On Algebraic Expressions of Directed Grid Graphs

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A two-terminal dag directed acyclic graph (*st-dag*) has only one source s and only one sink t. We consider a *labeled graph* which has labels attached to its edges. Each path between the source and the sink (a *spanning path*) in an st-dag can be presented by a product of all edge labels of the path. We define the sum of edge label products corresponding to all possible spanning paths of an st-dag G as the *canonical expression* of G. An algebraic expression is called an *st-dag expression* if it is algebraically equivalent to the canonical expression of an st-dag. An st-dag expression consists of labels, and the operators $+$ (disjoint union) and $\cdot$ (concatenation, also denoted by juxtaposition). We define the total number of labels in an algebraic expression, including all their appearances, as the *complexity of the algebraic expression.* We consider expressions with a minimum (or, at least, a polynomial in relation to an st-dag's size) complexity as a key to generating efficient algorithms on distributed systems.

This talk deals with a *directed grid graph* $G_{m,n}$ whose vertices correspond to pairs of integers x, y ($1 \leq x < m$, $1 \leq y < n$) and edges leaving (x, y) and entering $(x+1, y)$ and $(x, y+1)$. We construct expressions with $O(n^c)$ complexities for graphs $G_{c,n}$ ($c = 2, 3, ...$) and show that the numbers of plus operators in these expressions are $O(n^{c-1})$. Moreover, we describe a decomposition algorithm which generates expressions with complexities less than $O(n^c)$ for graphs $G_{c,n}$ and, correspondingly, with fewer numbers of plus operators. For the square graph $G_{n,n}$, this algorithm builds an expression whose complexity grows no faster than quasi-polynomially with n.

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