(Di)graphs products, labelings and related results

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Gallian's survey shows that there is a big variety of labelings of graphs. By means of (di)graphs products we can establish strong relations among some of them. Moreover, due to the freedom of one of the factors, we can also obtain enumerative results that provide lower bounds on the number of nonisomorphic labelings of a particular type. In this talk, we will focus in three of the (di)graphs products that have been used for these duties: the $\otimes_h$-product of digraphs, the weak tensor product of graphs and the weak $\otimes_h$-product of graphs.

A super edge-magic labeled digraph F is in the set $\mathcal{S}_n^k$ if $|V(F)| = |E(F)| = n$ and the minimum sum of the labels of the adjacent vertices is equal to k. A typical result in this context says that if D is any (super) edge-magic digraph and h is any function $h : E(D) \to \mathcal{S}_n^k$. Then $\mathrm{und}(D \otimes_h \mathcal{S}_n^k)$ is (super) edge-magic. Analogous results can be obtained when instead of assuming D (super) edge-magic we assume that D is one of the following types of labelings: (super) edge bi-magic, harmonious, sequential, partitional, cordial. With a slight modification we can also obtain an application to k-equitable digraphs. In particular, due to the relation existing between k-equitable labelings of $m\overrightarrow{K_1}$ and Langford sequences, we can construct an exponential number of Langford sequences with certain order and defect.

Snevily proved in 1997 that if G and F are two bipartite graphs that have α-labelings, with stable sets L_G, H_G, L_F and H_F, respectively, then, the graph $G\bar{\otimes}F$ also has an α-labeling. Using a similar proof, this result was extended to near α-labelings by El-Zanati et al. It turns out that these two applications (together with an application to bigraceful labelings) also hold when instead of considering the weak tensor product, we consider a generalization of it, inspired in the $\otimes_h$-product, namely, the weak $\otimes_h$-product.

In this talk, we will survey all the results mentioned above together with some enumerative results.

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