On cordial hypertrees

Michał Tuczyński

Faculty of Mathematics and Information Science, Warsaw University of Technology, Poland

Joint work with Przemysław Wenus, Krzysztof Węsek

Let $H = (V, E)$ be a hypergraph. A vertex labeling of H (with elements from $\mathbb{Z}_k$) is a function $f : V \to \mathbb{Z}_k$. A vertex labeling f induces an edge labeling (also denoted by f) $f : E \to \mathbb{Z}_k$ defined by $f(e) = \sum_{v \in e} f(v)$. A labeling is k-cordial if every element of $\mathbb{Z}_k$ is a label of exactly $\lfloor \frac{|V|}{k} \rfloor$ or $\lceil \frac{|V|}{k} \rceil$ vertices and exactly $\lfloor \frac{|E|}{k} \rfloor$ or $\lceil \frac{|E|}{k} \rceil$ edges. A hypergraph is called k-cordial if it admits a k-cordial labeling.

Cichacz, Görlich and Tuza [2] conjectured that all hypertrees (connected hypergraphs without cycles) are k-cordial for all k. We prove the conjecture for $k = 2, 3, 4$. These results generalize results on cordial labelins of graphs: Cahit's theorem [1] which states that every tree is 2-cordial and Hovey's theorem [3] which states that every tree is k-cordial for $k = 3, 4$. We also prove that every loose hypergraph (a hypergraph such that its every edge contains a vertex of degree 1) is 2-cordial.

References

[1] I. Cahit, Cordial graphs: a weaker version of graceful and harmonious graphs, Ars Combin. 23 (1987) 201–207.

[2] S.Cichacz, A.Görlich, Z.Tuza, Cordial labeling of hypertrees, Discrete Mathematics 313 (22) (2013) 2518–2524

[3] M. Hovey, A-cordial graphs, Discrete Math. 93 (1991) 183–194.

m.tuczynski@mini.pw.edu.pl