back to step 4;
- if the parent node does not exist, create it (a new root of the AVB+tree) and let it represent this middle element (representing the middle key together with its counter), and create new branches to the divided node representing the leftmost element and to the node pointed by the passed pointer to the new node representing the rightmost element. Next, finish this operation.

- ✓ The Remove operation allows to remove a key from the AVB+tree structure and next quickly rebalance and reorganize the structure automatically if necessary.
- ✓ If the removed key is duplicated in the current structure, then only the counter of the element which represents it is decremented.
- ✓ When the removed key is represented by the element which counter is equal one then the element is removed from the node.
- ✓ If this node is a leaf containing only a single element, then the leaf is removed as well, and a rebalancing operation of the AVB+tree is executed.

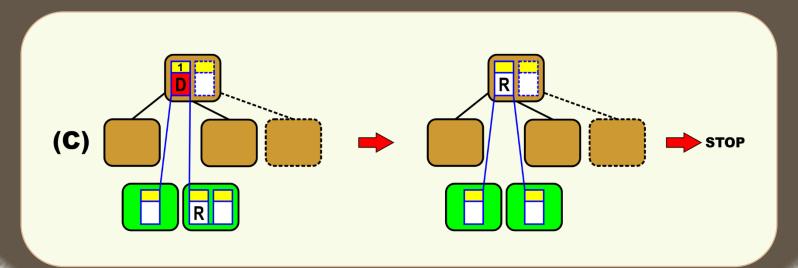
#### The Remove operation on the AVB+tree is processed as follows:

- 1. Use the search procedure to find an element containing the key intended for removal. If this key is not found in the tree, finish the delete operation with no effect;
- 2. Else if the counter of the element storing the removed key is greater than one, decrement this counter, and finish the delete operation.
- 3. Else if the element storing the removed key is a leaf, then remove the element storing this key from this leaf, switch pointers from its predecessor and successor to point themselves as direct neighbors. Next, if this leaf is not empty, finish the delete operation (Fig. A), else go to step 7 (Fig. B).



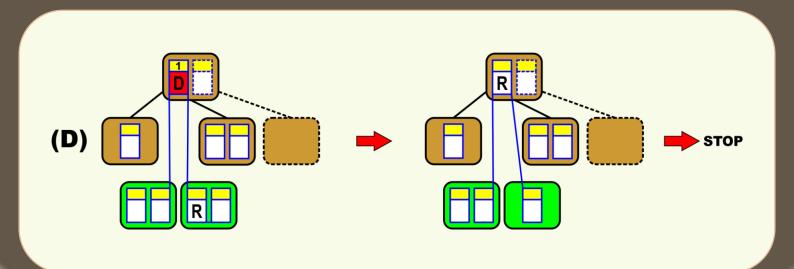
#### The Remove operation on the AVB+tree is processed as follows:

4. Else the element storing the removed key is a non-leaf node that must be replaced by one of the neighbor connected elements stored in one of two leaves. If only one leaf from the leaves containing neighbor elements to the removed element contains two elements, then replace the removed element in the non-leaf node by this connected neighbor element from the leaf containing two elements, and finish the delete operation (Fig. 14C), else go to step 5.



#### The Remove operation on the AVB+tree is processed as follows:

5. Here, both leaves containing a neighbor element to the removed one contain two elements or one element both. In this case, check which one of the neighbor child nodes contains more elements. Next, replace the removed element by the neighbor element stored in the leaf of the subtree which root contains more elements, and finish the delete operation (Fig. D) in case when no leaf left without any element or go to step 8 when there is left an empty node; else go to step 6.



#### The Remove operation on the AVB+tree is processed as follows:

6. Here, both neighbor child nodes contain the same number of elements. In this case, check whether the key stored in the rightmost element from the left neighbor child or the key stored in the leftmost element from the right neighbor child is more distant from the key stored in the removed element. The distance can be calculated differently dependently on compared data types. We can use different metrics for the string and numerical data types:

$$DISTANCE_{STR} = \sum_{i=1}^{max\{lengh(KEY_{1}), lengh(KEY_{2})\}} \frac{1}{|KEY_{1}[i] - KEY_{2}[i]| \cdot ||X||^{i-1}}$$

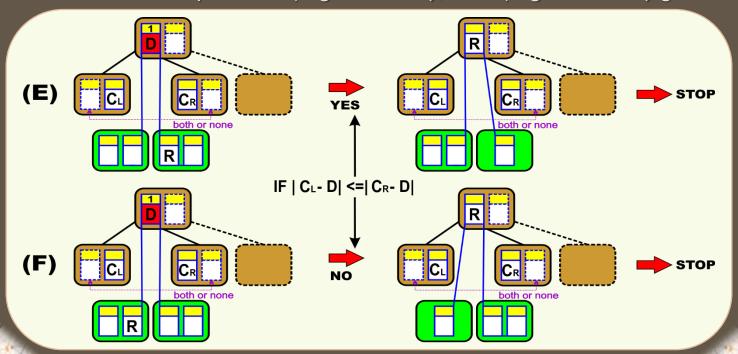
$$|KEY_1[i] - KEY_2[i]| = \begin{cases} distance(KEY_1[i], KEY_2[i]) \ in \ X \ if \ KEY_1[i] \in X \land KEY_2[i] \in X \\ ||X|| \qquad \qquad if \ KEY_1[i] \notin X \lor KEY_2[i] \notin X \end{cases}$$

$$DISTANCE_{NUM} = |KEY_1 - KEY_2|$$

where  $KEY_1[i] \in X$  means the i-th sign of the  $KEY_1$ -th string and  $|KEY_1[i] - KEY_2[i]|$  is equal to the number of signs (e.g. letters) between  $KEY_1[i]$  and  $KEY_2[i]$  in a given sign set X (e.g. ASCII), and ||X|| determines the number of signs in the set X.

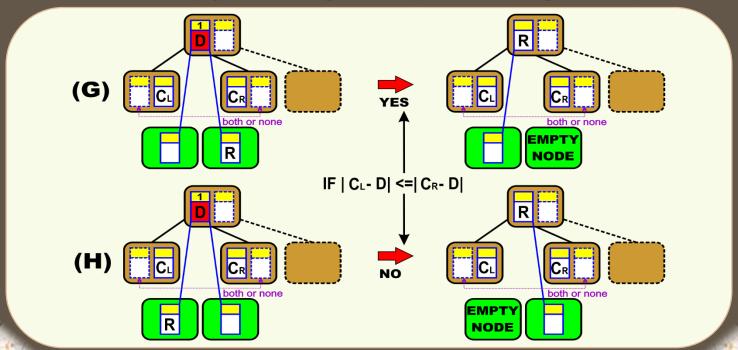
The Remove operation on the AVB+tree is processed as follows:

Next, replace the removed element by the right neighbor element when the distance of the key stored in this right neighbor child is greater or equal to the distance of the key stored in this left neighbor child (Fig. E or G), else replace it by the left neighbor element (Fig. F or H). If the leaves containing the neighbor elements contain two elements both, then finish the delete operation (Fig. E and F), else (Fig. G and H) go to step 7.



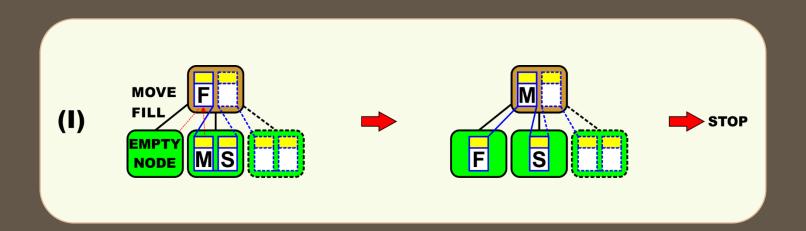
The Remove operation on the AVB+tree is processed as follows:

7. After the removal of the element from the leaf or after the replacement of the removed element from the non-leaf node by the leaf element, there is left an empty leaf (Fig. B, G, or H) that must be filled by at least one element or removed from the tree. Next, the tree must be rebalanced to meet the AVB+tree requirements. First, try to take an element from the nearest sibling. In these cases, remove the empty leaf and go to its parent, and go to step 8.



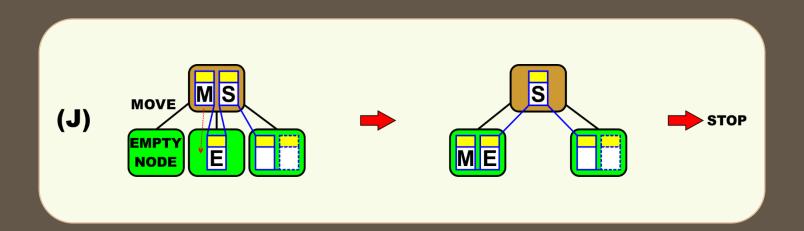
#### The Remove operation on the AVB+tree is processed as follows:

8. If the nearest sibling of the empty leaf contains more than a single element, then move the closest key (2 in Fig. I) to the removed one from the empty node to the parent, and move the neighbor element (1 in Fig. I) (to the removed one from the empty node) from the parent node to the empty leaf (Fig. I). Next, finish the delete operation.



#### The Remove operation on the AVB+tree is processed as follows:

9. Else if the nearest sibling of the empty leaf contains only a single element (2 in Fig. J), but its parent contains two elements, then move the closest parent element (1 in Fig. J) to the element removed from the empty node to this sibling in the right order, remove the empty node (Fig. J), and finish the delete operation.



#### The Remove operation on the AVB+tree is processed as follows:

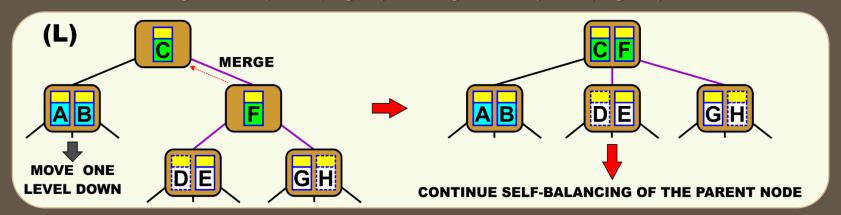
10. Else both the parent and the sibling contain only a single element. In this case, merge them in the parent node, moving the element from this sibling to its parent, and this parent node becomes to be a leaf which is placed one level higher than the other leaves (Fig. K). Hence, the tree must be rebalanced to meet the AVB+tree requirements in the subsequent routines described in the following steps.



11. In this and following steps, there is always one reduced subtree which is one level up, i.e. all its leaves are one level higher than the other leaves of the tree. The smallest subtree can consist of the leaf containing two elements. The rebalancing operation is started from the root of the reduced subtree in step 12.

#### The Remove operation on the AVB+tree is processed as follows:

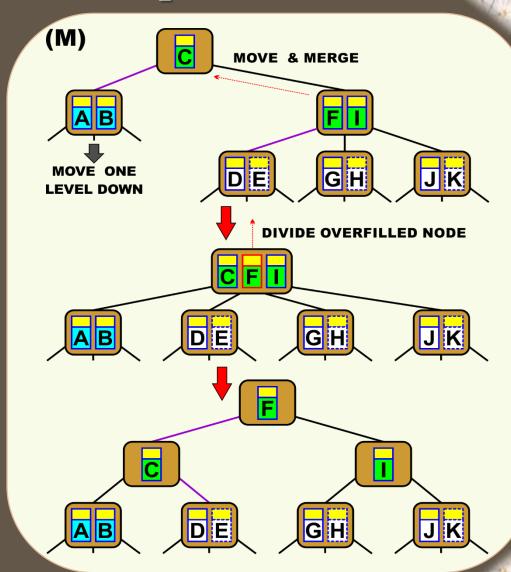
- 12. If the parent node of the root of the reduced subtree contains two elements go to step 16, else go to step 13.
- 13. If the second child of this parent contains a single element go to step 14 (Fig. L), else go to step 15 (Fig. M).



14. Merge this second child (containing a single element) with that parent as shown in Fig. L, and because the parent subtree of the reduced subtree has also lowered its height and must be rebalanced, go back to step 11 and rebalance the resultant subtree achieved after this transformation until the root of this subtree is not the root of the whole tree. If the main root is reached, it means that the tree is rebalanced and its height was lowered by one, therefore finish the deletion operation; else go to step 15.

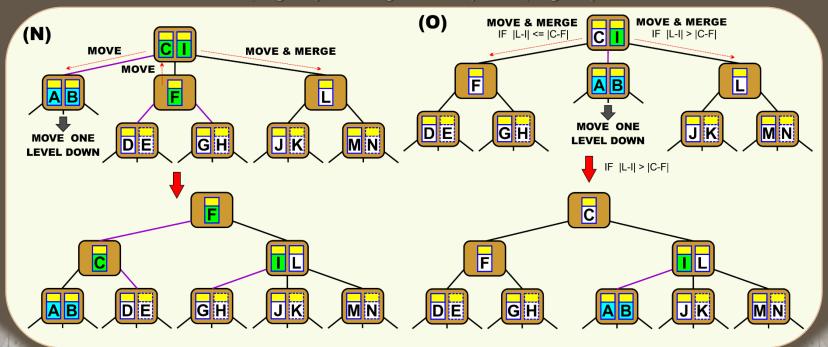
The Remove operation on the AVB+tree is processed as follows:

15. Merge this second child (containing two elements) with that parent as shown in Fig. M, and because the merged parent node is overfilled, divide it and create a new root of this subtree (Fig. M). Next, finish the delete operation.



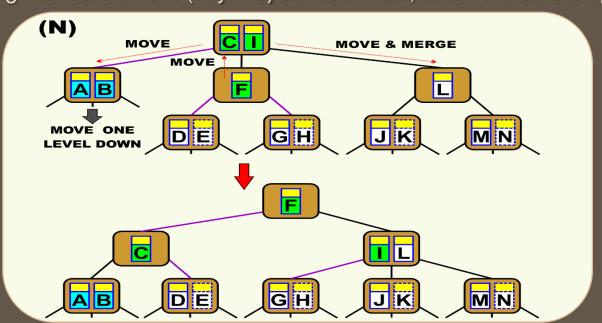
#### The Remove operation on the AVB+tree is processed as follows:

16. In this case, the parent node of the root of the reduced subtree contains two elements. If no one of the neighbor siblings of this reduced subtree root contains two elements, then go to step 17 (Figs. N and O), else go to step 20. 17. If this reduced subtree root is a left or right child of its parent, then go to step 18 (Fig. N), else go to step 19 (Fig. O).



The Remove operation on the AVB+tree is processed as follows:

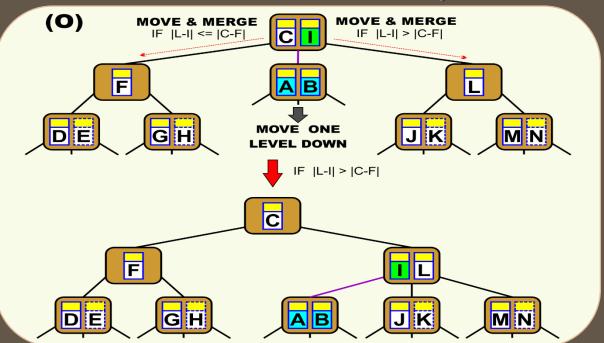
18. Move the element (key = 5) from the middle sibling to the parent node together with the pointers to the children of this node, and next move these pointers left and right together with the left (4|..) and right (6|..) nodes to the left and right children of the parent node appropriately as shown in Fig. N. Create a new parent (with key = 3) for the reduced subtree, also connecting this new parent to the node containing the passed left child node of the moved middle sibling (with key = 5). Connect this new parent (key = 3) to the parent node containing moved element (key = 5) as well. Next, finish the delete operation.



The Remove operation on the AVB+tree is processed as follows:

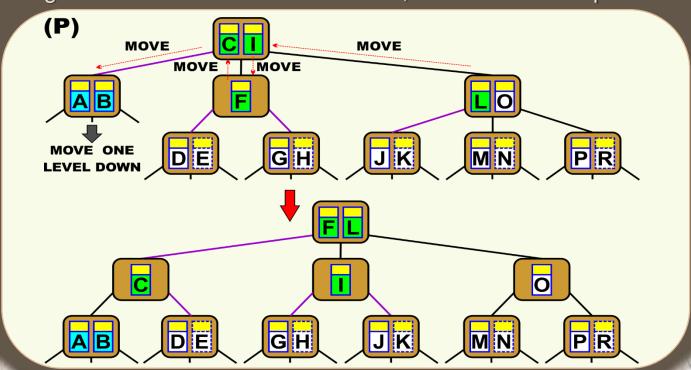
19. Merge the left or right element of the parent node of this reduced subtree root together with this subtree with the left or right child (that contains only a single element) as shown in Fig. O. Choose the child on the basis of the lower distance between the left parent element and the element of the left child or between the right parent element and the element of the right child.

The Fig. O shows the situation when the distance to the right child is lower than to the left one. The second situation is symmetrical.



#### The Remove operation on the AVB+tree is processed as follows:

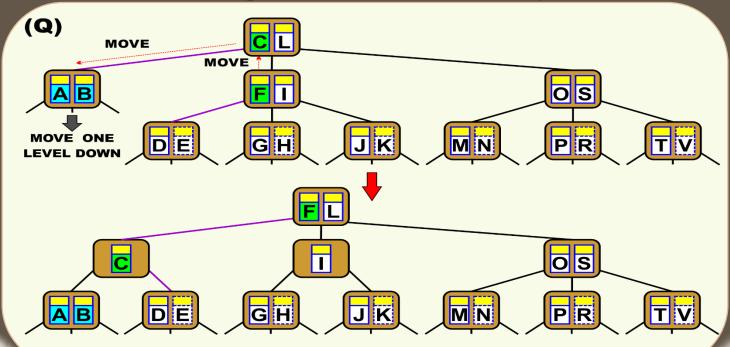
20. In this case, the parent node of the root of the reduced subtree contains two elements, and at least one of the siblings of this reduced subtree root contains two elements. If there is no direct sibling of the reduced subtree root that contains two elements, go to step 21 (Fig. P), else go to step 22 (Fig. Q and R). 21. Move elements between the parent node and both children in a way shown in Fig. P. to rebalance this subtree. Next, finish the delete operation.



The Remove operation on the AVB+tree is processed as follows:

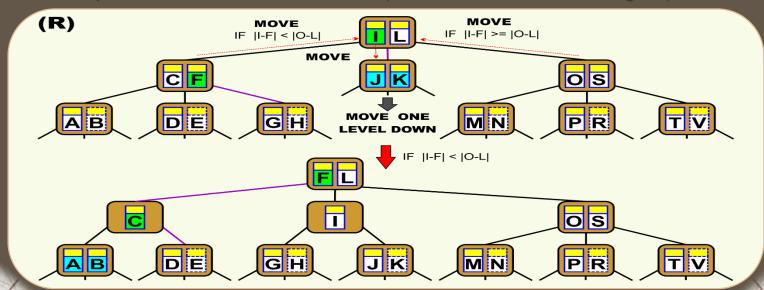
22. If the reduced subtree is placed in the left or right subtree of its parent, then go to step 23 (Fig. Q), else go to step 24 (Fig. R).

23. Move the closest element from the neighbor siblings containing two elements to the parent node and replace the closest element to the elements stored in the root of the reduced subtree, and this replaced element use to rebalance this subtree as shown in Fig. Q. Next, finish the delete operation.



#### The Remove operation on the AVB+tree is processed as follows:

24. In this case, the reduced subtree is the middle child of its parent. Therefore, move the rightmost element from the left sibling if its key is more distant to the key of the right parent element than the distance of the key of the leftmost element for the right sibling to the left parent element. In the symmetric case, move the leftmost element of the right sibling. The selected sibling is moved to the parent node, and the element from the parent node that is the closest to the elements of the reduced subtree is moved together with its closest child to the middle child where the reduced subtree is placed. Then, the new node (with the element 6 in Fig. R) is created.



Next, finish the delete operation.

## **Update Operation**

- ✓ The Update operation is a simple sequence of Remove and Insert operations because it is not possible to simply update a value in an element because of the structure of AVB+trees which represent various relations.
- ✓ Data can be easily updated (a value can be changed) only in those structures which do not represent relations, e.g. unsorted arrays, lists, or tables.
- ✓ The Update operation on an AVB+tree removes the old key (value) from this structure using the Remove operation and inserts an updated one using the Insert operation.

## **Search Operation**

#### The Search operation in the AVB+tree is processed as follows:

- 1. Start from the root and go recursively down along the branches to the descendants until the searched key or the leaf is not achieved after the following rules:
- If one of the keys stored in the elements of this node equals to the searched key, return the pointer to this element;
- else go to the left child node if the searched key is less than the key represented by the leftmost element in this node;
- else go to the right child node if the searched key is greater than the key represented by the rightmost key in this node;
- else go to the middle child node.
- 2. If the leaf is achieved and one of the stored elements in this leaf contains the searched key, return the pointer to this element, else return the null pointer.

# GetMin and GetMax Operations

The GetMin and GetMax operations can be implemented in two different ways dependently on how often extreme elements are used in other computations using an AVB+tree structure:

1. The first way is used when extreme keys are not often used. In this case, it is necessary to start from the root node and always go along the left tree branches until the leaf is achieved and in its leftmost element (if there are two) is the minimum key (value) stored in this tree.

Similarly, we go always along the right branches starting from the root node until the leaf is achieved and in its rightmost element (if there are two) is the maximum key (value) stored in this tree. These operations take log Ň time, where Ň is the number of elements stored in the tree, which is equal the number of unique keys (values) of the data.

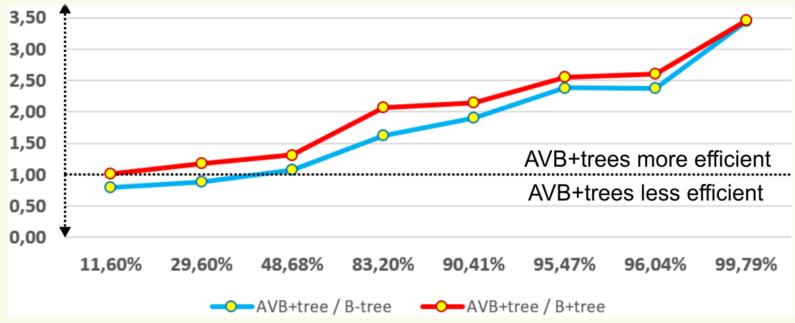
# GetMin and GetMax Operations

The GetMin and GetMax operations can be implemented in two different ways dependently on how often extreme elements are used in other computations using an AVB+tree structure:

2. The second way is used when extreme keys are often used and should be quickly available (in constant time). In this case, the leftmost (minimum) and rightmost (maximum) elements of the leftmost and rightmost leaves appropriately are additionally pointed from the class implementing the AVB+tree. If using these extra pointers they are automatically updated when the minimum or maximum element is changed, and the minimum and maximum element can be easily recognized because its neighbor connection to the left or right neighbor element is set to null.

## Comparison of Efficiencies

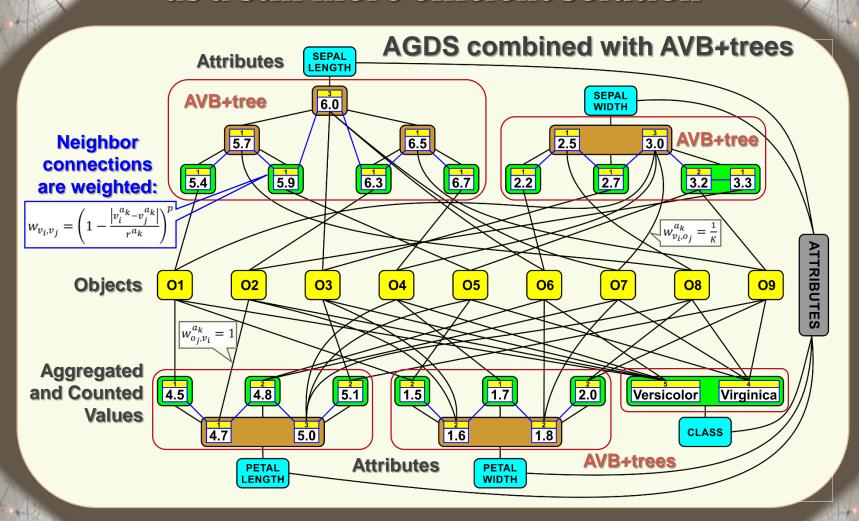
The efficiencies of the same operations on the same datasets from UCI ML Repository were compared on B-trees, B+trees, AVB-trees, and AVB+trees.



The achieved results proved the concept that AVB+trees are always faster than B+trees commonly used in databases, and AVB-trees are usually faster than B-trees when data contain more than 30% of duplicates.

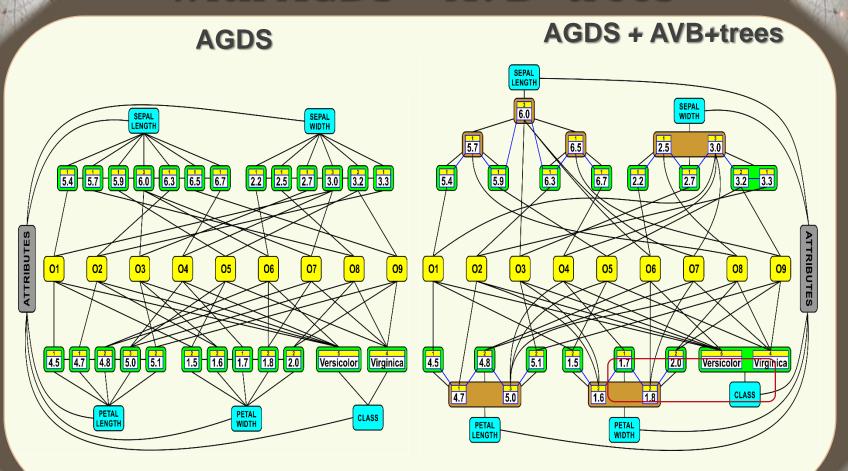
AVB-trees and AVB+trees outperform commonly used B-trees and B+trees in most cases!

## AGDS + AVB+trees as a still more efficient solution



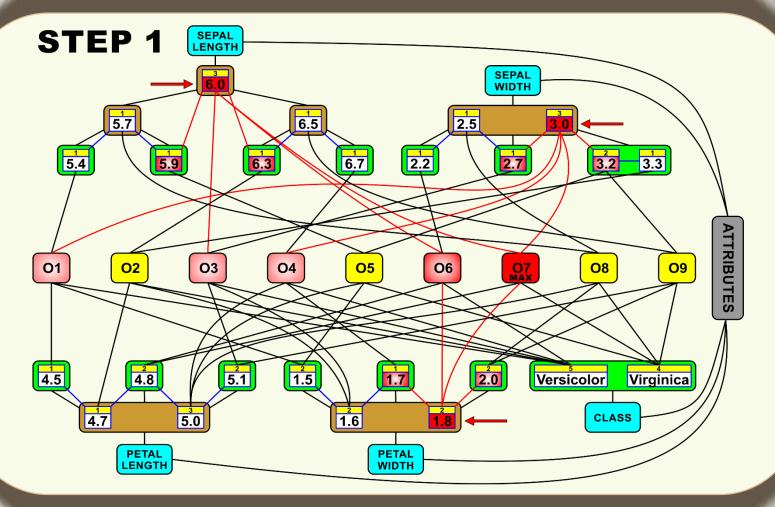
AVB+trees implemented to AGDS structures make the data access faster especially for Big Data datasets and databases.

## Comparison of AGDS with AGDS + AVB+trees



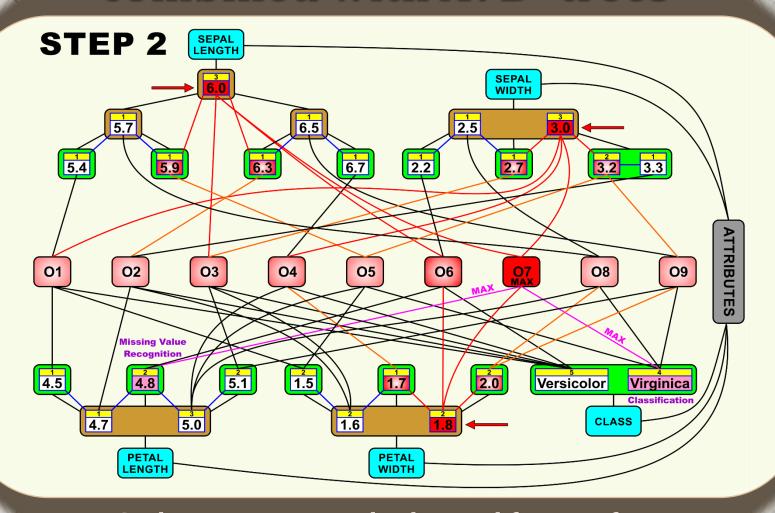
When data contain many duplicates we practically achieve the constant access to all data stored in AGDS + AVB+trees.

## Inferences on AGDS combined with AVB+trees



We do not need to search for common relations in many (nested) loops but we simply go along the connections and get results.

# Inferences on AGDS combined with AVB+trees



Such structures can also be used for very fast recognition, clustering, classification, searching for the most similar objects etc.



## Conclusions

- ✓ AGDS structures combined with AVB+trees provide incredibly fast access to any data stored and sorted for all attributes simultaneously.
- ✓ AGDS + AVB+trees stores data together with the most common vertical and horizontal relations, so there is no need to loop and search for these relations.
- ✓ Typical operations on AGDS + AVB+trees structures have pessimistically logarithmic time, but the expected complexity on typical real data is constant.





## **Questions or Remarks?**

- 1. A. Horzyk, J. A. Starzyk, J. Graham, *Integration of Semantic and Episodic Memories*, IEEE Transactions on Neural Networks and Learning Systems, Vol. 28, Issue 12, Dec. 2017, pp. 3084 3095, 2017, <u>DOI: 10.1109/TNNLS.2017.2728203</u>.
- 2. A. Horzyk, J.A. Starzyk, *Multi-Class and Multi-Label Classification Using Associative Pulsing Neural Networks*, IEEE Xplore, In: 2018 IEEE World Congress on Computational Intelligence (WCCI IJCNN 2018), 2018, (in print).
- **3. A. Horzyk**, J.A. Starzyk, *Fast Neural Network Adaptation with Associative Pulsing Neurons*, IEEE Xplore, In: 2017 IEEE Symposium Series on Computational Intelligence, pp. 339 -346, 2017, DOI: 10.1109/SSCI.2017.8285369.
- **4. A. Horzyk**, K. Gołdon, *Associative Graph Data Structures Used for Acceleration of K Nearest Neighbor Classifiers*, LNCS, In: 27th International Conference on Artificial Neural Networks (ICANN 2018), 2018, (in print).
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