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Chapter 1

# Measurement and Physical Quantities

Chapter 1 Lecture Notes

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# Contents

# Preface

This chapter opens the first volume of *Physics for Technical and Engineering Students – Volume I: Mechanics, Fluids, Waves, and Thermodynamics*. Its purpose is to establish the quantitative language that will be used throughout mechanics, fluids, waves and thermodynamics: physical quantities, units, dimensions, significant numerical reporting, uncertainty and model-based measurement.

Measurement is not a minor preliminary topic. It is the point at which observation becomes physics and engineering. A beam length used in a mechanical design, a temperature reported by a thermocouple, a distance recorded by a robot encoder, a pressure read from a sensor, or a velocity computed in a simulation is useful only if the quantity is defined, the unit is controlled, the measurement procedure is understood and the uncertainty is interpreted honestly.

The chapter is written for students in Technical Physics, engineering, applied-science and computer-science programmes. It is intended to support both lecture use and independent study. Core ideas are presented first, while advanced perspectives show stronger students how the same elementary language develops into scaling laws, calibration, uncertainty budgets, data acquisition and numerical modelling.

All explanations, examples, figures and exercises in this chapter are written as independent lecture-note material. The bibliography at the end lists sources that are useful for checking conventions and for further study, but the chapter is designed to stand on its own.

**The Guideline**

*Use this chapter as a working manual for quantitative reasoning. Every numerical result in physics, engineering and computation should be checked at four levels: what physical quantity is being computed, which unit is being used, whether the dimensions are consistent and whether the final magnitude is plausible.*

**Pedagogical boundary**

The core flow of this chapter remains introductory: it prepares students for the beginning of mechanics. At the same time, modern technical work requires early awareness of calibration, uncertainty budgets, sensors and simulation units. Advanced boxes introduce these ideas without replacing a full course on statistics, covariance matrices, Bayesian estimation, professional metrology standards or data-analysis methods.

# Conventions and notation

Throughout this chapter we use SI units unless stated otherwise. Numerical answers should normally be reported with an appropriate unit and with a number of significant figures justified by the input data or by an explicit uncertainty. A quantity without a unit is usually incomplete, except for dimensionless ratios, pure numbers and angles measured in radians.

**Table 1:** Notation used repeatedly in Chapter 1.

| **Symbol** | **Meaning** | **SI unit or dimension** |
|---|---|---|
| $L$ or $\ell$ | length scale | m, dimension $[L]$ |
| $t$ | time or time interval | s, dimension $[T]$ |
| $m$ | mass | kg, dimension $[M]$ |
| $A$ | area | $\mathrm{m}^2$, dimension $[L^2]$ |
| $V$ | volume | $\mathrm{m}^3$, dimension $[L^3]$ |
| $\rho$ | mass density | $\mathrm{kg\,m^{-3}}$, dimension $[ML^{-3}]$ |
| $Q$ | generic physical quantity | depends on context |
| $[Q]$ | physical dimension of $Q$ in dimensional analysis | product of powers of $[M]$, $[L]$, $[T]$, etc. |
| $\Delta Q$ | change or absolute uncertainty in $Q$ | same unit as $Q$ |
| $\Delta Q/Q$ | relative uncertainty | dimensionless |
| $f_s$ | sampling frequency in data acquisition | $\mathrm{s}^{-1}$ or Hz |
| $\Delta t_s$ | sampling interval | s |

**Note**

In this chapter $V$ denotes volume. Later in mechanics, potential energy will usually be denoted by $U$ rather than $V$ to avoid confusion. The square brackets $[Q]$ denote the dimension of a quantity in dimensional analysis; they should not be confused with the unit symbol written after a numerical value.

# Chapter 1

# Measurement and Physical Quantities

## 1.1 Motivation and roadmap

Physics begins with a simple but demanding question:

**How do we turn observations of the world into quantitative, testable statements?**

A visual impression is not yet physics. A statement such as “the object is heavy”, “the motion is fast”, or “the rod is long” becomes scientific only when it is connected to a measurable quantity, a unit, a procedure and an uncertainty. Measurement is therefore not a preliminary technicality. It is the first step in modelling nature.

For technical and engineering students this point is especially important. A design drawing, a laboratory report, a numerical simulation, a sensor reading and an industrial quality-control certificate all rely on the same foundation: quantities must be defined, units must be consistent and numerical precision must be meaningful.

**Learning goals**

After studying this chapter, the student should be able to:

- distinguish physical quantities, numerical values, units and dimensions;
- state the role of SI base units and construct common derived units;
- use scientific notation and SI prefixes correctly;
- convert units by treating units as algebraic factors;
- check equations by dimensional analysis;
- report numerical answers with sensible significant figures;
- estimate orders of magnitude without overusing a calculator;
- use density as a first example of a derived physical quantity;
- explain the elementary difference between accuracy, precision and uncertainty;
- recognise why units, calibration and uncertainty matter in engineering measurements and numerical simulations.

**Key message**

A physical measurement is never just a number. It is a number together with a unit, a definition of the quantity, a measurement procedure and, in careful work, an uncertainty.

### 1.1.1 Measurement in technical and engineering practice

In engineering practice a measurement often appears as a single value in a table: a length, a mass, a temperature, a voltage or a pressure. Behind that value there is a chain of decisions. One has to decide which physical quantity is relevant, which idealised model connects it to the problem,

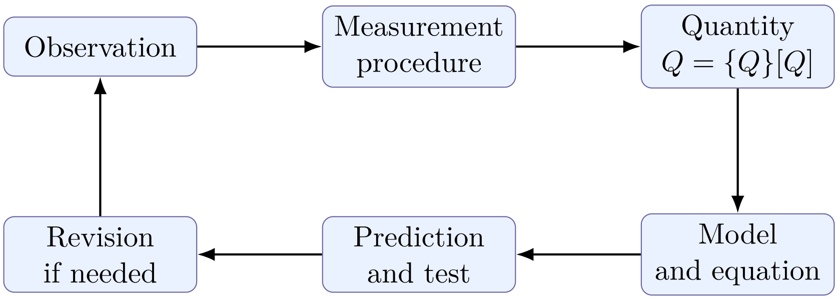


**Figure 1.1:** Roadmap from observation to quantitative physics. The loop is essential: a model becomes useful only when its predictions can be compared with measurements.

which instrument can measure it, how the instrument is calibrated and how much uncertainty can be tolerated.

For example, the phrase "measure the beam" is incomplete. Does the engineer need the total length, the distance between supports, the cross-sectional dimensions, the deflection under load, or the surface roughness? Each quantity requires a different instrument and a different uncertainty target. Similarly, a robot position sensor may output encoder counts, but the physical quantity needed by the control algorithm is usually a length or angle in SI units.

**Method**

A technical measurement can be organised as the following modelling chain:

1. define the quantity needed by the physical or engineering model;
2. choose an instrument whose range and resolution match the task;
3. calibrate or verify the instrument against a known reference;
4. record the numerical value, unit and uncertainty;
5. check dimensional consistency before using the value in a calculation or code;
6. compare the result with physical intuition and design tolerances.

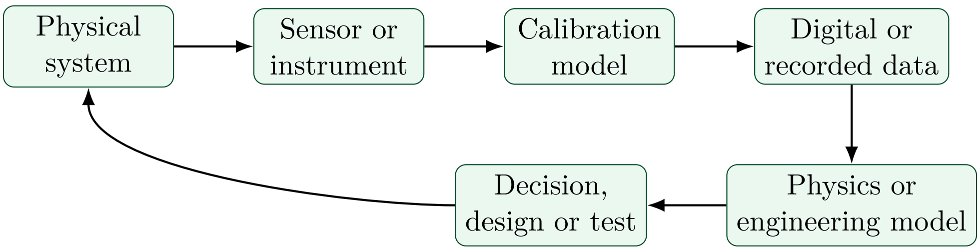


**Figure 1.2:** A measurement chain in engineering practice. The final number used in a model is the result of a physical procedure, calibration and interpretation.

## 1.2 Physical quantities, units and dimensions

### 1.2.1 What is a physical quantity?

A physical quantity is a measurable property of a system or process. Examples include length, time, mass, area, speed, force, pressure, temperature, energy and density. In calculations we normally write a measured quantity as

$$Q = \{Q\}\,[Q], \tag{1.1}$$

where $\{Q\}$ is the numerical value and $[Q]$ is the unit used to express the quantity. For example,

$$L = 2.50\,\mathrm{m} \tag{1.2}$$

means that the length is 2.50 times the unit metre.

**Definition 1.1: Physical quantity**

A physical quantity is a property that can be compared with a standard of the same kind. To specify it operationally, one must state both a numerical value and a unit.

**Caution**

Do not write isolated numbers for dimensional physical quantities. The statement "the length is 2.50" is incomplete. The statement "the length is 2.50 m" is meaningful.

### 1.2.2 Units versus dimensions

A unit is a chosen standard for measurement, such as metre, second or kilogram. A dimension describes the physical kind of a quantity, independently of the particular unit system. For example, a length may be expressed in metres, millimetres or kilometres, but its dimension is always length, denoted $[L]$.

**Definition 1.2: Dimension of a quantity**

The dimension of a physical quantity identifies how that quantity is built from fundamental kinds of measurement such as mass $[M]$, length $[L]$ and time $[T]$. It is independent of the numerical unit chosen.

**Table 1.1:** Examples distinguishing quantity, unit and dimension.

| Quantity | Example value | SI unit | Dimension |
|---|---|---|---|
| Length | $1.20\,\mathrm{m}$ | metre, m | $[L]$ |
| Time | $3.5\,\mathrm{s}$ | second, s | $[T]$ |
| Mass | $0.250\,\mathrm{kg}$ | kilogram, kg | $[M]$ |
| Area | $4.0\,\mathrm{m}^2$ | square metre, $\mathrm{m}^2$ | $[L^2]$ |
| Volume | $7.5\,\mathrm{m}^3$ | cubic metre, $\mathrm{m}^3$ | $[L^3]$ |
| Density | $1000\,\mathrm{kg\,m^{-3}}$ | kilogram per cubic metre | $[ML^{-3}]$ |
| Speed | $15\,\mathrm{m\,s^{-1}}$ | metre per second | $[LT^{-1}]$ |

**Guided checks**

- Are 1 m and 100 cm the same physical length?
- Are metre and second different units only, or different dimensions?
- Can two quantities with different dimensions be added?

## 1.3 The SI system: base and derived quantities

The International System of Units, abbreviated SI, is the standard unit system used in modern science and engineering. Physics 1 initially relies most heavily on the base quantities length, mass and time. Temperature will enter in the thermodynamics part of the course, and electric current will become central in Physics 2.

**Table 1.2:** The seven SI base quantities. The first three are the dominant base quantities in the mechanics chapters of Physics 1.

| **Base quantity** | **SI unit** | **Unit symbol** |
|---|---|---|
| Length | metre | m |
| Mass | kilogram | kg |
| Time | second | s |
| Electric current | ampere | A |
| Thermodynamic temperature | kelvin | K |
| Amount of substance | mole | mol |
| Luminous intensity | candela | cd |

**Note**

Modern SI base units are defined through fixed values of fundamental constants. Historically, standards were often material objects or astronomical cycles. The conceptual lesson for Physics 1 is simple: a unit must be reproducible, stable and internationally agreed.

### 1.3.1 Derived units

Most quantities in physics are not base quantities. They are derived from base quantities. For example,

$$\text{area:} \quad [A] = [L^2], \qquad A = \ell w, \tag{1.3}$$

$$\text{volume:} \quad [V] = [L^3], \qquad V = \ell w h, \tag{1.4}$$

$$\text{speed:} \quad [v] = [LT^{-1}], \qquad v = \frac{\text{distance}}{\text{time}}. \tag{1.5}$$

Derived SI units may have special names. For instance, later chapters will introduce the newton, joule and watt:

$$1\,\mathrm{N} = 1\,\mathrm{kg\,m\,s^{-2}}, \tag{1.6}$$

$$1\,\mathrm{J} = 1\,\mathrm{N\,m} = 1\,\mathrm{kg\,m^2\,s^{-2}}, \tag{1.7}$$

$$1\,\mathrm{W} = 1\,\mathrm{J\,s^{-1}} = 1\,\mathrm{kg\,m^2\,s^{-3}}. \tag{1.8}$$

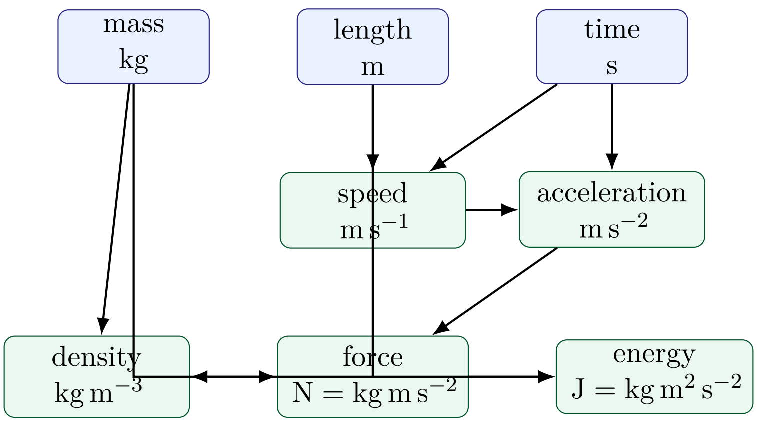


**Figure 1.3:** A simple map from base SI units to derived units that will appear later in mechanics, fluids and thermodynamics.

### 1.3.2 Engineering and computational examples of derived units

Derived units are especially important in technical work because they connect formulas to design requirements. A pressure in pascals is a force per area. A density in $\mathrm{kg\,m^{-3}}$ connects the geometry of a component to its mass. A power in watts connects energy use to time. In a computer simulation these quantities may appear only as floating-point numbers, but the physical units must still be controlled by the user.

**Table 1.3:** Derived quantities frequently encountered in engineering and applied science.

| Quantity | Definition idea | SI unit | Typical technical use |
|---|---|---|---|
| Speed | length per time | $\mathrm{m\,s^{-1}}$ | robotics, transport, flow, motion analysis |
| Acceleration | speed change per time | $\mathrm{m\,s^{-2}}$ | vibration, vehicle dynamics, sensors |
| Force | mass times acceleration | N | mechanics, structures, machines |
| Pressure | force per area | Pa | fluids, hydraulics, materials testing |
| Energy | force times distance | J | mechanics, thermodynamics, power systems |
| Power | energy per time | W | motors, heating, electronics |
| Density | mass per volume | $\mathrm{kg\,m^{-3}}$ | materials, fluids, component mass estimates |

**Example 1.1: Engineering pressure from force and area**

A hydraulic press applies a force of $F = 12.0\,\mathrm{kN}$ on a circular piston of area $A = 30.0\,\mathrm{cm}^2$. Estimate the pressure in pascals.
First convert the data to SI units:

$$F = 12.0\,\mathrm{kN} = 1.20 \times 10^4\,\mathrm{N}, \qquad A = 30.0\,\mathrm{cm}^2 = 30.0 \times 10^{-4}\,\mathrm{m}^2 = 3.00 \times 10^{-3}\,\mathrm{m}^2. \tag{1.9}$$

The pressure is

$$p = \frac{F}{A} = \frac{1.20 \times 10^4\,\mathrm{N}}{3.00 \times 10^{-3}\,\mathrm{m}^2} = 4.00 \times 10^6\,\mathrm{Pa} = 4.00\,\mathrm{MPa}. \tag{1.10}$$

The dimensional check is $[p] = [F]/[A] = [MLT^{-2}]/[L^2] = [ML^{-1}T^{-2}]$, the dimension of pressure.

## 1.4 Scientific notation and SI prefixes

Physics deals with quantities ranging from subatomic distances to astronomical scales. Scientific notation expresses a number as

$$a \times 10^n, \qquad 1 \le |a| < 10, \tag{1.11}$$

where $n$ is an integer. For example,

$$4500\,\mathrm{m} = 4.5 \times 10^3\,\mathrm{m}, \tag{1.12}$$

$$0.00072\,\mathrm{s} = 7.2 \times 10^{-4}\,\mathrm{s}. \tag{1.13}$$

SI prefixes are compact labels for powers of ten. The following subset is enough for most Physics 1 calculations.

**Table 1.4:** Common SI prefixes for Physics 1.

| Factor | Prefix | Symbol | Example |
|---|---|---|---|
| $10^{12}$ | tera | T | $1\,\mathrm{TW} = 10^{12}\,\mathrm{W}$ |
| $10^{9}$ | giga | G | $1\,\mathrm{GW} = 10^{9}\,\mathrm{W}$ |
| $10^{6}$ | mega | M | $1\,\mathrm{MJ} = 10^{6}\,\mathrm{J}$ |
| $10^{3}$ | kilo | k | $1\,\mathrm{km} = 10^{3}\,\mathrm{m}$ |
| $10^{-2}$ | centi | c | $1\,\mathrm{cm} = 10^{-2}\,\mathrm{m}$ |
| $10^{-3}$ | milli | m | $1\,\mathrm{mm} = 10^{-3}\,\mathrm{m}$ |
| $10^{-6}$ | micro | $\mu$ | $1\,\mu\mathrm{m} = 10^{-6}\,\mathrm{m}$ |
| $10^{-9}$ | nano | n | $1\,\mathrm{ns} = 10^{-9}\,\mathrm{s}$ |
| $10^{-12}$ | pico | p | $1\,\mathrm{pF} = 10^{-12}\,\mathrm{F}$ |

**Caution**

The symbol m means metre, but the prefix symbol m also means milli when attached to another unit. Thus m is metre, while mm is millimetre: $1\,\mathrm{mm} = 10^{-3}\,\mathrm{m}$.

**Example 1.2: Using prefixes without losing dimensions**

Convert $0.036\,\mathrm{km}$ to metres and millimetres.
Since $1\,\mathrm{km} = 10^3\,\mathrm{m}$,

$$0.036\,\mathrm{km} = 0.036 \times 10^3\,\mathrm{m} = 36\,\mathrm{m}. \tag{1.14}$$

Since $1\,\mathrm{m} = 10^3\,\mathrm{mm}$,

$$36\,\mathrm{m} = 36 \times 10^3\,\mathrm{mm} = 3.6 \times 10^4\,\mathrm{mm}. \tag{1.15}$$

The physical length has not changed; only the unit used to express it has changed.

## 1.5 Unit conversion as dimensional algebra

Unit conversion is not a separate trick. It is algebra applied to units. Multiplying by a conversion factor equal to one changes the unit while preserving the physical quantity.

**Method**

**Chain conversion method.**

1. Write the known quantity with its unit.
2. Multiply by conversion factors written as ratios equal to one.

3. Arrange each factor so unwanted units cancel.
4. Check that the remaining unit is the requested one.
5. Estimate whether the numerical result is reasonable.

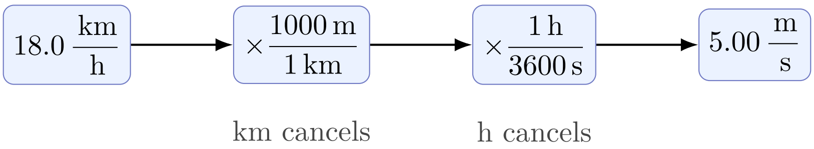


**Figure 1.4:** Unit conversion as algebraic cancellation. Conversion factors are equal to one, so they change the representation but not the physical quantity.

**Example 1.3: Converting a speed from kilometres per hour to metres per second**

A bicycle moves at $18.0\,\mathrm{km\,h^{-1}}$. Convert this speed to $\mathrm{m\,s^{-1}}$.

$$18.0\,\mathrm{km\,h^{-1}} = 18.0\frac{\mathrm{km}}{\mathrm{h}}\left(\frac{1000\,\mathrm{m}}{1\,\mathrm{km}}\right)\left(\frac{1\,\mathrm{h}}{3600\,\mathrm{s}}\right) \tag{1.16}$$

$$= 5.00\,\mathrm{m\,s^{-1}}. \tag{1.17}$$

The units km and h cancel algebraically, leaving $\mathrm{m\,s^{-1}}$.

**Example 1.4: Converting an area**

Convert $25.0\,\mathrm{cm^2}$ to square metres. The square applies to the unit as well as to the numerical conversion:

$$25.0\,\mathrm{cm^2} = 25.0\left(10^{-2}\,\mathrm{m}\right)^2 = 25.0\times10^{-4}\,\mathrm{m^2} = 2.50\times10^{-3}\,\mathrm{m^2}. \tag{1.18}$$

**Example 1.5: CAD units and component mass**

A CAD file gives a rectangular aluminium plate with dimensions $300\,\mathrm{mm}\times80\,\mathrm{mm}\times5\,\mathrm{mm}$. Estimate its mass using $\rho_{\mathrm{Al}} = 2700\,\mathrm{kg\,m^{-3}}$.
Convert all lengths to metres before computing volume:

$$300\,\mathrm{mm} = 0.300\,\mathrm{m}, \quad 80\,\mathrm{mm} = 0.080\,\mathrm{m}, \quad 5\,\mathrm{mm} = 0.005\,\mathrm{m}. \tag{1.19}$$

Thus

$$V = (0.300)(0.080)(0.005)\,\mathrm{m^3} = 1.20\times10^{-4}\,\mathrm{m^3}. \tag{1.20}$$

The estimated mass is

$$m = \rho V = (2700\,\mathrm{kg\,m^{-3}})(1.20\times10^{-4}\,\mathrm{m^3}) = 0.324\,\mathrm{kg}. \tag{1.21}$$

If the dimensions in millimetres were inserted directly as if they were metres, the computed volume would be wrong by a factor of $10^9$.

**Caution**

The most common area and volume conversion errors come from forgetting to square or cube the conversion factor. Since $1\,\mathrm{cm} = 10^{-2}\,\mathrm{m}$, it follows that $1\,\mathrm{cm}^2 = 10^{-4}\,\mathrm{m}^2$ and $1\,\mathrm{cm}^3 = 10^{-6}\,\mathrm{m}^3$.

**Guided checks**

- Which is larger: $1\,\mathrm{m}^2$ or $1\,\mathrm{cm}^2$?
- If a speed is written as $72\,\mathrm{km\,h^{-1}}$, should the answer in $\mathrm{m\,s^{-1}}$ be larger or smaller than 72?
- Why does a unit conversion never change the physical quantity itself?

## 1.6 Dimensional analysis

Dimensional analysis is one of the simplest and most powerful error checks in physics. Every physically meaningful equation must be dimensionally consistent: all terms added or equated must have the same dimension.

**Definition 1.3: Dimensional consistency**

An equation is dimensionally consistent if the left-hand side and right-hand side have the same dimension, and if every term in a sum or difference has the same dimension.

### 1.6.1 Basic dimensions

For the first mechanics chapters we mostly need mass, length and time:

$$[M], \qquad [L], \qquad [T]. \tag{1.22}$$

Some important derived dimensions are

$$[x] = [L], \tag{1.23}$$
$$[t] = [T], \tag{1.24}$$
$$[m] = [M], \tag{1.25}$$
$$[v] = [LT^{-1}], \tag{1.26}$$
$$[a] = [LT^{-2}], \tag{1.27}$$
$$[F] = [MLT^{-2}], \tag{1.28}$$
$$[K] = [ML^2T^{-2}], \tag{1.29}$$
$$[p] = [ML^{-1}T^{-2}], \tag{1.30}$$
$$[\rho] = [ML^{-3}]. \tag{1.31}$$

### 1.6.2 What dimensional analysis can and cannot do

Dimensional analysis can detect many algebraic mistakes, missing factors with dimensions and impossible additions. It cannot determine dimensionless numerical constants such as $1/2$, $2\pi$ or $\sqrt{2}$, and it cannot prove that a physically consistent equation is correct.

**Example 1.6: Checking a proposed equation**

Suppose a student proposes that the distance fallen from rest under constant gravitational acceleration $g$ after time $t$ is

$$y = gt^2. \tag{1.32}$$

Check the dimensions. Since $[g] = [LT^{-2}]$ and $[t^2] = [T^2]$,

$$[gt^2] = [LT^{-2}][T^2] = [L]. \tag{1.33}$$

The equation is dimensionally consistent. However, dimensional analysis cannot tell us the missing numerical factor. The constant-acceleration result for motion from rest is $y = (1/2)gt^2$ if downward is chosen positive.

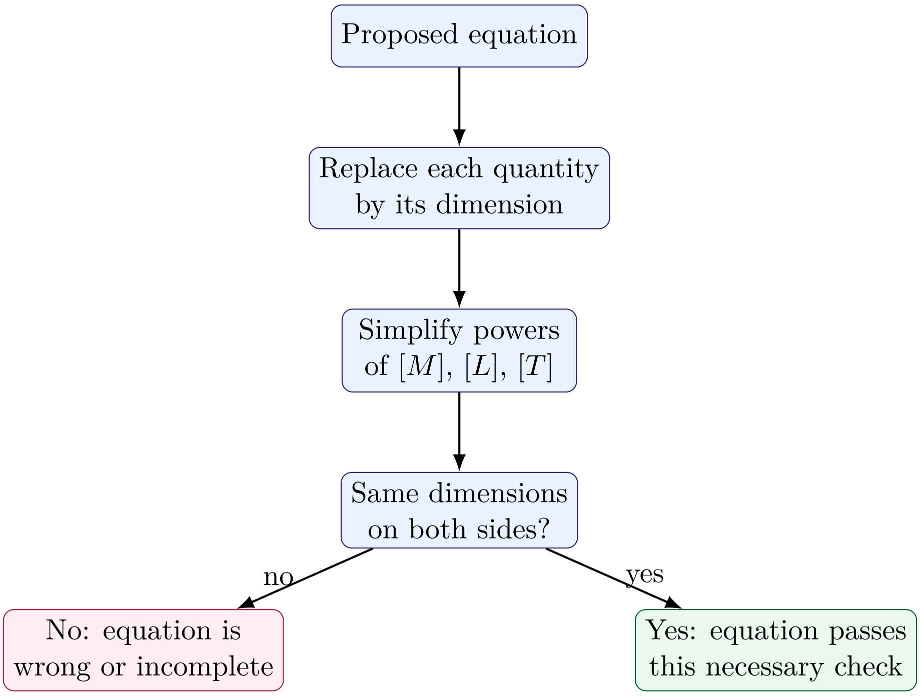


**Figure 1.5:** Dimensional-analysis workflow. Passing the test does not prove that an equation is correct; failing it proves that something is wrong.

**Example 1.7: Detecting an impossible equation**

A proposed expression for a distance is

$$x = v_0 t + at, \tag{1.34}$$

where $v_0$ is a speed and $a$ is an acceleration. The first term has dimension

$$[v_0 t] = [LT^{-1}][T] = [L], \tag{1.35}$$

while the second term has dimension

$$[at] = [LT^{-2}][T] = [LT^{-1}]. \tag{1.36}$$

A length cannot be added to a speed. The expression is dimensionally inconsistent.

**Example 1.8: Dimensions of kinetic energy and pressure**

The kinetic energy of a particle is $K = (1/2)mv^2$. Its dimension is

$$[K] = [M][LT^{-1}]^2 = [ML^2T^{-2}]. \tag{1.37}$$

Pressure is force per unit area, $p = F/A$. Therefore

$$[p] = \frac{[MLT^{-2}]}{[L^2]} = [ML^{-1}T^{-2}]. \tag{1.38}$$

This dimension will be useful later in fluids and thermodynamics.

**The Guideline**

When checking dimensions, treat dimensions like algebraic symbols. Replace every quantity by its dimension, simplify powers of $[M]$, $[L]$ and $[T]$, and compare terms. Never add terms with different dimensions.

**Key message**

Dimensional analysis is a necessary test, not a sufficient proof. If an equation fails dimensional analysis, it is wrong. If it passes, it may still be physically or algebraically wrong.

### 1.6.3 Advanced perspective: scaling laws and dimensionless groups

Dimensional analysis is not only an error-checking tool. It also helps engineers and physicists recognise how physical systems scale. Suppose all lengths of a solid object are multiplied by a factor $s$. Then areas scale as $s^2$ and volumes scale as $s^3$. If the material density remains the same, the mass also scales as $s^3$.

This simple observation explains why scale models require care. A bridge model that is one tenth the length of the real bridge has cross-sectional areas smaller by $10^2$ and volumes smaller by $10^3$. Forces, stiffness, weight and fluid effects do not all scale in the same way. Engineers therefore use dimensionless groups to compare systems of different size.

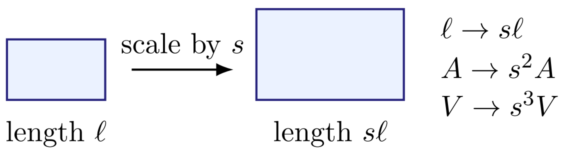


**Figure 1.6:** Scaling of length, area and volume. This is one reason why small models and full-size engineering systems are not automatically equivalent.

**Note**

**Advanced perspective: why dimensionless ratios matter.** A dimensionless ratio compares two quantities of the same dimension. Examples include aspect ratio $h/L$, strain $\Delta L/L$, relative uncertainty $\Delta Q/Q$ and efficiency. In fluid mechanics, a later example will be the Reynolds number, which compares inertial and viscous effects. Dimensionless parameters are central in engineering because they allow comparison between systems with different sizes, speeds or materials.

A first glimpse of the Buckingham Pi idea is this: if a physical problem contains dimensional variables, useful relations can often be rewritten in terms of dimensionless combinations of those variables. This does not solve the whole physics problem, but it reduces the number of independent combinations and clarifies what must be measured or simulated.

**Pedagogical boundary**

The Buckingham Pi theorem is a powerful general method in dimensional analysis, but a full treatment belongs later. At this point the important idea is qualitative: when possible, express comparisons through dimensionless ratios. They are often more portable across experiments, simulations and engineering designs than dimensional quantities alone.

**Example 1.9: Scaling of component mass**

A plastic component is redesigned so that every length is increased by a factor $s = 1.5$, while the same material is used. If the original mass is 0.80 kg, estimate the new mass.
The volume scales as $s^3$, and for the same material the mass scales in the same way:

$$m_{\text{new}} = s^3 m_{\text{old}} = (1.5)^3(0.80\,\text{kg}) = 2.7\,\text{kg}. \tag{1.39}$$

The length increased by only 50 percent, but the mass increased by more than a factor of three.

## 1.7 Significant figures and meaningful numerical reporting

Measurements have limited precision. Therefore, calculated results should not be reported with more meaningful digits than the input data justify. This is not merely a formatting issue: too many digits can falsely imply a precision that was never measured.

**Definition 1.4: Significant figures**

Significant figures are the digits in a measured or calculated value that carry meaningful information about the precision of the quantity. Leading zeros are not significant; zeros between nonzero digits usually are significant; trailing zeros require context or scientific notation to remove ambiguity.

**Table 1.5:** Examples of significant figures.

| Value | Significant figures | Comment |
|---|---|---|
| 3.42 m | 3 | all digits shown are meaningful |
| 0.0042 s | 2 | leading zeros only locate the decimal point |
| 5.00 kg | 3 | trailing zeros after a decimal point are significant |
| $1.20 \times 10^3$ m | 3 | scientific notation removes ambiguity |
| 1200 m | ambiguous | may mean 2, 3 or 4 significant figures unless context is given |

### 1.7.1 Practical reporting rules

For introductory Physics 1, use the following practical rules unless a laboratory instruction specifies otherwise.

**The Guideline**

1. For multiplication and division, the result should usually have the same number of significant figures as the input with the fewest significant figures.
2. For addition and subtraction, the result should usually be rounded to the least precise decimal place among the inputs.
3. Keep extra guard digits during intermediate calculations; round only the final answer.
4. Always report the unit.
5. When uncertainty is given explicitly, round the uncertainty sensibly and report the value to the same decimal place.

**Example 1.10: Rounding after multiplication**

A rectangular plate has measured sides

$$a = 2.4\,\text{cm}, \qquad b = 5.36\,\text{cm}. \tag{1.40}$$

The area is

$$A = ab = (2.4)(5.36)\,\text{cm}^2 = 12.864\,\text{cm}^2. \tag{1.41}$$

Since 2.4 cm has two significant figures, the result should normally be reported as

$$A \simeq 13\,\text{cm}^2. \tag{1.42}$$

The unrounded value may be kept internally, but it should not be presented as if all five digits were experimentally meaningful.

### 1.7.2 Numerical precision versus measurement precision

A calculator or computer can often display many digits. Those digits describe the numerical representation used by the machine, not necessarily the physical information in the measurement. In computational work, it is common to store numbers with high internal precision and then report results with lower physical precision.

**Caution**

A simulation result with many decimal places is not automatically more physically accurate. The accuracy of a numerical prediction is limited by the physical model, input data, numerical method, unit consistency and uncertainty, not only by floating-point precision.

**Note**

**Advanced perspective: floating-point output is not an uncertainty estimate.** A program may print 9.806650327, but this does not mean the physical acceleration is known to ten significant figures in the experiment being analysed. Floating-point precision, algorithmic error, model error and measurement uncertainty are different concepts.

## 1.8 Computational connection: units in code and simulations

Computer programs usually manipulate numbers, not physical quantities. Unless a unit-aware library is used, the program cannot know whether the number 10 means 10 m, 10 cm, 10 m s$^{-1}$ or 10 N. Unit consistency is therefore the responsibility of the programmer and the modeller.

In robotics, a control algorithm may combine encoder positions, wheel radii, angular velocities and time steps. In computational mechanics, a finite-element input file may contain lengths, densities, forces and elastic constants. In data acquisition, voltages must be converted into temperatures, pressures or accelerations through calibration relations. Unit mistakes in any of these steps can produce plausible-looking but physically meaningless output.

**Method**

**A practical unit policy for code.**

1. Convert all input data to SI units at the boundary of the program.
2. Use variable names or comments that record units, for example `length_m` or `velocity_m_s`.
3. Keep a short table of input and output units in the documentation.
4. Check dimensions of important formulas before coding them.
5. Format output with physically justified significant figures.

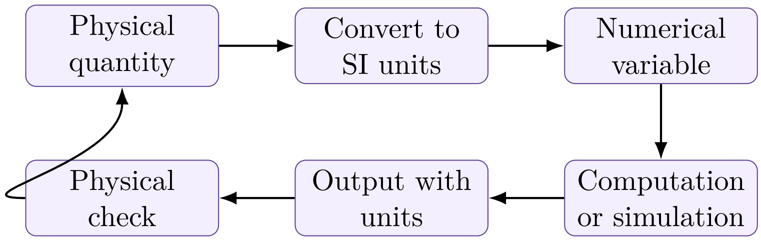


**Figure 1.7:** A unit-consistency workflow for numerical modelling. The numerical variable is only meaningful when its physical unit is controlled.

**Example 1.11: A unit error in a simple simulation**

A simulation computes the time for a robot to move a distance $d$ at constant speed $v$ using $t = d/v$. The intended inputs are $d = 120\,\mathrm{cm}$ and $v = 0.40\,\mathrm{m\,s^{-1}}$.
The distance must first be converted:

$$d = 120\,\mathrm{cm} = 1.20\,\mathrm{m}. \tag{1.43}$$

Then

$$t = \frac{1.20\,\mathrm{m}}{0.40\,\mathrm{m\,s^{-1}}} = 3.0\,\mathrm{s}. \tag{1.44}$$

If the code uses `d = 120` as if it were metres, it obtains $t = 300\,\mathrm{s}$, wrong by a factor of 100. The formula was correct; the unit handling was not.

**Example 1.12: Why `velocity = 10` is incomplete**

A line of code says `velocity = 10`. This is not a complete physical statement. It might mean $10\,\mathrm{m\,s^{-1}}$, $10\,\mathrm{km\,h^{-1}}$, $10\,\mathrm{cm\,s^{-1}}$ or some internal simulation unit. A safer convention is to write, for example, `velocity_m_s = 10.0` and to document that all speeds in the calculation are in SI units.

## 1.9 Order-of-magnitude estimates

An order-of-magnitude estimate asks for the nearest power of ten, or sometimes for a result accurate only within a factor of a few. This is useful when exact data are unavailable or unnecessary. In engineering, estimates are used to check whether a design calculation is reasonable before detailed modelling begins.

**Method**

**How to make an order-of-magnitude estimate.**

1. Identify the physical quantity to estimate.
2. Replace complicated shapes or processes by simple models.
3. Use approximate but reasonable values.
4. Keep powers of ten carefully.
5. State the assumptions, because they matter more than extra digits.

**Example 1.13: Estimating the mass of air in a lecture room**

Consider a lecture room of approximate dimensions

$$10\,\text{m} \times 8\,\text{m} \times 3\,\text{m}. \tag{1.45}$$

Its volume is approximately

$$V \sim 2.4 \times 10^2\,\text{m}^3. \tag{1.46}$$

Using the density of air as roughly $\rho_{\text{air}} \sim 1\,\text{kg}\,\text{m}^{-3}$, the mass of air is

$$m \sim \rho V \sim 2 \times 10^2\,\text{kg}. \tag{1.47}$$

The result is not meant to be precise; it tells us that the air in a room has a mass comparable to a few people, not a few grams and not many tonnes.

**Example 1.14: Rainfall on a flat roof**

During a storm, 20 mm of rain falls on a flat roof of area $600\,\text{m}^2$. Estimate the mass of water on the roof if drainage is temporarily blocked.
The rainfall height is $h = 20\,\text{mm} = 2.0 \times 10^{-2}\,\text{m}$. The volume is

$$V = Ah = (600\,\text{m}^2)(2.0 \times 10^{-2}\,\text{m}) = 12\,\text{m}^3. \tag{1.48}$$

Using $\rho_{\text{water}} \simeq 1000\,\text{kg}\,\text{m}^{-3}$,

$$m = \rho V \simeq 1.2 \times 10^4\,\text{kg}. \tag{1.49}$$

Even moderate rainfall can correspond to many tonnes of water over a large area. This is why drainage and load assumptions are engineering issues, not just weather observations.

**Guided checks**

- Which assumption dominates the air-mass estimate: room size or air density?
- Would using $\rho_{\text{air}} = 1.2\,\text{kg}\,\text{m}^{-3}$ change the order of magnitude?
- Why is an estimate often more useful than a calculator result with many digits?

## 1.10 Density as a first derived physical quantity

Density is the mass per unit volume. It is one of the simplest derived quantities and will reappear in fluids, buoyancy, pressure, waves in media, thermodynamics and materials engineering.

**Definition 1.5: Mass density**

For a uniform material of mass $m$ occupying volume $V$, the mass density is

$$\rho = \frac{m}{V}. \tag{1.50}$$

Its SI unit is $\mathrm{kg\,m^{-3}}$ and its dimension is $[ML^{-3}]$.

For a non-uniform material, density may vary from point to point. In that case one may define an average density,

$$\rho_{\text{avg}} = \frac{m_{\text{total}}}{V_{\text{total}}}, \tag{1.51}$$

while a more advanced local definition uses a small volume element,

$$\rho(\boldsymbol{r}) = \frac{\mathrm{d}m}{\mathrm{d}V}. \tag{1.52}$$

The local definition is included only to show the direction in which continuum physics develops later; most Chapter 1 problems use uniform or average density.

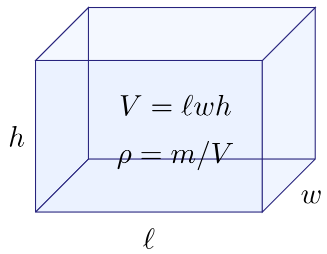


**Figure 1.8:** Density for a uniform rectangular block. Density connects mass, geometry and material composition.

**Example 1.15: Density of a metal block**

A rectangular metal block has dimensions

$$4.00\,\mathrm{cm} \times 3.00\,\mathrm{cm} \times 2.00\,\mathrm{cm} \tag{1.53}$$

and mass $m = 64.8\,\mathrm{g}$. Its volume is

$$V = (4.00)(3.00)(2.00)\,\mathrm{cm^3} = 24.0\,\mathrm{cm^3}. \tag{1.54}$$

Thus

$$\rho = \frac{64.8\,\mathrm{g}}{24.0\,\mathrm{cm^3}} = 2.70\,\mathrm{g\,cm^{-3}}. \tag{1.55}$$

In SI units,

$$2.70\,\mathrm{g\,cm^{-3}} = 2.70 \times 10^3\,\mathrm{kg\,m^{-3}}. \tag{1.56}$$

This value is characteristic of a light metal such as aluminium.

**Example 1.16: Mass of a steel shaft from density**

A cylindrical steel shaft has radius $r = 1.5\,\text{cm}$ and length $L = 0.80\,\text{m}$. Estimate its mass using $\rho_{\text{steel}} = 7.8 \times 10^3\,\text{kg}\,\text{m}^{-3}$.
Convert the radius to SI units: $r = 0.015\,\text{m}$. The volume is

$$V = \pi r^2 L = \pi(0.015\,\text{m})^2(0.80\,\text{m}) = 5.65 \times 10^{-4}\,\text{m}^3. \tag{1.57}$$

The mass is

$$m = \rho V = (7.8 \times 10^3\,\text{kg}\,\text{m}^{-3})(5.65 \times 10^{-4}\,\text{m}^3) \simeq 4.4\,\text{kg}. \tag{1.58}$$

The estimate is useful for checking whether the component can be handled safely and whether the support structure is reasonable.

**Caution**

Density is not the same as mass. A small object made of a dense material may have less mass than a large object made of a low-density material. Density is an intensive material property; mass and volume are extensive quantities.

**Note**

**Advanced perspective: local, average and effective density.** For a uniform solid, density is nearly constant. For a composite, porous material, biological tissue, foam, soil sample or granular medium, the density used in a model may be an average or effective density. The correct choice depends on the scale of the measurement and on the model being used.

## 1.11 Uncertainty, accuracy and precision

Every measurement has limited resolution and may contain systematic effects. In this chapter we need a first operational vocabulary, because uncertainty appears immediately in laboratory measurements and engineering tolerances.

**Definition 1.6: Accuracy and precision**

Accuracy describes closeness to the true or accepted value. Precision describes the reproducibility or spread of repeated measurements. A measurement can be precise but inaccurate if it is affected by a systematic error.

**Definition 1.7: Absolute and relative uncertainty**

If a measured value is reported as $Q \pm \Delta Q$, then $\Delta Q$ is the absolute uncertainty. The relative uncertainty is

$$\frac{\Delta Q}{Q}, \tag{1.59}$$

and the percentage uncertainty is

$$100\% \times \frac{\Delta Q}{Q}. \tag{1.60}$$

**Example 1.17: Relative uncertainty**

A length is measured as

$$L = (25.0 \pm 0.1)\,\text{cm}. \tag{1.61}$$

The relative uncertainty is

$$\frac{\Delta L}{L} = \frac{0.1}{25.0} = 0.004, \tag{1.62}$$

which is a percentage uncertainty of 0.4%.

### 1.11.1 Metrology, calibration and traceability

Metrology is the science and practice of measurement. Its purpose is not only to obtain numbers, but to make measurements reproducible, comparable and traceable to standards. In a laboratory or industrial setting, an instrument reading is trusted only if the measurement chain is understood.

A calibration relates an instrument output to a physical quantity. For example, a temperature sensor may output a voltage, and a calibration relation converts that voltage into temperature. A zero offset means the instrument reads a nonzero value even when the true input is zero. A scale-factor error means the slope of the calibration relation is incorrect. Both are systematic effects.

**Definition 1.8: Calibration**

Calibration is the process of comparing an instrument response with known reference values and establishing a relation between the instrument output and the physical quantity being measured.

**Note**

**Advanced perspective: calibration is a model.** A calibration curve is not just a correction table. It is a physical and statistical model of how an instrument responds. It may be linear only over a limited range, may drift with temperature and may have uncertainty of its own.

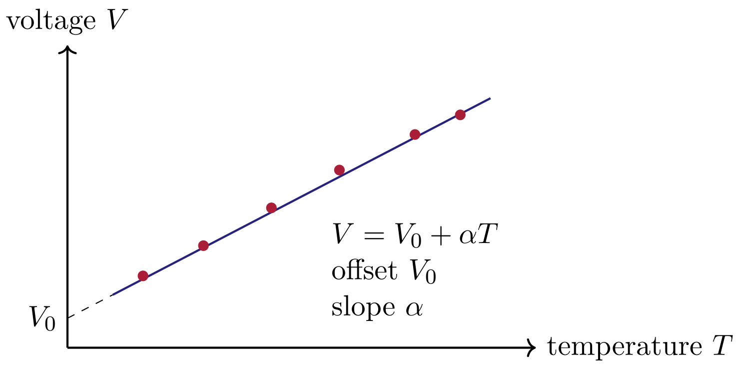


**Figure 1.9:** A simple linear sensor calibration curve. The measured voltage is converted to a physical quantity through a model with offset and slope.

**Example 1.18: Sensor calibration: voltage to temperature**

A temperature sensor has calibration relation

$$V = V_0 + \alpha T, \tag{1.63}$$

where $V_0 = 0.50\,\mathrm{V}$ and $\alpha = 10\,\mathrm{mV\,K^{-1}} = 0.010\,\mathrm{V\,K^{-1}}$. If the measured voltage is $V = 2.95\,\mathrm{V}$, find the temperature.
Solving for $T$ gives

$$T = \frac{V - V_0}{\alpha} = \frac{2.95\,\mathrm{V} - 0.50\,\mathrm{V}}{0.010\,\mathrm{V\,K^{-1}}} = 245\,\mathrm{K}. \tag{1.64}$$

The voltage unit cancels and the remaining unit is kelvin. This is a unit check on the calibration relation.

### 1.11.2 Simple propagation rules

For a first laboratory-level estimate, the following rules are useful.

**The Guideline**

- If $z = x + y$ or $z = x - y$, absolute uncertainties add approximately:

$$\Delta z \simeq \Delta x + \Delta y. \tag{1.65}$$

- If $z = xy$ or $z = x/y$, relative uncertainties add approximately:

$$\frac{\Delta z}{z} \simeq \frac{\Delta x}{x} + \frac{\Delta y}{y}. \tag{1.66}$$

- If $z = x^n$, then

$$\frac{\Delta z}{z} \simeq |n| \frac{\Delta x}{x}. \tag{1.67}$$

These are conservative introductory rules. More careful statistical propagation often combines independent random uncertainties in quadrature.

**Example 1.19: Uncertainty in density**

A student measures the mass and volume of a sample as

$$m = (120.0 \pm 0.2)\,\mathrm{g}, \qquad V = (15.0 \pm 0.1)\,\mathrm{cm^3}. \tag{1.68}$$

The density is

$$\rho = \frac{m}{V} = 8.00\,\mathrm{g\,cm^{-3}}. \tag{1.69}$$

Using relative uncertainties,

$$\frac{\Delta \rho}{\rho} \simeq \frac{0.2}{120.0} + \frac{0.1}{15.0} \simeq 0.0017 + 0.0067 = 0.0084. \tag{1.70}$$

Therefore

$$\Delta \rho \simeq 0.0084 \times 8.00\,\mathrm{g\,cm^{-3}} \simeq 0.07\,\mathrm{g\,cm^{-3}}. \tag{1.71}$$

A sensible report is

$$\rho = (8.00 \pm 0.07)\,\mathrm{g\,cm^{-3}}. \tag{1.72}$$

### 1.11.3 Uncertainty budgets in applied measurements

An uncertainty budget lists the main sources that contribute to the uncertainty of a final result. It is a practical engineering tool: it shows which part of a measurement should be improved if better accuracy is required.

**Table 1.6:** Common uncertainty sources in applied measurements.

| Source | Type | Typical origin | Possible mitigation |
|---|---|---|---|
| Instrument resolution | random or bounded | finite scale division or digital step | use higher-resolution instrument |
| Calibration offset | systematic | zero error or drift | calibrate with reference standard |
| Environmental conditions | systematic or random | temperature, humidity, vibration | control environment or correct model |
| Operator reading | random or systematic | parallax, alignment, reaction time | training, automated acquisition |
| Repeated-measurement scatter | random | noise, uncontrolled fluctuations | average repeated readings |
| Digital sensor noise | random | electronics and ADC quantisation | filtering, shielding, better ADC |
| Model approximation | systematic | idealised geometry or neglected effects | improve model or include correction |

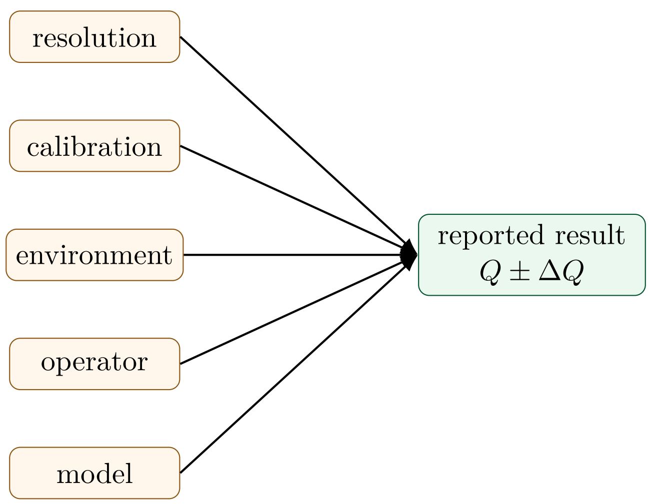


**Figure 1.10:** An uncertainty budget gathers several sources into a final reported result. The largest contribution often tells the engineer what to improve first.

**Example 1.20: Engineering uncertainty budget for a metal cylinder**

A metal cylinder is measured to estimate its density. The measured values are

$$m = (245.0 \pm 0.2)\,\mathrm{g}, \qquad d = (2.50 \pm 0.01)\,\mathrm{cm}, \qquad h = (6.00 \pm 0.02)\,\mathrm{cm}. \tag{1.73}$$

Assume the cylinder is well described by an ideal circular cylinder. The volume is

$$V = \pi \left(\frac{d}{2}\right)^2 h = \pi(1.25\,\mathrm{cm})^2(6.00\,\mathrm{cm}) = 29.45\,\mathrm{cm}^3. \tag{1.74}$$

The density is

$$\rho = \frac{245.0\,\mathrm{g}}{29.45\,\mathrm{cm}^3} = 8.32\,\mathrm{g\,cm^{-3}}. \tag{1.75}$$

For a conservative relative uncertainty estimate,

$$\frac{\Delta\rho}{\rho} \simeq \frac{\Delta m}{m} + 2\frac{\Delta d}{d} + \frac{\Delta h}{h} = \frac{0.2}{245.0} + 2\frac{0.01}{2.50} + \frac{0.02}{6.00} \simeq 0.012. \tag{1.76}$$

Thus $\Delta\rho \simeq 0.10\,\mathrm{g\,cm^{-3}}$, and a sensible report is

$$\rho = (8.32 \pm 0.10)\,\mathrm{g\,cm^{-3}}. \tag{1.77}$$

The diameter uncertainty is multiplied by 2 because the volume depends on $d^2$. This identifies the diameter measurement as a particularly important part of the uncertainty budget.

**Pedagogical boundary**

The formulas above are not a complete theory of measurement. They are included to help students understand why laboratory and engineering results are reported with uncertainties and why meaningful digits matter. A statistically rigorous treatment requires probability distributions, calibration models, covariance information and repeated-measurement analysis.

## 1.12 Laboratory connection: measuring length, time and mass

Chapter 1 connects directly to the first laboratory sessions. Students should become comfortable with instruments, resolution and data tables before more sophisticated mechanical experiments are introduced.

**Guided checks**

- If a ruler has millimetre marks, what is a reasonable reading uncertainty for a single length measurement?
- Why is the reaction time of a human operator relevant for stopwatch measurements?
- Why should a balance be tared before measuring the mass of a sample in a container?

### 1.12.1 Data acquisition and sensor units

Modern laboratories and engineering systems often measure quantities electronically. A sensor produces an analog or digital signal, and this signal must be converted into a physical quantity. The conversion may involve an offset, a scale factor, filtering and a sampling rate.

**Table 1.7:** Basic measuring instruments and typical quantities. The exact resolution depends on the specific device used in the laboratory.

| **Instrument** | **Typical quantity** | **Important habit** |
|---|---|---|
| Ruler or measuring tape | length | read scale carefully; avoid parallax |
| Vernier caliper | external/internal diameter, depth | record resolution and zero offset |
| Micrometer screw gauge | small thickness or diameter | avoid excessive force; check zero |
| Stopwatch or photogate | time interval | distinguish reaction-time uncertainty from device resolution |
| Balance | mass | tare the balance; avoid mixing mass and weight |
| Graduated cylinder | volume of liquid | read the meniscus at eye level |
| Digital sensor | voltage, acceleration, temperature, pressure | record calibration, sampling rate and units |

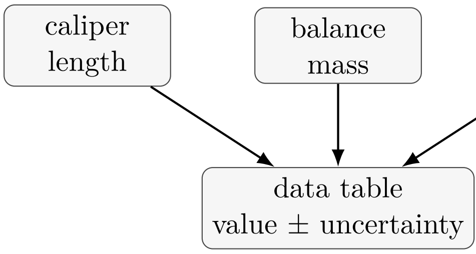


**Figure 1.11:** Basic laboratory measurements produce numerical values with units and uncertainties.

If a data-acquisition system samples at frequency $f_s$, the time interval between samples is

$$\Delta t_s = \frac{1}{f_s}. \tag{1.78}$$

The unit of $f_s$ is $\mathrm{s}^{-1}$, also called hertz (Hz). The sampling interval has unit seconds.

**Example 1.21: Sampling frequency and time interval**

An accelerometer is sampled at $f_s = 200\,\mathrm{Hz}$. The sampling interval is

$$\Delta t_s = \frac{1}{f_s} = \frac{1}{200\,\mathrm{s}^{-1}} = 0.005\,\mathrm{s} = 5\,\mathrm{ms}. \tag{1.79}$$

A data file containing 4000 samples therefore covers approximately

$$4000 \times 0.005\,\mathrm{s} = 20\,\mathrm{s} \tag{1.80}$$

of recorded motion, assuming no missing samples.

**Note**

**Advanced perspective: data are measured quantities.** A column of numbers in a file is not self-explanatory. A scientifically useful dataset should record the quantity, unit, calibration, sampling rate, uncertainty or resolution, and any processing already applied to the signal.

## 1.13 Common mistakes and how to avoid them

**Caution**

**Mistake 1: adding quantities with different dimensions.** Expressions such as $3\,\mathrm{m} + 4\,\mathrm{s}$ are meaningless unless a physical conversion relation is supplied.

**Caution**

**Mistake 2: dropping units during a calculation.** Units are not decoration; they are part of the algebra. Keeping them visible often reveals errors before the final answer.

**Caution**

**Mistake 3: confusing mass and weight.** Mass is an intrinsic measure of inertia, with SI unit kilogram. Weight is a gravitational force, with SI unit newton. The distinction becomes essential in Chapter 5.

**Caution**

**Mistake 4: overreporting digits.** A calculator may display many digits, but a measurement rarely justifies them. Report physically meaningful precision.

**Caution**

**Mistake 5: treating dimensional consistency as proof.** Dimensional analysis can rule out wrong equations, but it cannot determine dimensionless constants or guarantee physical correctness.

**Caution**

**Mistake 6: mixing engineering units in software.** If one part of a program uses millimetres and another assumes metres, errors in area, volume, density, force or stress can be enormous. Unit conversion should be explicit and documented.

## 1.14 Guided checks

**Guided checks**

1. A student writes $A = 3.2\,\mathrm{m}$. Is $A$ more likely to represent an area or a length? What is the problem with the notation?
2. Convert $5.0\,\mathrm{m}^3$ to cubic centimetres. Which power of 10 appears?
3. The proposed equation for the period of a pendulum is $T = 2\pi\sqrt{g/\ell}$. Use dimensional analysis to test it.
4. A length is measured as $L = 1.20\,\mathrm{m}$ and another as $d = 3\,\mathrm{cm}$. What should be done before adding them?
5. A density is reported as $\rho = 7.8364912\,\mathrm{g\,cm^{-3}}$ after measuring mass with a kitchen scale. What is suspicious about the report?
6. Why is $\rho = m/V$ dimensionally meaningful?
7. What is the difference between a unit conversion and a physical law?

8. A sensor outputs voltage, but the model requires temperature. What additional information is needed?
9. A code variable is named `time = 0.01`. What is missing from this statement?
10. If every length of a component is doubled, by what factor does its volume change?

## 1.15 Engineering applications and modelling checkpoints

Before a physical result is used in design, simulation or laboratory interpretation, it should pass several checks. These checks are simple, but they prevent many serious errors.

**Method**

**Modelling checkpoint for technical calculations.**

1. **Quantity:** What physical quantity is being calculated?
2. **System:** What object, material, fluid, sensor or process is being modelled?
3. **Assumptions:** What geometry, uniformity, calibration or idealisation is assumed?
4. **Units:** Are all inputs converted to a consistent unit system?
5. **Dimensions:** Does the formula have the correct dimensions?
6. **Uncertainty:** Which input dominates the uncertainty?
7. **Plausibility:** Is the final order of magnitude physically reasonable?

**Example 1.22: A compact modelling check: mass of a tabletop**

A laboratory tabletop has dimensions $2.0\,\mathrm{m} \times 0.80\,\mathrm{m} \times 0.040\,\mathrm{m}$. The effective density of the material is approximately $700\,\mathrm{kg\,m^{-3}}$. Estimate its mass.
The model is a rectangular block of uniform effective density. The volume is

$$V = (2.0)(0.80)(0.040)\,\mathrm{m}^3 = 6.4 \times 10^{-2}\,\mathrm{m}^3. \tag{1.81}$$

Then

$$m = \rho V = (700\,\mathrm{kg\,m^{-3}})(6.4 \times 10^{-2}\,\mathrm{m}^3) \simeq 45\,\mathrm{kg}. \tag{1.82}$$

The result is plausible: the tabletop is heavy enough to require careful handling but not as massive as a large machine component.

# Chapter 2

# Advanced Topics for Technical and Engineering Students

The main part of the chapter has established the working language of measurement: physical quantities, units, dimensions, significant numerical reporting, density and introductory uncertainty. The present section is deliberately more demanding. It is written for strong students and for technical programmes in which measurement is not only a classroom topic but also part of design, simulation, instrumentation, quality control and data analysis.

The purpose is not to replace later courses in statistics, metrology, numerical methods, sensor systems or engineering design. The purpose is to show, already in Chapter 1, how far the elementary ideas can be pushed when they are used carefully. The central message is that measurement is a model-based activity. We never simply read "the truth" from an instrument. We define a measurand, choose a model of the system, choose an instrument, calibrate it, estimate uncertainty, make approximations and then decide whether the result is useful for the question being asked.

**Pedagogical boundary**

The advanced material in this section is optional for a short Physics 1 course. It is included because technical students often meet units, calibration, sensors, tolerances and numerical models before they meet a formal course in experimental methods. The level is intentionally higher than the core chapter, but every idea remains connected to elementary quantities and dimensional reasoning.

### 2.0.1 Measurement as a model-building process

A measurement begins before an instrument is touched. The first step is to decide which physical quantity is intended. For example, the phrase "measure the length of the rod" sounds simple, but a professional measurement requires more detail. Is the rod straight? Is the required length the distance between end faces, the arc length along a curved centreline, or the projected length along a machine axis? At what temperature is the length required? Is surface roughness relevant? Is the rod under load? What uncertainty is acceptable?

This is why metrology uses the idea of a *measurand*: the quantity intended to be measured, including the conditions under which it is defined. In introductory physics we do not need a formal vocabulary for every case, but the habit is essential. A numerical value is useful only when the quantity, unit, method and context are clear.

**Method**

**Model-based measurement workflow.**

1. Define the measurand: what physical quantity is to be determined?
2. Choose an idealised model: point object, rigid body, uniform material, steady signal, linear sensor, or another approximation.

3. Choose the instrument and measurement procedure.
4. Establish calibration and zero reference.
5. Record raw readings with units and resolution.
6. Convert readings into the desired physical quantity.
7. Estimate uncertainty and identify dominant sources.
8. Check dimensions, plausibility and consistency with other information.

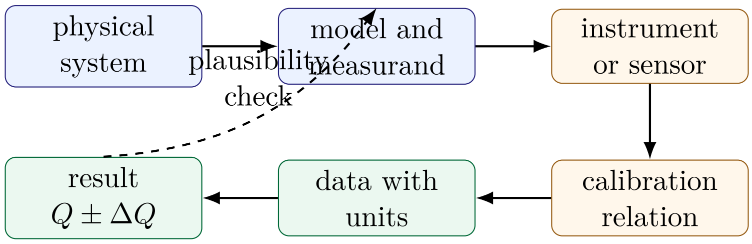


**Figure 2.1:** Measurement as a model-building process. A useful result comes from a chain of definitions, instruments, calibration and interpretation, not from a raw number alone.

**Example 2.1: Defining the measurand in a technical measurement**

A civil-engineering laboratory wants to measure the length of an aluminium beam before a thermal-expansion experiment. The beam is approximately 1.2 m long. A student proposes to report simply "length = 1.200 m".
**Analysis.** The value is incomplete because the measurand is not fully defined. A better statement is: the measured distance between the two end faces of the unloaded beam, along the beam axis, at room temperature, using a tape measure aligned with the axis. If the beam is later heated, the length depends on temperature. If the beam is under load, elastic strain may change the length. If the end faces are not perpendicular to the axis, different contact points may give slightly different readings.
**Interpretation.** The same number can be useful or misleading depending on the physical question. In a rough construction estimate, millimetre details may be irrelevant. In a thermal-expansion experiment, temperature, alignment and uncertainty matter.

### 2.0.2 Scaling laws and dimensionless groups

Dimensional analysis is not only a method for checking equations after they have been written. It is also a way to discover useful combinations of variables before the detailed theory is known. Engineering design uses this idea constantly. A wind-tunnel model of an aircraft, a small-scale model of a bridge, a laboratory flow experiment and a numerical simulation are useful only if the relevant dimensionless parameters are controlled.

A dimensionless quantity has no physical dimension. Examples include the ratio of two lengths, a strain $\Delta L/L$, an angle in radians, a coefficient of friction, a relative uncertainty and many engineering similarity parameters. Dimensionless quantities are powerful because they can compare systems of very different sizes.

**Key message**

A dimensionless ratio often tells us what is physically small, large or comparable. The statement $\Delta L/L = 10^{-4}$ is more informative than saying only that $\Delta L = 0.1$ mm, because it tells us the deformation relative to the size of the object.

### Geometric scaling

Suppose a solid object is scaled uniformly by a factor $s$: every length becomes $s$ times larger. Then areas scale as $s^2$ and volumes scale as $s^3$. If density remains the same, mass also scales as $s^3$:

$$L \to sL, \qquad A \to s^2 A, \qquad V \to s^3 V, \qquad m \to s^3 m. \tag{2.1}$$

This simple observation is extremely important. If the linear size of a component is doubled, its volume and mass become eight times larger, not two times larger. Forces, stresses, thermal time scales and structural limits may not scale in the same way.

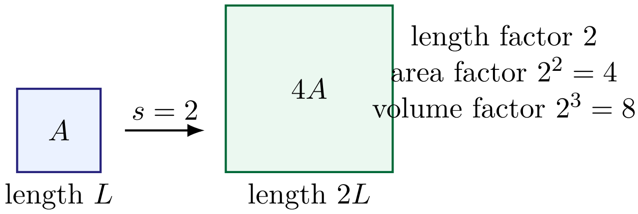


**Figure 2.2:** Uniform geometric scaling. Linear dimensions, areas and volumes scale with different powers of the scale factor.

**Example 2.2: Scaling a component mass**

A small aluminium bracket has mass 0.35 kg. A geometrically similar bracket is made with all lengths larger by a factor $s = 1.5$, using the same material. Estimate its mass.
**Model.** The brackets are geometrically similar and have the same density. Therefore mass scales as volume.
**Calculation.**

$$m_2 = s^3 m_1 = (1.5)^3 (0.35\,\mathrm{kg}) = 1.18\,\mathrm{kg}. \tag{2.2}$$

**Interpretation.** A 50% increase in every length produces more than a threefold increase in mass. This is why scaling arguments are central in design.

### A first Buckingham-Pi viewpoint

A general theorem of dimensional analysis states that if a physical problem involves $n$ dimensional variables built from $k$ independent base dimensions, then the problem can often be rewritten using $n - k$ independent dimensionless groups. A full treatment is beyond this chapter, but the idea is simple: physics should not depend on our arbitrary choice of metre, kilogram or second.

For example, suppose a phenomenon involves a characteristic length $L$, a speed $v$ and a time $t$. The combination

$$\Pi = \frac{vt}{L} \tag{2.3}$$

is dimensionless. It compares the distance travelled in time $t$ with the reference length $L$. If $\Pi \ll 1$, the object has travelled only a small fraction of $L$; if $\Pi \sim 1$, the motion is comparable to the system size; if $\Pi \gg 1$, it has travelled many system lengths.

**Note**

**Advanced perspective: dimensionless groups are compressed physics.** A dimensionless group is often a ratio of two competing effects. In later chapters, examples include strain $\Delta L/L$, Mach number $v/c_s$, Reynolds number for fluid flow, and efficiency for heat engines. The exact physical interpretation depends on the problem, but the logic begins here.

### 2.0.3 Dimensional analysis as a design tool

Dimensional analysis can help build candidate formulas when the detailed physical law is not yet known. It cannot determine dimensionless constants, and it cannot decide which variables are physically relevant. Those choices require physical modelling. Nevertheless, it can restrict the possible form of a relation very strongly.

#### Case study: time scale for falling through a height

Suppose the time $t$ for an object to fall through a height $h$ near Earth's surface depends mainly on $h$ and the gravitational acceleration magnitude $g$. We seek a combination of $h$ and $g$ with dimension of time. Let

$$t \propto h^a g^b. \tag{2.4}$$

The dimensions are

$$[T] = [L]^a [LT^{-2}]^b = [L^{a+b} T^{-2b}]. \tag{2.5}$$

Equating powers of $L$ and $T$ gives

$$a + b = 0, \qquad -2b = 1. \tag{2.6}$$

Thus $b = -1/2$ and $a = 1/2$, so

$$t \propto \sqrt{\frac{h}{g}}. \tag{2.7}$$

The exact result for free fall from rest is $t = \sqrt{2h/g}$. Dimensional analysis found the dependence on $h$ and $g$ but not the numerical factor $\sqrt{2}$.

**Caution**

Dimensional analysis can miss physics if the variable list is incomplete. For a feather falling through air, the time does not depend only on $h$ and $g$; air resistance, shape, area, mass and fluid properties matter.

#### Case study: pendulum period as a preview

A simple pendulum will be studied later in oscillations. At the level of dimensions, suppose its small-amplitude period $T$ depends on length $\ell$ and gravitational acceleration $g$, but not on the bob mass. The only time scale that can be formed is

$$\sqrt{\frac{\ell}{g}}. \tag{2.8}$$

Thus dimensional analysis suggests

$$T = C\sqrt{\frac{\ell}{g}}, \tag{2.9}$$

where $C$ is dimensionless. The full small-angle theory gives $C = 2\pi$. Again, dimensional analysis gives the scaling but not the dimensionless coefficient.

**Example 2.3: Using scaling to compare pendulums**

Two pendulums have lengths $\ell_1$ and $\ell_2 = 4\ell_1$. Without deriving the pendulum equation, estimate the ratio of their small-amplitude periods.

**Solution.** Since $T \propto \sqrt{\ell/g}$ for the same gravitational field,

$$\frac{T_2}{T_1} = \sqrt{\frac{\ell_2}{\ell_1}} = \sqrt{4} = 2. \tag{2.10}$$

**Interpretation.** A pendulum four times longer has twice the period. The result is a scaling statement and does not require knowing the factor $2\pi$.

### 2.0.4 Metrology, calibration and traceability

Metrology is the science of measurement. In engineering, metrology provides the link between a local measurement and accepted standards. Traceability means that a measurement can be related, through an unbroken chain of calibrations, to recognised standards, with uncertainties stated at each step.

For a beginner, the practical message is modest but important: instruments are not magic. A caliper, balance, voltmeter or sensor must be zeroed, calibrated and used within its range. Calibration is not merely a label on an instrument; it is a physical model connecting a reading to a quantity.

**Definition 2.1: Calibration relation**

A calibration relation is a mathematical rule that converts an instrument reading into an estimate of a physical quantity. In a simple linear model,

$$Q = aR + b, \tag{2.11}$$

where $R$ is the raw reading, $a$ is a scale factor and $b$ is an offset. The constants $a$ and $b$ must be determined by comparison with reference values.

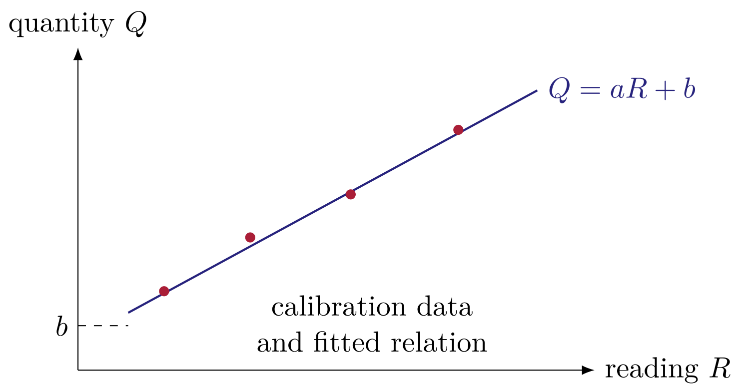


**Figure 2.3:** A simple linear calibration model. The intercept represents an offset and the slope represents sensitivity.

**Example 2.4: Linear calibration of a force sensor**

A force sensor outputs a voltage $V$. Calibration with known loads gives the approximate relation

$$F = (20.0\,\mathrm{N\,V^{-1}})(V - 0.050\,\mathrm{V}). \tag{2.12}$$

During an experiment the measured voltage is $V = 0.735\,\mathrm{V}$. Estimate the force.

**Calculation.**

$$F = (20.0\,\mathrm{N\,V^{-1}})(0.735\,\mathrm{V} - 0.050\,\mathrm{V}) = 13.7\,\mathrm{N}. \tag{2.13}$$

**Interpretation.** Subtracting the offset is part of the physical measurement model. If the offset were ignored, the reported force would be 14.7 N, a systematic error of about 1 N.

Method

**Calibration checklist for introductory laboratories.**

1. Check zero reading before applying the quantity to be measured.
2. Record the range over which the calibration is valid.
3. Use at least two reference points for a linear calibration; use more if linearity is uncertain.
4. Keep units with the slope and offset.
5. Recheck calibration if temperature, supply voltage or instrument configuration changes.

### 2.0.5 Uncertainty propagation using derivatives

The core chapter introduced conservative uncertainty rules. A more systematic first-order method uses derivatives. If a calculated quantity

$$Q = f(x_1, x_2, \ldots, x_n) \tag{2.14}$$

depends on measured inputs $x_i$, then a small change in the inputs produces approximately

$$\delta Q \simeq \frac{\partial f}{\partial x_1}\delta x_1 + \frac{\partial f}{\partial x_2}\delta x_2 + \cdots + \frac{\partial f}{\partial x_n}\delta x_n. \tag{2.15}$$

If independent random uncertainties are represented by standard uncertainties $u(x_i)$, a common first-order estimate is

$$u^2(Q) \simeq \left(\frac{\partial f}{\partial x_1}u(x_1)\right)^2 + cdots + \left(\frac{\partial f}{\partial x_n}u(x_n)\right)^2. \tag{2.16}$$

This expression is called combination in quadrature. It is less conservative than simply adding relative uncertainties, but it requires stronger assumptions about independence and the meaning of the reported uncertainties.

**Pedagogical boundary**

This derivative formula is included as enrichment. A professional uncertainty analysis must define probability distributions, coverage factors, correlations and calibration uncertainties. In this chapter we use the formula only to show how sensitivity to inputs is measured mathematically.

#### Cylinder density with derivative sensitivities

For a cylinder of mass $m$, diameter $d$ and height $h$, the density is

$$\rho = \frac{m}{V} = \frac{4m}{\pi d^2 h}. \tag{2.17}$$

Taking logarithmic differentials gives

$$\frac{\delta\rho}{\rho} = \frac{\delta m}{m} - 2\frac{\delta d}{d} - \frac{\delta h}{h}. \tag{2.18}$$

For uncertainty magnitudes, this shows why diameter matters strongly: the coefficient is 2 because volume depends on $d^2$.

**Example 2.5: Quadrature uncertainty for cylinder density**

A cylinder has

$$m = (245.0 \pm 0.2)\,\mathrm{g}, \quad d = (2.50 \pm 0.01)\,\mathrm{cm}, \quad h = (6.00 \pm 0.02)\,\mathrm{cm}. \tag{2.19}$$

Estimate the relative uncertainty of its density using quadrature.
**Solution.** The relative standard uncertainty estimate is

$$\left(\frac{u(\rho)}{\rho}\right)^2 \simeq \left(\frac{0.2}{245.0}\right)^2 + \left(2\frac{0.01}{2.50}\right)^2 + \left(\frac{0.02}{6.00}\right)^2 \tag{2.20}$$

$$\simeq (0.00082)^2 + (0.0080)^2 + (0.0033)^2. \tag{2.21}$$

Thus

$$\frac{u(\rho)}{\rho} \simeq 0.0087. \tag{2.22}$$

The density itself is $8.32\,\mathrm{g\,cm^{-3}}$, so

$$u(\rho) \simeq 0.072\,\mathrm{g\,cm^{-3}}. \tag{2.23}$$

A compact report is approximately

$$\rho = (8.32 \pm 0.07)\,\mathrm{g\,cm^{-3}}, \tag{2.24}$$

with the meaning of the uncertainty stated by the laboratory convention.

**Key message**

Sensitivity coefficients tell us where effort matters. If the final result depends on $d^2$, improving the diameter measurement may be more valuable than improving the mass measurement.

### 2.0.6 Digital measurement: resolution, quantisation and sampling

Many modern measurements enter a computer through an analog-to-digital converter. The converter maps a continuous voltage range onto a finite set of digital levels. This introduces quantisation: two slightly different voltages may be assigned the same digital value. Quantisation is not the only uncertainty in a digital measurement, but it is often the easiest to estimate.

If an $N$-bit converter spans a voltage range 0 to $V_{mmax}$, the ideal voltage step is approximately

$$\Delta V \simeq \frac{V_{mmax}}{2^N}. \tag{2.25}$$

A single reading is then limited by at least a fraction of this step, even before sensor noise, calibration and environmental effects are considered.

**Example 2.6: ADC resolution in a temperature measurement**

A temperature sensor has sensitivity $10\,\mathrm{mV\,K^{-1}}$. Its output is read by a 12-bit converter over a 0 to 5.0 V range. Estimate the temperature interval corresponding to one digital step.
**Voltage step.**

$$\Delta V \simeq \frac{5.0\,\mathrm{V}}{2^{12}} = 1.22\,\mathrm{mV}. \tag{2.26}$$

**Temperature step.**

$$\Delta T \simeq \frac{1.22\,\text{mV}}{10\,\text{mV}\,\text{K}^{-1}} = 0.122\,\text{K}. \tag{2.27}$$

**Interpretation.** The digital conversion alone cannot resolve temperature changes much smaller than about 0.1 K unless additional techniques or a narrower voltage range are used. Calibration and sensor accuracy may impose larger limits.

#### Sampling and aliasing as a preview

Sampling frequency determines how often a signal is recorded. A sampling rate of $f_s$ gives a sampling interval $\Delta t_s = 1/f_s$. If a signal changes rapidly between samples, important information may be missed. A full treatment of sampling theory is beyond Physics 1, but the practical warning is simple: a data file can look smooth and still fail to capture fast motion.

**Caution**

Do not confuse the sampling interval with the uncertainty of each sample. The interval $\Delta t_s$ tells when samples are taken. The uncertainty of each sample depends on timing accuracy, calibration, sensor noise, resolution and data processing.

### 2.0.7 Units in software and numerical simulations

A computer variable usually stores a number, not a physical quantity. The program does not automatically know whether a variable called `length` is measured in metres, millimetres or pixels. This is why unit discipline in code is part of scientific computing.

There are several safe strategies:

1. convert all inputs to SI units at the boundary of the program;
2. include units in variable names when unit-aware libraries are not used, for example `length_m` and `time_s`;
3. document units in data files and function descriptions;
4. use unit-aware libraries when available and appropriate;
5. perform dimensional sanity checks on intermediate and final results;
6. format output according to physical uncertainty, not merely machine precision.

**Method**

**Unit-safe simulation workflow.**

1. Read raw input values and their declared units.
2. Convert all dimensional inputs to an internal unit convention, usually SI.
3. Compute using variables whose units are documented.
4. Check that each equation combines compatible dimensions.
5. Convert outputs only at the presentation stage, if desired.
6. Report numerical precision consistent with uncertainty or model accuracy.

**Example 2.7: A millimetre–metre error in a volume calculation**

A CAD file lists a cube side as 40 mm. A simulation script interprets the number 40 as metres. By what factor is the computed volume wrong?
**Correct side length.**

$$40\,\text{mm} = 0.040\,\text{m}. \tag{2.28}$$

The incorrect length is 40 m, so the length factor error is

$$\frac{40\,\text{m}}{0.040\,\text{m}} = 1000. \tag{2.29}$$

Volume scales as length cubed, so the volume error factor is

$$1000^3 = 10^9. \tag{2.30}$$

**Interpretation.** A small-looking unit mistake in a length field can become a billion-fold error in volume and mass. This is why unit conversion must be explicit in software.

**Note**

**Advanced perspective: numerical precision is not physical precision.** Double-precision arithmetic can store many significant digits, but a laboratory measurement may justify only two or three. A simulation can produce many decimal places even when the model assumptions are crude. Physical precision is limited by measurement, modelling and numerical error, not only by floating-point representation.

### 2.0.8 Quality control, tolerances and acceptance bands

Engineering measurement often asks a yes-or-no question: is a manufactured part acceptable? A drawing may specify a nominal diameter $d_0$ and a tolerance band. For example,

$$d = (10.00 \pm 0.05)\,\text{mm} \tag{2.31}$$

means that parts may be accepted if their diameter lies between 9.95 mm and 10.05 mm, depending on the inspection standard. The measurement uncertainty of the inspection instrument must be small enough compared with the tolerance. Otherwise the decision can be ambiguous.

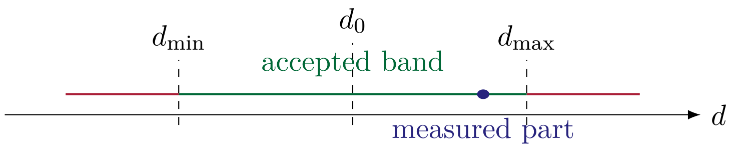


**Figure 2.4:** Tolerance band for a manufactured dimension. Measurement uncertainty should be considered when the reading is close to an acceptance boundary.

**Example 2.8: Inspection near a tolerance boundary**

A shaft diameter is specified as $d = (20.00 \pm 0.03)$ mm, meaning an acceptable interval from 19.97 mm to 20.03 mm. A digital caliper reads 20.028 mm with an estimated measurement uncertainty of 0.010 mm. Is the acceptance decision obvious?
**Discussion.** The central reading lies inside the tolerance band, but it is only 0.002 mm below the upper limit. The measurement uncertainty is larger than this margin. A responsible inspection procedure may require repeated measurements, a more accurate instrument or a

specified decision rule.
**Interpretation.** Tolerance is a design requirement; measurement uncertainty describes how well the inspection procedure can determine whether that requirement is met.

### 2.0.9 Density beyond a uniform block

The core chapter introduced density as $\rho = m/V$. This is enough for many introductory calculations, but technical work often distinguishes several meanings of density.

- **Average density** is total mass divided by total volume.
- **Local density** describes how mass is distributed near a point in a continuum model.
- **Effective density** may include voids, pores, composite structure or packaging.
- **Bulk density** may refer to granular material including spaces between grains.

For a continuum body with local density $\rho(\boldsymbol{r})$, the mass of a region is written formally as

$$m = \int_V \rho(\boldsymbol{r})\,\mathrm{d}V. \tag{2.32}$$

This is not needed for simple Chapter 1 calculations, but it prepares the idea that density can vary with position. In fluids and thermodynamics, density may also vary with pressure and temperature.

**Example 2.9: Average density of a porous material**

A porous ceramic sample has external volume $50\,\mathrm{cm}^3$ and mass $90\,\mathrm{g}$. The solid ceramic material itself has density about $3.0\,\mathrm{g\,cm^{-3}}$. Find the average density of the sample and estimate the volume fraction occupied by solid material.
**Average density.**

$$\rho_{\mathrm{avg}} = \frac{90\,\mathrm{g}}{50\,\mathrm{cm}^3} = 1.8\,\mathrm{g\,cm^{-3}}. \tag{2.33}$$

**Solid volume estimate.**

$$V_{\mathrm{solid}} = \frac{90\,\mathrm{g}}{3.0\,\mathrm{g\,cm^{-3}}} = 30\,\mathrm{cm}^3. \tag{2.34}$$

Thus the solid volume fraction is

$$\frac{V_{\mathrm{solid}}}{V_{\mathrm{external}}} = \frac{30}{50} = 0.60. \tag{2.35}$$

**Interpretation.** The average density is lower than the material density because the sample includes pore volume.

### 2.0.10 Extended advanced worked examples

**Example 2.10: Design estimate: mass of a scaled aluminium housing**

An aluminium electronics housing has external dimensions roughly $120\,\mathrm{mm} \times 80\,\mathrm{mm} \times 30\,\mathrm{mm}$ and wall thickness much smaller than its other dimensions. A geometrically similar housing is designed with all external dimensions larger by a factor 1.25 and the same relative wall thickness. If the original housing mass is 0.42 kg, estimate the new mass.
**Model.** Geometric similarity with the same material and relative wall thickness means all volumes scale as $s^3$.

**Calculation.**

$$m_2 = s^3 m_1 = (1.25)^3(0.42\,\text{kg}) = 0.82\,\text{kg}. \tag{2.36}$$

**Interpretation.** The mass almost doubles even though each linear dimension increases by only 25%.

### Example 2.11: Nondimensionalising a straight-line motion estimate

A small vehicle moves at speed $v$ along a track of length $L$. The travel time is $t = L/v$. Write the relation in dimensionless form using a reference time $t_0 = L/v$.
**Solution.** Define the dimensionless time

$$\tau = \frac{t}{t_0} = \frac{vt}{L}. \tag{2.37}$$

The end of the track corresponds to $\tau = 1$. If $\tau = 0.25$, the vehicle has travelled one quarter of the reference distance in the constant-speed model.
**Interpretation.** Dimensionless variables often reveal the structure of a problem more clearly than dimensional variables.

### Example 2.12: Unit check in a pressure sensor model

A pressure sensor calibration is proposed as

$$p = A(V - V_0), \tag{2.38}$$

where $p$ is pressure and $V$ is voltage. What unit must the calibration constant $A$ have?
**Solution.** The voltage difference has unit volt. Pressure has unit pascal. Therefore

$$[A] = \text{Pa}\,\text{V}^{-1}. \tag{2.39}$$

**Interpretation.** A calibration slope without units is incomplete. In software documentation it should be written explicitly, for example $A = 250\,\text{kPa}\,\text{V}^{-1}$.

### Example 2.13: Uncertainty contribution from a calibration offset

A temperature sensor is calibrated as

$$T = \frac{V - V_0}{S}, \tag{2.40}$$

where $S = 10.0\,\text{mV}\,\text{K}^{-1}$. The zero offset has uncertainty $u(V_0) = 0.5\,\text{mV}$. Estimate the corresponding temperature uncertainty.
**Solution.** The sensitivity of $T$ to $V_0$ is

$$\left|\frac{\partial T}{\partial V_0}\right| = \frac{1}{S}. \tag{2.41}$$

Therefore

$$u(T)_{V_0} = \frac{0.5\,\text{mV}}{10.0\,\text{mV}\,\text{K}^{-1}} = 0.05\,\text{K}. \tag{2.42}$$

**Interpretation.** Offset uncertainty appears directly as a temperature uncertainty through the sensor sensitivity.

**Example 2.14: Choosing a sampling rate for a slow thermal signal**

A temperature in a laboratory water bath changes on a time scale of minutes. A student suggests sampling at 1000 Hz. Is this necessary?

**Discussion.** A rate of 1000 Hz gives 1000 samples per second. For a signal changing over minutes, this is far more data than needed for the physical variation, although it may still capture electrical noise. A lower sampling rate, combined with averaging and calibration, may be more appropriate.

**Interpretation.** Sampling rate should be chosen from the time scale of the physical process and the purpose of the measurement, not from the largest rate available in the device.

### 2.0.11 Advanced guided checks

**Guided checks**

1. If a model is dimensionally consistent, is it necessarily correct? No. Dimensional consistency is necessary but not sufficient.
2. If all lengths of a solid part are doubled, does the mass double? No. For constant density and geometric similarity, mass increases by a factor of eight.
3. Why can a calibration offset create systematic error? Because it shifts all readings in the same direction if uncorrected.
4. Why is a dimensionless strain useful? It compares deformation with the original size of the object.
5. Does an ADC with more bits guarantee an accurate sensor? No. It improves digital resolution, but calibration, sensor physics and noise also matter.
6. Why should units be documented in data files? A column of numbers without units cannot be interpreted unambiguously.
7. Why can a tolerance decision be ambiguous? Because the measured value may lie near the tolerance boundary with uncertainty comparable to the remaining margin.

### 2.0.12 Advanced exercises for enrichment

#### A. Scaling and dimensionless reasoning

1. A geometrically similar model bridge is built at scale $1 : 20$. Assuming the same material, by what factor is its mass smaller than the full bridge? Discuss why this does not mean the model has the same structural behaviour.

2. A robot arm link is scaled by a factor $s$. Assuming constant density, how do its length, cross-sectional area, volume and mass scale?

3. Construct a dimensionless group from a speed $v$, a time scale $t$ and a length $L$. Interpret the cases where the group is much smaller than, comparable to and much larger than one.

4. A pendulum period is assumed to depend only on length $\ell$, mass $m$ and gravitational acceleration $g$. Use dimensional reasoning to show that mass cannot enter the period through any dimensional combination.

5. A pressure drop in a pipe depends on density $\rho$, speed $v$ and perhaps a dimensionless coefficient. Show that $\rho v^2$ has the dimensions of pressure.

**B. Calibration and sensors**

1. A displacement sensor has calibration $x = (2.00\,\text{mm}\,\text{V}^{-1})(V - 0.10\,\text{V})$. Find $x$ for $V = 1.85\,\text{V}$.
2. A force sensor has slope $50\,\text{N}\,\text{V}^{-1}$ and zero offset $0.020\,\text{V}$. What force error results if the zero offset is ignored?
3. A 10-bit ADC covers 0 to $3.3\,\text{V}$. Estimate the voltage step. If a sensor sensitivity is $0.50\,\text{V}\,\text{m}^{-1}$, what displacement step corresponds to one ADC count?
4. A calibration curve is linear over 0 to $100\,\text{N}$ but visibly nonlinear above that range. Explain why extrapolating it to $200\,\text{N}$ is risky.
5. List the metadata that should accompany a sensor dataset used in a laboratory report.

**C. Uncertainty and tolerance**

1. A rectangular plate has dimensions $a = (10.0 \pm 0.1)\,\text{cm}$ and $b = (5.0 \pm 0.1)\,\text{cm}$. Estimate the relative uncertainty in its area using both conservative addition and quadrature.
2. A measured shaft diameter is $12.014\,\text{mm}$ with uncertainty $0.006\,\text{mm}$. The tolerance interval is $12.000\,\text{mm}$ to $12.020\,\text{mm}$. Discuss the acceptance decision.
3. A density depends on diameter as $d^{-2}$. If the diameter is biased high by 0.5%, what is the approximate percentage bias in density?
4. A balance has a zero offset of $0.03\,\text{g}$. For which measurement is the relative effect larger: a $2\,\text{g}$ sample or a $200\,\text{g}$ sample?
5. Explain why repeated measurements can reduce random scatter but cannot automatically remove calibration bias.

**D. Computational unit discipline**

1. A data file stores time in milliseconds and position in centimetres. Write conversion formulas to SI variables $t_\text{s}$ and $x_\text{m}$.
2. A simulation computes kinetic energy using $K = mv^2/2$. If $m$ is in grams and $v$ is in metres per second, what unit does the raw result have? What conversion is needed to obtain joules?
3. A function takes an angle argument. Explain why documentation should state whether the angle is in degrees or radians.
4. A program prints every computed value with twelve decimal places. Explain why this is not a substitute for uncertainty analysis.
5. Propose variable names for a unit-safe calculation of density from mass in kilograms and volume in cubic metres.

### 2.0.13 Selected solutions to advanced enrichment problems

**Example 2.15: Selected solution: model bridge scaling**

For scale 1 : 20, every length in the model is 1/20 of the full size. If the same material is used and the geometry is similar, mass scales as volume:

$$\frac{m_{\text{model}}}{m_{\text{full}}} = \left(\frac{1}{20}\right)^3 = \frac{1}{8000}. \tag{2.43}$$

The model is much lighter, but structural behaviour does not necessarily scale in the same way because loads, self-weight, stiffness, buckling and material behaviour may scale differently.

**Example 2.16: Selected solution: pressure dimension from $\rho v^2$**

Density has dimension $[\rho] = [ML^{-3}]$ and speed squared has dimension $[v^2] = [L^2T^{-2}]$. Therefore

$$[\rho v^2] = [ML^{-3}][L^2T^{-2}] = [ML^{-1}T^{-2}], \tag{2.44}$$

which is the dimension of pressure because pressure is force per area:

$$[p] = \frac{[MLT^{-2}]}{[L^2]} = [ML^{-1}T^{-2}]. \tag{2.45}$$

**Example 2.17: Selected solution: area uncertainty of a rectangular plate**

For $A = ab$, the central value is

$$A = (10.0\,\text{cm})(5.0\,\text{cm}) = 50.0\,\text{cm}^2. \tag{2.46}$$

The conservative relative uncertainty estimate is

$$\frac{\Delta A}{A} \simeq \frac{0.1}{10.0} + \frac{0.1}{5.0} = 0.010 + 0.020 = 0.030. \tag{2.47}$$

Thus $\Delta A \simeq 1.5\,\text{cm}^2$. The quadrature estimate is

$$\frac{u(A)}{A} \simeq \sqrt{(0.010)^2 + (0.020)^2} = 0.022, \tag{2.48}$$

so $u(A) \simeq 1.1\,\text{cm}^2$. The two estimates differ because they represent different assumptions about how uncertainties combine.

**Example 2.18: Selected solution: kinetic energy with grams**

If $m$ is inserted in grams and $v$ in metres per second, the raw expression $mv^2/2$ has unit

$$\text{g}\,\text{m}^2\,\text{s}^{-2}. \tag{2.49}$$

Since $1\,\text{g} = 10^{-3}\,\text{kg}$ and $1\,\text{J} = 1\,\text{kg}\,\text{m}^2\,\text{s}^{-2}$, the raw numerical result must be multiplied by $10^{-3}$ to obtain joules. A safer policy is to convert mass to kilograms before computing.

### 2.0.14 Extended technical tutorials for strong students

The following tutorials are designed for tutorial classes, project work or self-study. They are longer than ordinary examples because they show how a technical question is translated into a sequence of modelling decisions. The calculations remain elementary, but the reasoning is closer to what students will later need in laboratories, design projects and computational work.

### Tutorial 1: auditing units in a simulation input file

Consider a simple simulation of a falling object. The code expects SI units: length in metres, mass in kilograms and time in seconds. A user prepares the following input table:

| Input name | Value | Intended meaning |
|---|---|---|
| height | 250 | 250 cm |
| mass | 75 | 75 g |
| time_step | 0.01 | 0.01 s |
| gravity | 9.81 | 9.81 m $s^{-2}$ |

If the program silently interprets all dimensional numbers as SI, then the height is treated as 250 m instead of 2.50 m, and the mass is treated as 75 kg instead of 0.075 kg. The first error changes the predicted fall time by a factor

$$\sqrt{\frac{250}{2.50}} = 10, \tag{2.50}$$

because $t \propto \sqrt{h/g}$ for free fall from rest. The mass error may not change ideal free fall, but it would severely affect drag, impact energy or structural loading.

**Method**

**Unit audit of a computational input file.**

1. Write a unit column for every dimensional input.
2. Convert every input to the internal unit convention before simulation.
3. Check at least one output against an analytic estimate.
4. Include a warning if a variable name has no documented unit.
5. Store converted values and raw values separately if traceability is needed.

A clean input table for the same simulation would use:

| Input name | Raw value | Raw unit | SI value |
|---|---|---|---|
| height | 250 | cm | 2.50 m |
| mass | 75 | g | 0.075 kg |
| time_step | 0.01 | s | 0.01 s |
| gravity | 9.81 | m $s^{-2}$ | 9.81 m $s^{-2}$ |

### Tutorial 2: full measurement worksheet for density of a cylinder

A useful first laboratory exercise is to determine the density of a metal cylinder. The calculation is simple, but the measurement chain is rich: the result depends on mass, diameter, height, geometric assumptions and instrument calibration. The worksheet below is intentionally detailed.

**Method**

**Density measurement worksheet.**

1. Inspect the object. Is it well approximated by a right circular cylinder?
2. Measure the mass several times after taring the balance.
3. Measure the diameter at several angular positions to test circularity.
4. Measure the height at several points to test flatness of the end faces.

5. Record instrument resolutions and possible zero offsets.
6. Compute the mean mass, mean diameter and mean height.
7. Compute the volume $V = \pi d^2 h/4$.
8. Compute $\rho = m/V$.
9. Estimate uncertainty and identify the dominant contribution.
10. Compare the result with reference material densities only after uncertainty is considered.

Suppose the data are:

| Quantity | Readings | Mean | Instrument resolution |
|---|---|---|---|
| mass | 156.42, 156.44, 156.43 g | 156.43 g | 0.01 g |
| diameter | 18.02, 18.04, 18.01 mm | 18.02 mm | 0.01 mm |
| height | 45.10, 45.12, 45.11 mm | 45.11 mm | 0.01 mm |

Convert to SI:

$$m = 0.15643\,\mathrm{kg}, \qquad d = 0.01802\,\mathrm{m}, \qquad h = 0.04511\,\mathrm{m}. \tag{2.51}$$

The volume is

$$V = \frac{\pi d^2 h}{4} = \frac{\pi(0.01802\,\mathrm{m})^2(0.04511\,\mathrm{m})}{4} = 1.151 \times 10^{-5}\,\mathrm{m}^3. \tag{2.52}$$

Thus

$$\rho = \frac{0.15643\,\mathrm{kg}}{1.151 \times 10^{-5}\,\mathrm{m}^3} = 1.36 \times 10^4\,\mathrm{kg\,m^{-3}}. \tag{2.53}$$

The numerical result suggests a dense metal. Before identifying the material, students should ask whether the cylinder is solid, whether the balance was tared, whether the dimensions were converted correctly and whether the object is actually cylindrical.

**Caution**

Material identification from density alone is not always reliable. Different alloys can have similar densities, a sample may contain voids, and surface coatings or internal structure can bias a simple estimate.

### Tutorial 3: tolerance, uncertainty and a decision rule

A mechanical part is specified to have length between 49.90 mm and 50.10 mm. A measurement gives

$$L = (50.08 \pm 0.04)\,\mathrm{mm}. \tag{2.54}$$

The central value lies inside the tolerance interval, but the upper end of the uncertainty interval is 50.12 mm, slightly outside the allowed range. What should be done?

The answer depends on the inspection standard, but the physics lesson is clear. A measurement result is not a single point on the number line. It is a region of plausible values. Near a boundary, the decision depends on the uncertainty and on the rule chosen by the laboratory or manufacturer.

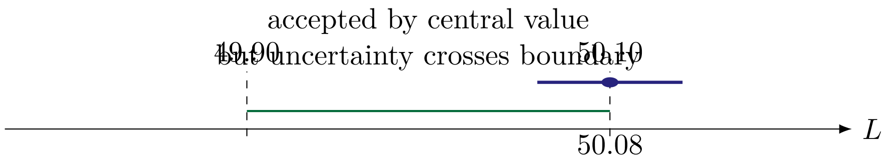


**Figure 2.5:** A tolerance decision near a boundary. The central value alone may not be enough for a reliable decision.

**Key message**

In quality control, uncertainty is not a philosophical detail. It affects accept–reject decisions, safety margins and cost.

### Tutorial 4: building a calibration curve from two reference points

A pressure sensor is tested with two reference pressures. At $p = 0\,\mathrm{kPa}$ it gives $V = 0.20\,\mathrm{V}$. At $p = 100\,\mathrm{kPa}$ it gives $V = 2.70\,\mathrm{V}$. Assume a linear relation

$$p = aV + b. \tag{2.55}$$

The slope is

$$a = \frac{100\,\mathrm{kPa} - 0\,\mathrm{kPa}}{2.70\,\mathrm{V} - 0.20\,\mathrm{V}} = 40.0\,\mathrm{kPa\,V^{-1}}. \tag{2.56}$$

Using the zero-pressure point,

$$0 = a(0.20\,\mathrm{V}) + b, \tag{2.57}$$

so

$$b = -8.0\,\mathrm{kPa}. \tag{2.58}$$

The calibration relation is therefore

$$p = (40.0\,\mathrm{kPa\,V^{-1}})V - 8.0\,\mathrm{kPa}. \tag{2.59}$$

If a later reading is $V = 1.35\,\mathrm{V}$, then

$$p = (40.0)(1.35)\,\mathrm{kPa} - 8.0\,\mathrm{kPa} = 46.0\,\mathrm{kPa}. \tag{2.60}$$

**Note**

A two-point calibration assumes linearity. More reference points are needed to test whether the sensor response is actually linear over the required range.

### Tutorial 5: order-of-magnitude estimate for laboratory safety

A laboratory demonstration uses a small steel sphere of radius 1.0 cm. Estimate its mass using $\rho_{\text{steel}} \approx 8 \times 10^3\,\mathrm{kg\,m^{-3}}$. The sphere volume is

$$V = \frac{4}{3}\pi r^3 \approx 4(10^{-2}\,\mathrm{m})^3 = 4 \times 10^{-6}\,\mathrm{m}^3, \tag{2.61}$$

where we have rounded $\frac{4}{3}\pi$ to about 4. The mass is then

$$m \approx (8 \times 10^3)(4 \times 10^{-6})\,\mathrm{kg} = 3.2 \times 10^{-2}\,\mathrm{kg}. \tag{2.62}$$

The mass is about 30 g. This is small, but if the sphere is launched at several metres per second, its kinetic energy may still be relevant for safety planning. Thus even Chapter 1 estimates can support responsible experimental design.

### Tutorial 6: writing a measurement result in a data file

A useful data file should be understandable without the person who created it standing nearby. For example, a file containing temperature measurements should not contain only a column called `T`. Better metadata include:

- quantity measured: temperature of water bath;

- unit: degree Celsius or kelvin, stated explicitly;
- sensor type and identification number;
- calibration date or calibration relation;
- sampling interval or time stamps;
- estimated uncertainty or resolution;
- experimental conditions;
- any filtering or averaging already applied.

The same principle applies to position data from video tracking, force data from a load cell, acceleration data from a smartphone or pressure data from a transducer.

### 2.0.15 Extended challenge problems

These problems are more open-ended than the standard practice problems. They are intended for strong students, project groups or computer-lab extensions.

1. **Unit audit project.** Design a one-page checklist for checking units in a numerical simulation of projectile motion. Include input variables, internal variables, output variables and at least three dimensional tests.
2. **Calibration project.** Suppose a sensor has a nonlinear calibration $Q = aR + bR^2 + c$. Explain how many reference points are minimally needed to determine the constants, and discuss why more points are preferable.
3. **Density and geometry project.** Compare the density uncertainty of a rectangular block and a cylinder when all length measurements have the same absolute uncertainty. Which geometry is more sensitive to a diameter or thickness measurement?
4. **Tolerance decision project.** Propose a decision rule for accepting or rejecting parts when measurement uncertainty overlaps a tolerance boundary. Discuss the trade-off between rejecting good parts and accepting bad parts.
5. **Scaling project.** Choose a technical object such as a drone frame, bridge model, robot arm or heat sink. Discuss qualitatively how mass, surface area, stiffness, heat transfer area and cost might change when all dimensions are scaled by a factor $s$.
6. **Data-file project.** Create a template header for a CSV file containing position measurements from video tracking. Include units, calibration, frame rate and uncertainty information.

### 2.0.16 Selected solutions to extended tutorials

**Example 2.19: Selected solution: two-point pressure calibration**

Using the two reference points $(V_1, p_1) = (0.20\,\mathrm{V}, 0\,\mathrm{kPa})$ and $(V_2, p_2) = (2.70\,\mathrm{V}, 100\,\mathrm{kPa})$, the linear slope in the relation $p = aV + b$ is

$$a = \frac{p_2 - p_1}{V_2 - V_1} = \frac{100\,\mathrm{kPa}}{2.50\,\mathrm{V}} = 40.0\,\mathrm{kPa\,V^{-1}}. \quad (2.63)$$

The intercept is

$$b = p_1 - aV_1 = 0 - (40.0\,\mathrm{kPa\,V^{-1}})(0.20\,\mathrm{V}) = -8.0\,\mathrm{kPa}. \quad (2.64)$$

Thus a voltage of 1.35 V corresponds to

$$p = (40.0)(1.35)\,\mathrm{kPa} - 8.0\,\mathrm{kPa} = 46.0\,\mathrm{kPa}. \tag{2.65}$$

**Example 2.20: Selected solution: tolerance interval with uncertainty**

The specified interval is $49.90\,\mathrm{mm} \leq L \leq 50.10\,\mathrm{mm}$. The measurement result is $L = (50.08 \pm 0.04)\,\mathrm{mm}$, so the plausible interval represented by the measurement is approximately 50.04 mm to 50.12 mm. Since this interval crosses the upper specification limit, a responsible procedure should not treat the part as unambiguously acceptable without a defined decision rule. Repeated measurements or a more accurate instrument may be needed.

**Example 2.21: Selected solution: unit audit of falling-object input**

The intended height 250 cm equals 2.50 m, while the program would interpret the raw number 250 as 250 m. The height error factor is 100. Since the free-fall time from rest scales as $\sqrt{h}$, the predicted time becomes too large by a factor of $\sqrt{100} = 10$. The intended mass 75 g equals 0.075 kg, while the program would interpret 75 as 75 kg. The mass error factor is 1000. This may not affect ideal free fall, but it affects any model involving force, drag, energy or momentum.

**Example 2.22: Selected solution: geometry sensitivity for a cylinder**

For a cylinder,

$$V = \frac{\pi d^2 h}{4}. \tag{2.66}$$

A fractional error in diameter contributes twice as strongly to the fractional error in volume:

$$\frac{\delta V}{V} = 2\frac{\delta d}{d} + \frac{\delta h}{h}. \tag{2.67}$$

Thus, for the same fractional uncertainty, the diameter measurement matters twice as much as the height measurement. For the same absolute uncertainty, the smaller dimension may dominate because $\delta d/d$ or $\delta h/h$ is larger.

### 2.0.17 Additional solved examples for chapter consolidation

The following examples consolidate the main and advanced parts of the chapter. They are deliberately written in a slow, explanatory style. The goal is to model the type of reasoning expected in a technical solution: define the quantity, choose units, compute, check dimensions and interpret the answer.

**Example 2.23: Converting a manufacturing speed to SI units**

A conveyor belt in a production line moves at $18\,\mathrm{km\,h^{-1}}$. A control algorithm requires the speed in metres per second.
**Solution.** Use $1\,\mathrm{km} = 1000\,\mathrm{m}$ and $1\,\mathrm{h} = 3600\,\mathrm{s}$:

$$18\,\mathrm{km\,h^{-1}} = 18\frac{1000\,\mathrm{m}}{3600\,\mathrm{s}} = 5.0\,\mathrm{m\,s^{-1}}. \tag{2.68}$$

**Interpretation.** The value is moderate for a vehicle but large for many indoor conveyor or robot systems. The conversion factor $1000/3600 = 1/3.6$ is useful, but it should not replace dimensional reasoning.

**Example 2.24: Area conversion in a stress estimate**

A force of 600 N acts on a contact patch of area $4.0\,\mathrm{cm}^2$. Find the average pressure in pascals.
**Common trap.** Since $1\,\mathrm{cm} = 10^{-2}\,\mathrm{m}$, one square centimetre is not $10^{-2}\,\mathrm{m}^2$ but

$$1\,\mathrm{cm}^2 = 10^{-4}\,\mathrm{m}^2. \tag{2.69}$$

Thus

$$A = 4.0\,\mathrm{cm}^2 = 4.0 \times 10^{-4}\,\mathrm{m}^2. \tag{2.70}$$

The pressure is

$$p = \frac{F}{A} = \frac{600\,\mathrm{N}}{4.0 \times 10^{-4}\,\mathrm{m}^2} = 1.5 \times 10^6\,\mathrm{Pa}. \tag{2.71}$$

**Interpretation.** A seemingly small contact area can produce a large pressure. Unit conversion for area must square the length conversion factor.

**Example 2.25: Volume conversion in a material-density calculation**

A polymer sample has mass 37.5 g and volume $25.0\,\mathrm{cm}^3$. Compute its density in $\mathrm{kg\,m^{-3}}$.
**Solution in cgs-like units.**

$$\rho = \frac{37.5\,\mathrm{g}}{25.0\,\mathrm{cm}^3} = 1.50\,\mathrm{g\,cm^{-3}}. \tag{2.72}$$

Now convert:

$$1\,\mathrm{g\,cm^{-3}} = \frac{10^{-3}\,\mathrm{kg}}{10^{-6}\,\mathrm{m}^3} = 10^3\,\mathrm{kg\,m^{-3}}. \tag{2.73}$$

Therefore

$$\rho = 1.50 \times 10^3\,\mathrm{kg\,m^{-3}}. \tag{2.74}$$

**Interpretation.** The factor 1000 is common when converting from $\mathrm{g\,cm^{-3}}$ to $\mathrm{kg\,m^{-3}}$.

**Example 2.26: Order-of-magnitude estimate: rainwater load on a flat roof**

A flat roof has area about $600\,\mathrm{m}^2$. During a storm, 20 mm of rain falls. Estimate the mass of water delivered to the roof.
**Model.** Treat rainfall depth as a uniform water layer of thickness $h = 20\,\mathrm{mm} = 2.0 \times 10^{-2}\,\mathrm{m}$. Use water density $\rho \approx 10^3\,\mathrm{kg\,m^{-3}}$.
**Volume.**

$$V = Ah = (600\,\mathrm{m}^2)(2.0 \times 10^{-2}\,\mathrm{m}) = 12\,\mathrm{m}^3. \tag{2.75}$$

**Mass.**

$$m = \rho V \approx (10^3\,\mathrm{kg\,m^{-3}})(12\,\mathrm{m}^3) = 1.2 \times 10^4\,\mathrm{kg}. \tag{2.76}$$

**Interpretation.** The water mass is of order ten tonnes. Drainage and structural loading are therefore engineering issues, not minor details.

**Example 2.27: Dimensional analysis of a braking-distance estimate**

A student proposes that braking distance $d$ for a vehicle might scale as

$$d = C\frac{v^2}{a}, \tag{2.77}$$

where $v$ is initial speed, $a$ is a characteristic deceleration magnitude and $C$ is dimensionless. Check the dimensions.

**Solution.**

$$\left[\frac{v^2}{a}\right] = \frac{[L^2T^{-2}]}{[LT^{-2}]} = [L]. \tag{2.78}$$

The expression has dimension length, so it is dimensionally possible.

**Interpretation.** Dimensional consistency does not prove the formula, but it supports the idea that $v^2/a$ is a natural length scale for stopping motion. The dimensionless coefficient depends on the detailed model.

**Example 2.28: Checking a proposed formula for pressure**

A formula in a draft report states that pressure is $p = \rho v$, where $\rho$ is density and $v$ is speed. Is this dimensionally valid?

**Solution.**

$$[\rho v] = [ML^{-3}][LT^{-1}] = [ML^{-2}T^{-1}]. \tag{2.79}$$

Pressure has dimension

$$[p] = [ML^{-1}T^{-2}]. \tag{2.80}$$

The proposed expression is not dimensionally consistent.

**Interpretation.** A more plausible pressure-like combination involving density and speed is $\rho v^2$, which has dimension $[ML^{-1}T^{-2}]$.

**Example 2.29: Reporting a result with meaningful digits**

A length and width are measured as

$$L = (2.40 \pm 0.02)\,\mathrm{m}, \qquad W = (1.20 \pm 0.01)\,\mathrm{m}. \tag{2.81}$$

A calculator gives the area as $2.879999999\,\mathrm{m}^2$. How should it be reported?

**Central value.**

$$A = LW = (2.40)(1.20)\,\mathrm{m}^2 = 2.88\,\mathrm{m}^2. \tag{2.82}$$

The relative uncertainty is roughly

$$\frac{\Delta A}{A} \simeq \frac{0.02}{2.40} + \frac{0.01}{1.20} \simeq 0.0167. \tag{2.83}$$

Thus

$$\Delta A \simeq 0.0167(2.88\,\mathrm{m}^2) \simeq 0.05\,\mathrm{m}^2. \tag{2.84}$$

A sensible report is

$$A = (2.88 \pm 0.05)\,\mathrm{m}^2. \tag{2.85}$$

**Interpretation.** The long calculator output is a numerical artefact, not physical information.

**Example 2.30: Mass and weight in an elevator experiment**

A student says that a 70 kg person has "weight 70 kg". Explain the problem and compute the gravitational force near Earth's surface.

**Solution.** Kilogram is the SI unit of mass, not force. The gravitational force magnitude near Earth's surface is approximately

$$W = mg = (70\,\mathrm{kg})(9.81\,\mathrm{m\,s^{-2}}) = 6.9 \times 10^{2}\,\mathrm{N}. \tag{2.86}$$

**Interpretation.** Everyday language often uses "weight" loosely, but physics and engineering must distinguish mass from force. This distinction becomes central in Newtonian dynamics.

### 2.0.18 Supplementary conceptual questions

1. Why is a unit conversion factor equal to one even when it changes the numerical value of a measurement?
2. Why can a dimensionless quantity still have physical meaning?
3. Give an example of a quantity that is dimensionless but not unimportant.
4. Explain why a measurement procedure is part of the definition of a reported result.
5. Why is a zero offset usually a systematic effect rather than a random effect?
6. Why can a high-resolution digital display still be inaccurate?
7. Why should raw data and processed data both keep unit information?
8. Why does volume conversion require cubing a length conversion factor?
9. Why can two materials with the same density still have different mechanical properties?
10. Why is order-of-magnitude estimation useful even when a calculator is available?
11. Why is it dangerous to mix millimetres and metres in a finite-element model?
12. Why does a relative uncertainty have no unit?
13. Explain why the symbol $m$ can mean metre in unit notation but mass in an equation, and how context avoids ambiguity.
14. Why is dimensional consistency a useful debugging tool for code?
15. Why does calibration need to be repeated or checked over time?

### 2.0.19 Supplementary engineering problems

1. A rectangular aluminium plate has dimensions $0.80\,\mathrm{m} \times 0.30\,\mathrm{m} \times 6.0\,\mathrm{mm}$. Estimate its mass using $\rho = 2.7 \times 10^{3}\,\mathrm{kg\,m^{-3}}$.
2. A hydraulic piston has diameter 40 mm and carries a force of 3.0 kN. Estimate the pressure in pascals.
3. A cylindrical tank has radius 0.75 m and height 1.8 m. Estimate the mass of water it can hold.
4. A sensor calibration is $x = (0.50\,\mathrm{mm\,V^{-1}})V + 0.20\,\mathrm{mm}$. Find $x$ for $V = 3.0\,\mathrm{V}$.

5. A temperature sensor has sensitivity $5.0\,\mathrm{mV\,K^{-1}}$. What temperature change corresponds to a voltage change of $25\,\mathrm{mV}$?

6. A time-of-flight measurement gives $t = (12.4 \pm 0.2)\,\mathrm{ms}$. Convert the value and uncertainty to seconds.

7. A load cell gives a reading with relative uncertainty 1.5%. What is the absolute uncertainty for a measured force of $240\,\mathrm{N}$?

8. A data logger records at $50\,\mathrm{Hz}$ for $3.0\,\mathrm{min}$. How many samples are collected?

9. A cube side length is measured with 0.5% uncertainty. Estimate the relative uncertainty of its volume.

10. A simulation uses density in $\mathrm{kg\,m^{-3}}$, but a user enters 7.8 intending $\mathrm{g\,cm^{-3}}$. By what factor is the density input too small?

**Take-home message**

The advanced lesson is the same as the elementary one, but sharper: physical numbers live inside a chain of modelling assumptions, unit conventions, calibration relations, uncertainty estimates and computational choices. Technical reliability comes from controlling the whole chain.

## 2.1 Chapter summary

**Summary**

- Physics begins with measurement: a physical quantity requires a numerical value, a unit and a clear operational meaning.
- SI units provide a common international language for quantitative science and engineering.
- Derived units are built from base units; their dimensions reveal how quantities depend on mass, length, time and other base dimensions.
- Unit conversion is algebra with conversion factors equal to one.
- Dimensional analysis is a powerful necessary check on equations, but it cannot prove that an equation is physically correct.
- Significant figures and uncertainties prevent us from claiming false precision.
- Order-of-magnitude estimates help us reason physically before and after detailed calculation.
- Density, $\rho = m/V$, is a first example of a derived quantity and will reappear in fluids, waves and thermodynamics.
- Engineering measurements require calibration, traceability, uncertainty budgets and clear reporting.
- Computational work requires explicit unit control; numbers in code do not automatically carry physical meaning.

**Formula summary**

$$Q = \{Q\}\,[Q] \qquad \text{quantity = numerical value times unit,} \tag{2.87}$$

$$\rho = \frac{m}{V} \qquad \text{mass density,} \tag{2.88}$$

$$[\rho] = [ML^{-3}] \qquad \text{dimension of density,} \tag{2.89}$$

$$[v] = [LT^{-1}] \qquad \text{dimension of speed,} \tag{2.90}$$

$$[a] = [LT^{-2}] \qquad \text{dimension of acceleration,} \tag{2.91}$$

$$[F] = [MLT^{-2}] \qquad \text{dimension of force,} \tag{2.92}$$

$$[K] = [ML^2T^{-2}] \qquad \text{dimension of kinetic energy,} \tag{2.93}$$

$$[p] = [ML^{-1}T^{-2}] \qquad \text{dimension of pressure,} \tag{2.94}$$

$$1\,\mathrm{cm}^2 = 10^{-4}\,\mathrm{m}^2, \qquad 1\,\mathrm{cm}^3 = 10^{-6}\,\mathrm{m}^3, \tag{2.95}$$

$$\frac{\Delta Q}{Q} = \text{relative uncertainty,} \tag{2.96}$$

$$\Delta t_s = \frac{1}{f_s} \qquad \text{sampling interval from sampling frequency.} \tag{2.97}$$

**Take-home message**

Before using a formula, check what every symbol means, what unit it carries, whether the dimensions match, which assumptions are being made and whether the numerical precision is justified. This habit will prevent many errors in mechanics, fluids, waves, thermodynamics, laboratory work and numerical simulations.

## 2.2 Original practice problems

The following problems are newly written for these notes. They are intended to practise the core skills of the chapter and are not copied from any textbook.

### A. Units and scientific notation

1. Write the following quantities in scientific notation: 0.00045 m, 720000 s and 0.0032 kg.
2. Convert 2.5 km to metres, centimetres and millimetres.
3. Convert $36\,\mathrm{km\,h^{-1}}$ to $\mathrm{m\,s^{-1}}$.
4. Convert $12.0\,\mathrm{cm^3}$ to $\mathrm{m^3}$.
5. Convert $0.85\,\mathrm{g\,cm^{-3}}$ to $\mathrm{kg\,m^{-3}}$.

### B. Dimensional analysis

1. Check whether $x = v_0 t + (1/2)at^2$ is dimensionally consistent.
2. Check whether $v^2 = v_0^2 + 2ax$ is dimensionally consistent.
3. A student proposes $F = mv^2$. Determine the dimension of the right-hand side and explain why this cannot be a force.

4. The drag force on an object moving through a fluid is sometimes modelled as $F = C\rho A v^2$, where $\rho$ is fluid density, $A$ is area and $v$ is speed. What dimension must $C$ have?

5. A formula for oscillation period is proposed as $T = C\sqrt{m/k}$, where $m$ is mass and $k$ has unit $\mathrm{N\,m^{-1}}$. Determine whether the dimensions are correct.

## C. Density and estimates

1. A cube of side 2.00 cm has mass 21.6 g. Find its density in $\mathrm{g\,cm^{-3}}$ and $\mathrm{kg\,m^{-3}}$.

2. Estimate the mass of water in a cylindrical bottle of radius 3 cm and height 20 cm.

3. Estimate the number of seconds in one year. Keep only one significant figure.

4. A laboratory table has approximate dimensions $2\,\mathrm{m} \times 0.8\,\mathrm{m} \times 0.04\,\mathrm{m}$ and density roughly $700\,\mathrm{kg\,m^{-3}}$. Estimate its mass.

5. Estimate the mass of air in a corridor of dimensions $30\,\mathrm{m} \times 2.5\,\mathrm{m} \times 3\,\mathrm{m}$.

## D. Uncertainty and reporting

1. A length is measured as $(15.0 \pm 0.1)$ cm. Find the relative and percentage uncertainty.

2. A block has mass $(80.0 \pm 0.2)$ g and volume $(10.0 \pm 0.1)\,\mathrm{cm^3}$. Estimate the density and its relative uncertainty.

3. A calculator gives 9.806650327. Discuss when it would be appropriate to report 9.8, 9.81 or more digits.

4. A student measures a time interval three times and obtains 1.22 s, 1.25 s and 1.23 s. Compute the average and comment qualitatively on the spread.

## E. Engineering applications

1. A steel plate has dimensions $500\,\mathrm{mm} \times 200\,\mathrm{mm} \times 8\,\mathrm{mm}$. Estimate its mass using $\rho_{\mathrm{steel}} = 7.8 \times 10^3\,\mathrm{kg\,m^{-3}}$.

2. A CAD drawing gives a cube side as 40 mm. Compute the volume in $\mathrm{m^3}$. Explain the error made if 40 is interpreted as metres.

3. A force of 2.5 kN acts on an area of $12\,\mathrm{cm^2}$. Find the pressure in pascals and megapascals.

4. A roof of area $900\,\mathrm{m^2}$ receives 15 mm of rain. Estimate the mass of water.

5. A support cable is specified to tolerate a maximum force of 5.0 kN. Express this force in newtons and identify its dimensions.

## F. Computational and data-analysis checks

1. A program line reads `distance = 25`. List at least three possible physical meanings and explain why the line is incomplete.

2. A simulation expects SI input, but a length is entered as 12 when the intended value is 12 cm. By what factor is the length wrong? By what factor would an area based on this length be wrong?

3. A sensor is sampled at 500 Hz. Find the sampling interval.

4. A numerical routine prints 3.1415926535 m for a length measured with a ruler. Explain why the output format is physically misleading.

5. In a robot simulation, wheel radius is stored in millimetres but angular speed is used with SI units. Describe a safe unit-conversion policy.

## G. Advanced dimensional and scaling problems

1. If all lengths of an object are multiplied by $s = 3$, by what factors do area, volume and mass change, assuming constant density?

2. Identify which of the following are dimensionless: $h/L$, $\Delta L/L$, $mv$, $v/c$, $\rho V/m$.

3. The force on a small sphere moving through a viscous fluid is proposed to depend on viscosity $\eta$, radius $r$ and speed $v$ as $F = C\eta rv$. Given $[\eta] = [ML^{-1}T^{-1}]$, check the dimensions.

4. A relation contains a length $L$, speed $v$ and time $t$. Construct a dimensionless group from these quantities.

5. Explain why a geometrically scaled model may not reproduce all forces of the full-size system.

## H. Measurement uncertainty and calibration

1. A sensor has calibration $V = 0.20\,\mathrm{V} + (5.0\,\mathrm{mV\,K^{-1}})T$. Find $T$ when $V = 1.45\,\mathrm{V}$.

2. A cylinder has $m = (100.0 \pm 0.1)\,\mathrm{g}$, $d = (1.00 \pm 0.01)\,\mathrm{cm}$ and $h = (5.00 \pm 0.02)\,\mathrm{cm}$. Estimate the relative uncertainty of its density using the conservative rules in this chapter.

3. Classify the following as mainly systematic or mainly random: zero offset of a balance, repeated electronic noise, parallax in reading a scale, temperature drift of a sensor.

4. A measured voltage fluctuates between 2.01 V and 2.04 V during repeated readings. How would you report a reasonable central value and qualitative spread?

5. Describe what information should be included in a data file containing accelerometer measurements.

# Chapter A

# Notation dictionary

| Notation | Meaning |
|---|---|
| $Q$ | Generic physical quantity. |
| $\{Q\}$ | Numerical value of $Q$ in the chosen unit. |
| $[Q]$ | Dimension of $Q$ when used in dimensional analysis; unit of $Q$ when explicitly stated in the quantity-unit expression. The meaning should be clear from context. |
| $[M]$ | Dimension of mass. |
| $[L]$ | Dimension of length. |
| $[T]$ | Dimension of time. |
| $\Delta Q$ | Change in a quantity or absolute uncertainty, depending on context. |
| $\Delta Q/Q$ | Relative uncertainty. |
| $\rho$ | Mass density. |
| $m$ | Mass. |
| $V$ | Volume. |
| $A$ | Area. |
| $p$ | Pressure. |
| $f_s$ | Sampling frequency. |
| $\Delta t_s$ | Sampling interval. |

# Chapter B

# Compact formula sheet

**Formula summary**

$$Q = \{Q\}\,[Q], \tag{B.1}$$

$$\rho = \frac{m}{V}, \tag{B.2}$$

$$[\rho] = [ML^{-3}], \tag{B.3}$$

$$[v] = [LT^{-1}], \tag{B.4}$$

$$[a] = [LT^{-2}], \tag{B.5}$$

$$[F] = [MLT^{-2}], \tag{B.6}$$

$$[K] = [ML^2T^{-2}], \tag{B.7}$$

$$[p] = [ML^{-1}T^{-2}], \tag{B.8}$$

$$\frac{\Delta Q}{Q} = \text{relative uncertainty}, \tag{B.9}$$

$$\text{percentage uncertainty} = 100\% \times \frac{\Delta Q}{Q}, \tag{B.10}$$

$$\Delta t_s = \frac{1}{f_s}. \tag{B.11}$$

**Table B.1:** Selected powers of ten and SI prefixes.

| **Factor** | **Prefix** | **Symbol** |
|---|---|---|
| $10^{12}$ | tera | T |
| $10^{9}$ | giga | G |
| $10^{6}$ | mega | M |
| $10^{3}$ | kilo | k |
| $10^{-2}$ | centi | c |
| $10^{-3}$ | milli | m |
| $10^{-6}$ | micro | $\mu$ |
| $10^{-9}$ | nano | n |
| $10^{-12}$ | pico | p |

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