Distance-constrained labeling of trees

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Let $G = (V, E)$ be a graph, and d a natural number. An $L(d, d-1, \ldots, 1)$-labeling of G with span s is a mapping $f : V \to \{0, 1, \ldots, s\}$ with the following property: if the distance $d(u, v)$ of two vertices $u, v \in V$ is at most d, then

$$|f(u) - f(v)| \geq d + 1 - d(u, v).$$

The smallest s for which G admits an $L(d, d-1, \ldots, 1)$-labeling is denoted by $\lambda_{d,d-1,\ldots,1}(G)$.

Conjecture (V. Halász and Zs. Tuza) For every fixed d, if T is a tree with maximum degree m, then

$$\lambda_{d,d-1,\ldots,1}(T) \leq \begin{cases} m^h + o(m^h) & \text{if } d = 2h \\ 2m^h + o(m^h) & \text{if } d = 2h+1 \end{cases}$$

as $m \to \infty$.

In a stronger form we conjecture that $o(m^h)$ above can be replaced with $O(m^{h-1})$, for both even and odd d. An upper bound of this kind is known to be valid for $d = 2$ and $d = 3$, by the results of [GY] and [CGSzT], respectively. For larger d we proved it in [HT] only for trees with diameter d.

References

[CGSzT] J. Clipperton, J. Gehrtz, Zs. Szaniszló, and D. Torkorno, $L(3, 2, 1)$-labeling of simple graphs. VERUM, Valpariso University, 2006. (unpublished manuscript)

[GY] J. R. Griggs and R. K. Yeh, Labelling graphs with a condition at distance 2. SIAM J. Discrete Math. 5 (1992), 586–595.

[HT] V. Halász and Zs. Tuza, Distance-constrained labeling of complete trees. Discrete Math. 338 (2015), 1398–1406.