Ironic Labeling of Graphs

Jarosław Grytczuk

Jagiellonian University, Kraków, Warsaw University of Technology, Warszawa

I will present some graph labeling problems with intensive number theoretic flavor. One of my favorite is due to Wiktor Żelazny, and goes as follows. Suppose that each vertex v of a graph G is assigned with a positive integer n_v. For an unknown reason we multiply n_v by the degree d_v of a vertex v, thereby obtaining a new label $m_v = n_v \cdot d_v$. If we did so for all vertices, then it may happen that new labels give a proper coloring of G, that is $m_v \neq m_u$ whenever v and u are adjacent. The initial labeling with numbers n_v is then called ironic. Now, what is the least possible upper bound $\varphi(G)$ for the maximum label in an ironic labeling of G? We conjecture that $\varphi(G) \leq \chi(G)$, where $\chi(G)$ is the chromatic number of a graph G. I will demonstrate that the conjecure is asymptotically true, and also point on its unexpected (or expected?) connections to some famous number theoretic problems.

grytczuk@gmail.com