Strong Chromatic Index of Unit Distance Graphs

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The *strong chromatic index* of a graph G, denoted $s'(G)$, is the minimum possible number of colors in a coloring of edges of G such that each color class is an induced matching (or: if edges e and f have the same color, then both vertices of e are not adjacent to any vertex of f).

A graph G is a *unit distance graph in* $\mathbb{R}^n$ if vertices of G can be uniquely indentified with points in $\mathbb{R}^n$ so that uv is an edge of G if and only if the Euclidean distance between the points indentified with u and v is 1.

We try to estimate the largest possible value $s'(G)$, where G is a unit distance graph (in $\mathbb{R}^2$ or $\mathbb{R}^{\nVdash}$) of maximum degree Δ. It is related to the problem posed by Erdős and Nešetřil in 1985 (they conjectured that $s'(G) \leq \frac{5}{4}\Delta^2$ for every graph G, while it is easy to prove that $s'(G) \leq 2\Delta^2$).

We still do not know the correct order of magnitude. We show that $s'(G) \leq c\frac{\Delta^2}{\ln \Delta}$ (where G is a unit distance graph in $\mathbb{R}^3$ of maximum degree Δ). However, some considerations suggest that the correct answer may be much lower, maybe even linear in Δ.

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