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Akademia Górniczo-Hutnicza
im. Stanisława Staszica w Krakowie

AGH University of Krakow

Physics I Tutorial

Chapters 1–3: Measurement, One-Dimensional Motion, and Vectors

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R1 — Quantities, SI units, dimensions and models

A quantity is a number multiplied by a unit

$q = \{q\}_u\, u$; a change of unit changes the numerical value but not the physical quantity.

$$1\,\mathrm{mm}^2 = 10^{-6}\,\mathrm{m}^2, \qquad 1\,\mathrm{mm}^3 = 10^{-9}\,\mathrm{m}^3, \qquad 1\,\mathrm{g/cm}^3 = 10^3\,\mathrm{kg/m}^3.$$

Dimensions constrain, but do not prove, a model

$[v] = LT^{-1}$, $[a] = LT^{-2}$, $[\rho] = ML^{-3}$, and homogeneous sums require matching dimensions.

$$m = \rho V, \quad V_{\mathrm{spacer}} = \pi L(R_o^2 - R_i^2), \quad \frac{m_{\mathrm{scaled}}}{m} = s^3 \quad \text{(similar shape, same density).}$$

Model check: Define the material boundary, assumptions and admissible limits first. A dimensionally correct answer can still describe the wrong object.

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Distinct questions require distinct indicators

Resolution: discernible increment. Precision: repeated agreement. Accuracy: closeness assessed against references. Calibration: indication–reference relationship, not necessarily an instrument adjustment.

$$\bar{x} = \frac{1}{N}\sum_{j=1}^{N} x_j, \quad r_j = x_j - x_{\mathrm{ref}}, \quad s_x = \sqrt{\frac{\sum_j (x_j - \bar{x})^2}{N-1}}.$$

A model needs an independent test

$y = ax + b$: the slope a has units $[y]/[x]$, intercept b has units of y, and test residual $r = y_{\mathrm{test}} - y_{\mathrm{model}}$. Plot reference and observed quantities with labelled axes and units.

Do not confuse: display increment, tolerance, sample spread and standard uncertainty. A common offset does not disappear by averaging more readings.

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Position, displacement and path length

Fix a coordinate axis, positive direction, origin and a common clock.

$$\Delta x = x_f - x_i, \qquad \ell = \sum_j |x_{j+1} - x_j| \quad \text{(monotone legs)}.$$

$$\bar{v} = \frac{\Delta x}{\Delta t}, \qquad \overline{\text{speed}} = \frac{\ell}{\Delta t}, \quad \ell \geq |\Delta x|.$$

Instantaneous rates and accumulated changes

$v(t) = \mathrm{d}x/\mathrm{d}t$, $a(t) = \mathrm{d}v/\mathrm{d}t$; signed area $\int_{t_i}^{t_f} v\,\mathrm{d}t = \Delta x$; actual path length is $\int_{t_i}^{t_f} |v|\,\mathrm{d}t$.

Reading graphs: $x(t)$ slope is v, $v(t)$ slope is a; v and a of opposite sign mean slowing only when $v \neq 0$.

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R4 — Constant acceleration and vertical launch

One constant-acceleration phase

With local time $\tau = t - t_0$ and prescribed x_0, v_0:

$$v = v_0 + a\tau, \qquad x = x_0 + v_0\tau + \tfrac{1}{2}a\tau^2, \qquad v^2 = v_0^2 + 2a(x - x_0).$$

These relations apply within a phase of constant a; at a phase boundary, carry x and v forward consistently.

Free fall, upward-positive convention

For ideal no-drag near-Earth motion $g > 0$ and $a_y = -g$:

$$v_y = w - g\tau, \qquad y = y_0 + w\tau - \tfrac{1}{2}g\tau^2.$$

Event check: Select roots within the actual time domain. A zero velocity at the apex is not a zero acceleration.

R5 — Vector addition, projections and products

Magnitude and signed components

$\boldsymbol{a} = a_x\hat{\imath} + a_y\hat{\jmath} + a_z\hat{k}$ in a specified orthonormal right-handed frame:

$$\|\boldsymbol{a}\| = \sqrt{a_x^2 + a_y^2 + a_z^2}, \quad \boldsymbol{r} = \sum_i \boldsymbol{a}_i, \quad \theta = \operatorname{atan2}(a_y, a_x)\ (\boldsymbol{a} \neq \boldsymbol{0}).$$

Scalar, vector and oriented products

For $\boldsymbol{b} \neq \boldsymbol{0}$, $\hat{b} = \boldsymbol{b}/\|\boldsymbol{b}\|$:

$$\boldsymbol{a} \cdot \boldsymbol{b} = ab\cos\phi, \qquad a_{\|b} = \frac{\boldsymbol{a} \cdot \boldsymbol{b}}{\|\boldsymbol{b}\|}, \qquad \boldsymbol{a}_{\|} = \frac{\boldsymbol{a} \cdot \boldsymbol{b}}{\boldsymbol{b} \cdot \boldsymbol{b}}\boldsymbol{b}.$$

$$\|\boldsymbol{a} \times \boldsymbol{b}\| = ab\sin\phi; \quad \boldsymbol{b} \times \boldsymbol{a} = -\boldsymbol{a} \times \boldsymbol{b}.$$

Before calculating: predict component signs and units; normalising a zero vector is undefined.

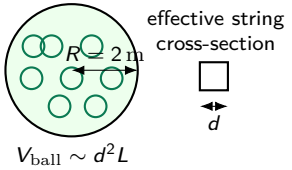
P01 — A ball of string: an order-of-magnitude estimate

Guided worked example — Chapter 1

The world's largest ball of string is about $2\,\mathrm{m}$ in radius. To the nearest order of magnitude, what is the total length L of the string in the ball?

Tasks

Estimate L and justify a geometric model for the string and its voids.



Source: Halliday, Resnick and Walker, *Fundamentals of Physics*, Vol. 1, 12th ed.; Ch. 1, Sample Problem 1.1.1, p. 5.

P01 — Solution 1 of 2: model and derivation

1. Represent and choose a method

The volume of one long string of effective cross-sectional area d^2 is $V_s = d^2 L$. Model the packed ball as a sphere, with voids included in the effective cross-section.

2. Work through the governing relations

Take $R = 2\,\mathrm{m}$ and an effective square cross-section with $d \simeq 4\,\mathrm{mm}$, as suggested in Halliday's estimation model. Then $V_b = \frac{4}{3}\pi R^3$ and $V_s \sim V_b$. The estimated length scales as R^3/d^2, so the uncertain effective string thickness strongly influences the result.

Source: Halliday, Resnick and Walker, *Fundamentals of Physics*, Vol. 1, 12th ed.; Ch. 1, Sample Problem 1.1.1, p. 5.

3. Evaluate each requested quantity

$$L \sim \frac{(4\pi/3)(2\,\mathrm{m})^3}{(4\times10^{-3}\,\mathrm{m})^2} = 2.09\times10^{6}\,\mathrm{m}.$$

Since this is an estimate, rounding to the nearest decade gives $L \sim 10^{6}\,\mathrm{m} \sim 10^{3}\,\mathrm{km}$.

4. Interpret, verify, and avoid a common error

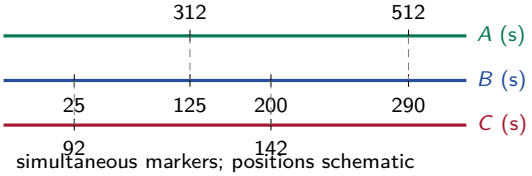
The implied string length is of order one thousand kilometres. The estimate is not a direct measurement: a different packing fraction or effective diameter changes it. The dimensions check: $\mathrm{m}^3/\mathrm{m}^2 = \mathrm{m}$.

Source: Halliday, Resnick and Walker, *Fundamentals of Physics*, Vol. 1, 12th ed.; Ch. 1, Sample Problem 1.1.1, p. 5.

P02 — Clock readings (statement 1 of 2)

Halliday Problem 13: simultaneous clock readings

Three digital clocks A, B, and C run at different rates and do not have simultaneous readings of zero. Figure 1.3 shows simultaneous readings on pairs of the clocks for four occasions. (At the earliest occasion, for example, B reads 25.0 s and C reads 92.0 s.)



simultaneous markers; positions schematic

Source: Halliday, Resnick and Walker, *Fundamentals of Physics*, Vol. 1, 12th ed.; Ch. 1, Problem 13, p. 9.

P02 — Clock readings (statement 2 of 2)

Original question and all subparts

If two events are 600 s apart on clock A, how far apart are they on (a) clock B and (b) clock C? (c) When clock A reads 400 s, what does clock B read? (d) When clock C reads 15.0 s, what does clock B read? (Assume negative readings for prezero times.)

Paired observations read from the source figure

$$(A, B) = (312, 125),\ (512, 290)\ \text{s}; \qquad (B, C) = (25, 92),\ (200, 142)\ \text{s}.$$

These are simultaneous observations, not equal numerical clock readings. Each clock may have a different zero offset.

Source: Halliday, Resnick and Walker, *Fundamentals of Physics*, Vol. 1, 12th ed.; Ch. 1, Problem 13, p. 9.

P02 — Solution 1 of 2: model and derivation

1. Represent and choose a method

Paired readings of simultaneously observed events define affine changes of scale, not simple equality of clock numbers: $B = m_{BA}A + b$ and $C = m_{CB}B + c$.

2. Work through the governing relations

From the graph: $(A, B) = (312, 125)$ and $(512, 290)$, and $(B, C) = (25, 92)$ and $(200, 142)$, in seconds. Subtract simultaneous pairs to eliminate offsets: $m_{BA} = 165/200 = 0.825$ and $m_{CB} = 50/175 = 2/7$.

Source: Halliday, Resnick and Walker, *Fundamentals of Physics*, Vol. 1, 12th ed.; Ch. 1, Problem 13, p. 9.

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3. Evaluate each requested quantity

$$\Delta B = 0.825(600) = 495\,\text{s}, \qquad \Delta C = \frac{2}{7}(495) = 141.43\,\text{s}.$$

$$B(400) = 125 + 0.825(400 - 312) = 197.6\,\text{s}.$$

$$B(C = 15) = 25 + \frac{175}{50}(15 - 92) = -244.5\,\text{s}.$$

4. Interpret, verify, and avoid a common error

All four answers concern clock readings or differences, not physical time dilation. The negative extrapolated B reading is permitted explicitly in the question. Substitution into the two affine relations reproduces the plotted pairs.

Source: Halliday, Resnick and Walker, *Fundamentals of Physics*, Vol. 1, 12th ed.; Ch. 1, Problem 13, p. 9.

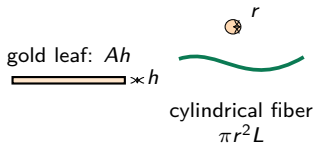
P03 — Gold transformed into foil or a fine wire

Guided worked example — Chapter 1

Gold, which has a density of $19.32\,\mathrm{g/cm^3}$, is the most ductile metal and can be pressed into a thin leaf or drawn out into a long fiber.

Tasks

(a) If a sample of gold, with a mass of $27.63\,\mathrm{g}$, is pressed into a leaf of $1.000\,\mu\mathrm{m}$ thickness, what is the area of the leaf? (b) If, instead, the gold is drawn out into a cylindrical fiber of radius $2.500\,\mu\mathrm{m}$, what is the length of the fiber?



Source: Halliday, Resnick and Walker, *Fundamentals of Physics*, Vol. 1, 12th ed.; Ch. 1, Problem 22, p. 10.

P03 — Solution 1 of 2: model and derivation

1. Represent and choose a method

Mass is unchanged in both transformations. First obtain the volume $V = m/\rho$, then use $V = Ah$ or $V = \pi r^2 L$. Convert micrometres to centimetres before substitution.

2. Work through the governing relations

$$V = \frac{27.63}{19.32}\,\mathrm{cm}^3 = 1.4301\,\mathrm{cm}^3.$$

For the leaf $h = 1.000\,\mu\mathrm{m} = 10^{-4}\,\mathrm{cm}$. For the fiber $r = 2.500\,\mu\mathrm{m} = 2.500 \times 10^{-4}\,\mathrm{cm}$.

Source: Halliday, Resnick and Walker, *Fundamentals of Physics*, Vol. 1, 12th ed.; Ch. 1, Problem 22, p. 10.

P03 — Solution 2 of 2: results and checks

3. Evaluate each requested quantity

$$A = \frac{1.4301}{10^{-4}}\,\mathrm{cm}^2 = 1.4301\,\mathrm{m}^2,$$

$$L = \frac{1.4301}{\pi(2.5\times 10^{-4})^2}\,\mathrm{cm} \simeq 72.84\,\mathrm{km}.$$

4. Interpret, verify, and avoid a common error

A very small thickness can produce a surprisingly large area or length. Substituting back into Ah and $\pi r^2 L$ recovers $1.4301\,\mathrm{cm}^3$; all powers of the micrometre conversion are accounted for.

Source: Halliday, Resnick and Walker, *Fundamentals of Physics*, Vol. 1, 12th ed.; Ch. 1, Problem 22, p. 10.

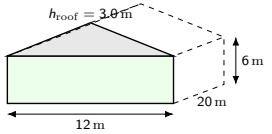
P04 — A doll house and a miniature house

Student exercise — Chapter 1

In the United States, a doll house has the scale of 1:12 of a real house (that is, each length of the doll house is 1/12 that of the real house) and a miniature house (a doll house to fit within a doll house) has the scale of 1:144 of a real house. Suppose a real house (Fig. 1.4) has a front length of 20 m, a depth of 12 m, a height of 6.0 m, and a standard sloped roof (vertical triangular faces on the ends) of height 3.0 m.

Tasks

In cubic meters, what are the volumes of the corresponding (a) doll house and (b) miniature house?



Source: Halliday, Resnick and Walker, *Fundamentals of Physics*, Vol. 1, 12th ed.; Ch. 1, Problem 32, p. 10.

P04 — Solution 1 of 2: model and derivation

1. Represent and choose a method

For an extruded gable roof the model volume is rectangular body plus triangular prism. Every length scales by s, so geometrically similar volumes scale by s^3.

2. Work through the governing relations

$$V_{\text{body}} = 20(12)(6) = 1440\,\text{m}^3, \quad V_{\text{roof}} = \tfrac{1}{2}(12)(3)(20) = 360\,\text{m}^3.$$

$$V_{\text{real}} = 1800\,\text{m}^3.$$

The same roof geometry and overall proportions are retained for both models.

Source: Halliday, Resnick and Walker, *Fundamentals of Physics*, Vol. 1, 12th ed.; Ch. 1, Problem 32, p. 10.

P04 — Solution 2 of 2: results and checks

3. Evaluate each requested quantity

$$V_{\text{doll}} = \frac{1800}{12^3} = 1.0417\,\text{m}^3, \qquad V_{\text{mini}} = \frac{1800}{144^3} = 6.03 \times 10^{-4}\,\text{m}^3.$$

4. Interpret, verify, and avoid a common error

The mini-house has one-twelfth of the doll-house linear dimensions, hence 1/1728 of its volume. The sloping roof is included; discarding it would undercount the house by 20% of the total model volume.

Source: Halliday, Resnick and Walker, *Fundamentals of Physics*, Vol. 1, 12th ed.; Ch. 1, Problem 32, p. 10.

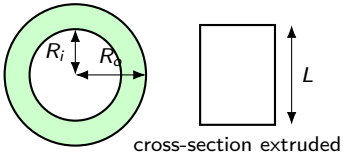
P05 — An annular spacer: mass by two unit routes

Guided worked example — Chapter 1

A uniform through-bored spacer has outer radius $R_o = 12.0\,\mathrm{mm}$, bore radius $R_i = 8.0\,\mathrm{mm}$, length $L = 30.0\,\mathrm{mm}$ and nominal density $\rho = 7.80\,\mathrm{g/cm^3}$. Treat the values as stipulated model inputs.

Tasks

Find its material volume and mass in grams and kilograms. Compare with the solid-cylinder prediction and explain the difference.



Source: Manuscript Ch. 1 Example 1.3

P05 — Solution 1 of 2: model and derivation

1. Represent and choose a method

The material boundary is an annulus extruded through length L: $V = \pi L(R_o^2 - R_i^2)$. Both radii appear squared. Do not include the empty bore in the mass.

2. Work through the governing relations

$$R_o^2 - R_i^2 = (144 - 64)\,\mathrm{mm}^2 = 80\,\mathrm{mm}^2,$$

$$V = 2400\pi\,\mathrm{mm}^3 = 2.4\pi\,\mathrm{cm}^3 = 7.540\,\mathrm{cm}^3.$$

Converting the same model to SI gives $V = 7.540 \times 10^{-6}\,\mathrm{m}^3$ and $\rho = 7800\,\mathrm{kg/m}^3$.

Source: Manuscript Ch. 1 Example 1.3

3. Evaluate each requested quantity

$$m = (7.80)(2.4\pi)\,\mathrm{g} = 58.81\,\mathrm{g} = 0.05881\,\mathrm{kg}.$$

$$m_{\mathrm{solid}} = \rho\pi L R_o^2 = 105.86\,\mathrm{g}, \qquad \frac{m}{m_{\mathrm{solid}}} = 1 - \left(\frac{2}{3}\right)^2 = \frac{5}{9}.$$

4. Interpret, verify, and avoid a common error

A solid model overpredicts because it counts empty space. Both unit routes agree to rounding. The limit $R_i \to 0$ recovers the solid cylinder; $R_i \to R_o$ gives zero material mass.

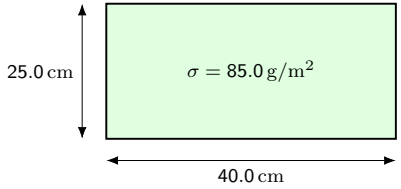
Source: Manuscript Ch. 1 Example 1.3

Student exercise — Chapter 1

A rectangular panel has sides $25.0\,\mathrm{cm}$ and $40.0\,\mathrm{cm}$. A uniform coating contributes areal mass $\sigma = 85.0\,\mathrm{g/m^2}$ on one face only. Neglect edge coating and nonuniformity. A report claims the coating mass is $85\,\mathrm{kg}$.

Tasks

Find the coating mass and diagnose the reporting error. Check using two unit routes.



Source: Manuscript Ch. 1 Example 1.4

P06 — Solution 1 of 2: model and derivation

1. Represent and choose a method

The model is $m_c = \sigma A$, with $A = ab$. An area unit must be converted with the square of the length factor: $1\,\mathrm{cm}^2 = 10^{-4}\,\mathrm{m}^2$.

2. Work through the governing relations

$$A = (0.250\,\mathrm{m})(0.400\,\mathrm{m}) = 0.100\,\mathrm{m}^2.$$

Alternatively $A = 1000\,\mathrm{cm}^2 = 1000 \times 10^{-4}\,\mathrm{m}^2$. Both routes return the same area.

Source: Manuscript Ch. 1 Example 1.4

P06 — Solution 2 of 2: results and checks

3. Evaluate each requested quantity

$$m_c = (85.0\,\mathrm{g/m^2})(0.100\,\mathrm{m^2}) = 8.50\,\mathrm{g}.$$

The proposed $85\,\mathrm{kg} = 85000\,\mathrm{g}$ is a factor 10^4 too large.

4. Interpret, verify, and avoid a common error

The model supplies a one-face coating mass. Multiplying the number 85 by the bare area number 1000 while ignoring $\mathrm{cm^2/m^2}$ produces a plausible-looking but incorrect unitless calculation.

Source: Manuscript Ch. 1 Example 1.4

P07 — A dimensionally admissible model need not be true

Guided worked example — Chapter 1

A design note proposes the magnitude relation $F = C\rho A v^{q}$ where $[F] = MLT^{-2}$, $[\rho] = ML^{-3}$, $[A] = L^{2}$, $[v] = LT^{-1}$ and C is dimensionless. Assume $v > 0$.

Tasks

Determine q. Under the proposed monomial, what is $F(2v)/F(v)$? Has dimensional analysis proved a force law?

Source: Manuscript Ch. 1 Example 1.5

P07 — Solution 1 of 2: model and derivation

1. Represent and choose a method

Match exponents of M, L, T on both sides of a proposed quantity equation. Dimensional consistency is a necessary but not sufficient condition for a correct physical law.

2. Work through the governing relations

$$[C\rho A v^q] = ML^{-3}L^2(LT^{-1})^q = ML^{q-1}T^{-q}.$$

Thus $q - 1 = 1$ and $-q = -2$, consistently yielding $q = 2$.

Source: Manuscript Ch. 1 Example 1.5

3. Evaluate each requested quantity

$$\frac{F(2v)}{F(v)} = \frac{C\rho A(2v)^2}{C\rho Av^2} = 4.$$

But $C = 0.3$ and $C = 0.9$ both preserve dimensions. Adding a further dimensionless factor such as $1 + (v/v_*)^2$ also preserves homogeneity.

4. Interpret, verify, and avoid a common error

Only the stipulated functional form and chosen variable list lead to $q = 2$. Neither C, the speed regime nor missing physical variables can be inferred from dimensions alone; do not mistake an audit for empirical proof.

Source: Manuscript Ch. 1 Example 1.5

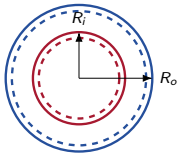
P08 — Defensible mass bounds from toleranced geometry

Student exercise — Chapter 1

A spacer model uses $R_o \in [11.9, 12.1]\,\mathrm{mm}$, $R_i \in [7.8, 8.2]\,\mathrm{mm}$, $L \in [29.8, 30.2]\,\mathrm{mm}$ and $\rho \in [7.7, 7.9]\,\mathrm{g/cm^3}$. Treat all endpoint combinations as admissible; these are deterministic bounds, not standard uncertainties.

Tasks

Find a guaranteed mass enclosure and identify which individual dimension has the largest influence over its stated range.



bounds shown exaggerated; not a probability band

Source: Manuscript Ch. 1 Example 1.8

P08 — Solution 1 of 2: model and derivation

1. Represent and choose a method

For $m = \rho\pi L(R_o^2 - R_i^2)$, the mass increases with ρ, L, R_o and decreases with R_i. Check $R_{i,\max} < R_{o,\min}$ before forming extrema.

2. Work through the governing relations

$$m_{\min} = 7.7\pi(29.8)(11.9^2 - 8.2^2)10^{-3}\,\mathrm{g},$$

$$m_{\max} = 7.9\pi(30.2)(12.1^2 - 7.8^2)10^{-3}\,\mathrm{g}.$$

The 10^{-3} converts mm^3 to cm^3.

Source: Manuscript Ch. 1 Example 1.8

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3. Evaluate each requested quantity

$$m_{\min} \simeq 53.611\,\mathrm{g}, \qquad m_{\max} \simeq 64.137\,\mathrm{g}.$$

An outward-rounded enclosure is $[53.6, 64.2]\,\mathrm{g}$. One-at-a-time sensitivity gives bore-radius variation about -4.05% to $+3.95\%$, larger than the outer-radius variation near $\pm 3\%$.

4. Interpret, verify, and avoid a common error

These results are bounds for the declared rectangular input set, not a 95% interval. The widest useful improvement is the bore measurement in this particular specification.

Source: Manuscript Ch. 1 Example 1.8

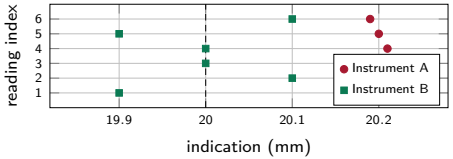
P09 — High precision is not the same as high accuracy

Guided worked example — Chapter 1

Six constructed readings of a 20.000 mm reference are shown below. Instrument A (increment 0.01 mm) reads 20.18, 20.20, 20.19, 20.21, 20.20, 20.19. Instrument B (increment 0.10 mm) reads 19.9, 20.1, 20.0, 20.0, 19.9, 20.1; a calibration model gives A a common +0.20 mm offset.

Tasks

Find both means, sample spreads and mean reference residuals. Correct A for the offset and interpret what does and does not improve.



Source: Manuscript Ch. 1 Example 1.10

P09 — Solution 1 of 2: model and derivation

1. Represent and choose a method

Use $\bar{x} = N^{-1}\sum_i x_i$ and $s = \sqrt{\sum_i (x_i - \bar{x})^2/(N-1)}$. Subtract a common offset from every indication; its effect on spread must be checked algebraically.

2. Work through the governing relations

$$\bar{x}_A = 20.195, \quad s_A \simeq 0.01049\,\mathrm{mm};$$

$$\bar{x}_B = 20.000, \quad s_B \simeq 0.08944\,\mathrm{mm}.$$

Mean residuals relative to $20.000\,\mathrm{mm}$ are $+0.195$ and $0.000\,\mathrm{mm}$, respectively.

Source: Manuscript Ch. 1 Example 1.10

P09 — Solution 2 of 2: results and checks

3. Evaluate each requested quantity

$$\bar{x}_{A,\mathrm{corr}} = 20.195 - 0.200 = 19.995\,\mathrm{mm}, \quad s_{A,\mathrm{corr}} = s_A.$$

Since $(x_i - b) - (\bar{x} - b) = x_i - \bar{x}$, all deviations about the mean are unchanged.

4. Interpret, verify, and avoid a common error

A is more repeatable but uncorrected A is more biased. Calibration shifts its mean; it does not reduce its scatter or erase uncertainty in the correction. All data are constructed instructional values.

Source: Manuscript Ch. 1 Example 1.10

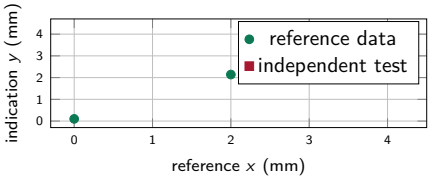
P10 — Test a straight-line calibration with new data

Student exercise — Chapter 1

A constructed sensor calibration gives reference $x = 0, 2, 4\,\mathrm{mm}$ and indications $y = 0.10, 2.14, 4.18\,\mathrm{mm}$. An independent test input $x = 3.00\,\mathrm{mm}$ produces the indication $y_{\mathrm{test}} = 3.19\,\mathrm{mm}$. Assume an affine model $y = ax + b$ over this range.

Tasks

Determine a and b, predict y at the test input and compute the signed data-minus-model residual. Is one small residual proof of universal linearity?



Source: Original from manuscript Ch. 1 graph-model concepts

P10 — Solution 1 of 2: model and derivation

1. Represent and choose a method

A line through the three calibration points has slope $a = \Delta y/\Delta x$ and intercept $b = y(0)$. An independent point tests, rather than defines, the fitted prediction.

2. Work through the governing relations

$$a = \frac{4.18 - 0.10}{4.00 - 0.00} = 1.02, \qquad b = 0.10\,\mathrm{mm}.$$

The middle reference value predicts $1.02(2.00) + 0.10 = 2.14\,\mathrm{mm}$, so all three stipulated points lie exactly on this line.

Source: Original from manuscript Ch. 1 graph-model concepts

P10 — Solution 2 of 2: results and checks

3. Evaluate each requested quantity

$$y_{\text{model}}(3.00) = 1.02(3.00) + 0.10 = 3.16\,\text{mm},$$

$$r = y_{\text{test}} - y_{\text{model}} = +0.03\,\text{mm}.$$

The slope is a ratio of like lengths and is dimensionless; the intercept has units of length.

4. Interpret, verify, and avoid a common error

The positive residual indicates an indication above the prediction. A finite data range and one check point cannot establish validity outside the calibrated range or separate drift from other model failures.

Source: Original from manuscript Ch. 1 graph-model concepts

P11 — A polynomial trajectory and its reversal

Guided worked example — Chapter 2

A particle's position on the x axis of Fig. 2.1.1 is given by $x = 4 - 27t + t^3$, with x in meters and t in seconds.

Tasks

(a) Because position x depends on time t, the particle must be moving. Find the particle's velocity function $v(t)$ and acceleration function $a(t)$. (b) Is there ever a time when $v = 0$? (c) Describe the particle's motion for $t \geq 0$.

Sketch the system or its graph before revealing the solution.

Source: Halliday, Resnick and Walker, *Fundamentals of Physics*, Vol. 1, 12th ed.; Ch. 2, Sample Problem 2.3.1, p. 22–23.

P11 — Solution 1 of 2: model and derivation

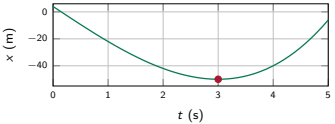
1. Represent and choose a method

Differentiate the stated continuous model: $v = dx/dt$ and $a = dv/dt$. Velocity sign determines direction; the product va determines speed change when $v \neq 0$.

2. Work through the governing relations

$$v(t) = (-27 + 3t^2)\,\mathrm{m/s}, \qquad a(t) = 6t\,\mathrm{m/s^2}.$$

Stationarity requires $-27 + 3t^2 = 0$, giving $t = \pm 3\,\mathrm{s}$. Only $t = 3\,\mathrm{s}$ is within the requested nonnegative-time description.



Source: Halliday, Resnick and Walker, *Fundamentals of Physics*, Vol. 1, 12th ed.; Ch. 2, Sample Problem 2.3.1, p. 22–23.

P11 — Solution 2 of 2: results and checks

3. Evaluate each requested quantity

$$x(0) = 4\,\mathrm{m}, \quad v(0) = -27\,\mathrm{m/s}, \quad a(0) = 0;$$

$$x(3) = 4 - 81 + 27 = -50\,\mathrm{m}.$$

For $0 < t < 3$, $v < 0$ and $a > 0$: leftward and slowing. For $t > 3$, both are positive: rightward and speeding up.

4. Interpret, verify, and avoid a common error

The root $t = 3\,\mathrm{s}$ is a true reversal because v changes sign; $a(3) = 18\,\mathrm{m/s^2}$, not zero. Negative $t = -3\,\mathrm{s}$ solves the analytic equation but lies outside the requested evolution.

Source: Halliday, Resnick and Walker, *Fundamentals of Physics*, Vol. 1, 12th ed.; Ch. 2, Sample Problem 2.3.1, p. 22–23.

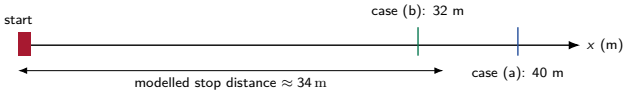
P12 — Yellow light: brake, continue, either or neither?

Student exercise — Chapter 2

You are driving toward a traffic signal when it turns yellow. Your speed is the legal speed limit of $v_0 = 55\,\mathrm{km/h}$; your best deceleration rate has the magnitude $a = 5.18\,\mathrm{m/s^2}$. Your best reaction time to begin braking is $T = 0.75\,\mathrm{s}$.

Tasks

To avoid having the front of your car enter the intersection after the light turns red, should you brake to a stop or continue to move at $55\,\mathrm{km/h}$ if the distance to the intersection and the duration of the yellow light are (a) $40\,\mathrm{m}$ and $2.8\,\mathrm{s}$, and (b) $32\,\mathrm{m}$ and $1.8\,\mathrm{s}$? Give an answer of brake, continue, either (if either strategy works), or neither (if neither strategy works and the yellow duration is inappropriate).



Source: Halliday, Resnick and Walker, *Fundamentals of Physics*, Vol. 1, 12th ed.; Ch. 2, Problem 40, p. 36.

P12 — Solution 1 of 2: model and derivation

1. Represent and choose a method

During reaction, speed remains v_0; after reaction, constant braking $-a$ applies until rest. Stopping requires $D_{\text{stop}} = v_0 T + v_0^2/(2a)$; continuing clears the stop line during yellow if $v_0 t_y \geq D$.

2. Work through the governing relations

$$v_0 = \frac{55}{3.6} = 15.278\,\text{m/s}.$$

$$D_{\text{reaction}} = 15.278(0.75) = 11.458\,\text{m},$$

$$D_{\text{braking}} = \frac{15.278^2}{2(5.18)} \simeq 22.53\,\text{m}.$$

Therefore $D_{\text{stop}} \simeq 34.0\,\text{m}$.

Source: Halliday, Resnick and Walker, *Fundamentals of Physics*, Vol. 1, 12th ed.; Ch. 2, Problem 40, p. 36.

P12 — Solution 2 of 2: results and checks

3. Evaluate each requested quantity

	$D = 40\,\mathrm{m},\ 2.8\,\mathrm{s}$	$D = 32\,\mathrm{m},\ 1.8\,\mathrm{s}$
$D_{\mathrm{stop}} \le D$	yes	no
$v_0 t_y \ge D$	$42.78 \ge 40$	$27.50 < 32$

Case (a): **either**. Case (b): **neither**.

4. Interpret, verify, and avoid a common error

This is a two-phase ideal kinematic model, not a guarantee for actual intersections. In (b) the yellow duration is incompatible with both modelled strategies. Distance and time criteria must be checked separately.

Source: Halliday, Resnick and Walker, *Fundamentals of Physics*, Vol. 1, 12th ed.; Ch. 2, Problem 40, p. 36.

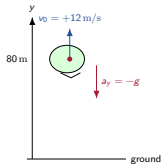
P13 — A package released from an ascending balloon

Guided worked example — Chapter 2

A hot-air balloon is ascending at the rate of $12\,\mathrm{m/s}$ and is $80\,\mathrm{m}$ above the ground when a package is dropped over the side.

Tasks

(a) How long does the package take to reach the ground? (b) With what speed does it hit the ground?



Source: Halliday, Resnick and Walker, *Fundamentals of Physics*, Vol. 1, 12th ed.; Ch. 2, Problem 49, p. 37.

P13 — Solution 1 of 2: model and derivation

1. Represent and choose a method

Choose upward $+y$ with ground at $y = 0$. At release, $y_0 = 80\,\mathrm{m}$ and $v_0 = +12\,\mathrm{m/s}$; acceleration is $-g$, not zero initial velocity.

2. Work through the governing relations

$$y(t) = 80 + 12t - 4.9t^2, \quad v_y(t) = 12 - 9.8t.$$

Setting $y(t) = 0$ gives $4.9t^2 - 12t - 80 = 0$, so

$$t = \frac{12 \pm \sqrt{12^2 + 4(4.9)(80)}}{9.8}.$$

The negative root predates the release event and is inadmissible.

Source: Halliday, Resnick and Walker, *Fundamentals of Physics*, Vol. 1, 12th ed.; Ch. 2, Problem 49, p. 37.

P13 — Solution 2 of 2: results and checks

3. Evaluate each requested quantity

$$t_{\text{hit}} = \frac{12 + \sqrt{1712}}{9.8} \simeq 5.447\,\text{s},$$

$$v_y(t_{\text{hit}}) = 12 - 9.8 t_{\text{hit}} = -\sqrt{1712} \simeq -41.38\,\text{m/s}.$$

Thus impact speed is $41.4\,\text{m/s}$ downward.

4. Interpret, verify, and avoid a common error

As an independent check, $v_y^2 = v_0^2 + 2(-g)(0 - 80) = 1712\,\text{m}^2/\text{s}^2$. At release the package initially rises, despite being described as dropped.

Source: Halliday, Resnick and Walker, *Fundamentals of Physics*, Vol. 1, 12th ed.; Ch. 2, Problem 49, p. 37.

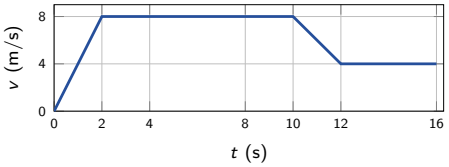
P14 — Distance travelled from a runner velocity graph

Student exercise — Chapter 2

How far does the runner whose velocity–time graph is shown in Fig. 2.25 travel in 16 s? The figure's vertical scaling is set by $v_s = 8.0\,\mathrm{m/s}$.

Tasks

Find the total travelled distance using the complete piecewise velocity graph, with the given axis scales.



Source: Halliday, Resnick and Walker, *Fundamentals of Physics*, Vol. 1, 12th ed.; Ch. 2, Problem 69, p. 38.

P14 — Solution 1 of 2: model and derivation

1. Represent and choose a method

The area under $v(t)$ gives displacement. Because the supplied velocity is never negative, each area contributes positively to distance. Split the graph at every slope change.

2. Work through the governing relations

$$D_1 = \tfrac{1}{2}(2)(8) = 8\,\mathrm{m}, \quad D_2 = (10-2)(8) = 64\,\mathrm{m},$$

$$D_3 = \tfrac{1}{2}(8+4)(12-10) = 12\,\mathrm{m}, \quad D_4 = (16-12)(4) = 16\,\mathrm{m}.$$

Source: Halliday, Resnick and Walker, *Fundamentals of Physics*, Vol. 1, 12th ed.; Ch. 2, Problem 69, p. 38.

P14 — Solution 2 of 2: results and checks

3. Evaluate each requested quantity

$$D = D_1 + D_2 + D_3 + D_4 = 8 + 64 + 12 + 16 = \boxed{100\,\mathrm{m}}.$$

Mean speed is $D/16 = 6.25\,\mathrm{m/s}$. The velocity graph has non-negative ordinates throughout.

4. Interpret, verify, and avoid a common error

The axes are seconds and metres per second: every area has units of metres. A position–time graph would require an initial position, but the distance does not.

Source: Halliday, Resnick and Walker, *Fundamentals of Physics*, Vol. 1, 12th ed.; Ch. 2, Problem 69, p. 38.

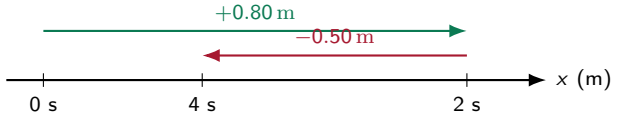
P15 — A translation stage returns toward its start

Student exercise — Chapter 2

A translation-stage marker is at $x = -0.20\,\mathrm{m}$ when $t = 0$, at $x = +0.60\,\mathrm{m}$ when $t = 2.0\,\mathrm{s}$, and at $x = +0.10\,\mathrm{m}$ when $t = 4.0\,\mathrm{s}$. It travels monotonically between observations and reverses at the middle observation.

Tasks

Find total signed displacement, total path length, average velocity and average speed. Shift the origin to $x = +0.60\,\mathrm{m}$ and repeat the endpoint check.



Source: Manuscript Ch. 2 Example 2.1

P15 — Solution 1 of 2: model and derivation

1. Represent and choose a method

Split the two monotone travel legs. For each, $\Delta x_j = x_{j+1} - x_j$; total distance is $\sum_j |\Delta x_j|$ and average velocity is $\Delta x/\Delta t$.

2. Work through the governing relations

$$\Delta x_1 = +0.60 - (-0.20) = +0.80\,\mathrm{m},$$

$$\Delta x_2 = +0.10 - 0.60 = -0.50\,\mathrm{m}.$$

$$\Delta x = +0.30\,\mathrm{m}, \qquad \ell = 0.80 + 0.50 = 1.30\,\mathrm{m}.$$

Source: Manuscript Ch. 2 Example 2.1

P15 — Solution 2 of 2: results and checks

3. Evaluate each requested quantity

$$\bar{v} = \frac{0.30}{4.0} = +0.075\,\mathrm{m/s}, \quad \bar{s} = \frac{1.30}{4.0} = 0.325\,\mathrm{m/s}.$$

New coordinates are $(-0.80, 0, -0.50)\,\mathrm{m}$, returning the same $\Delta x = +0.30\,\mathrm{m}$.

4. Interpret, verify, and avoid a common error

Translating the origin does not change displacements or speeds. Without the stated monotonicity, three sampled positions would supply only a lower bound on total distance.

Source: Manuscript Ch. 2 Example 2.1

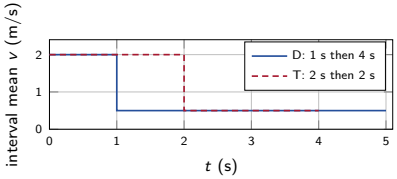
P16 — Two velocities: equal distances versus equal times

Guided worked example — Chapter 2

A marker makes two forward legs with respective interval-average velocities $v_1 = 2.00\,\mathrm{m/s}$ and $v_2 = 0.50\,\mathrm{m/s}$. Journey D covers $2.00\,\mathrm{m}$ on each leg. Journey T spends $2.00\,\mathrm{s}$ on each leg.

Tasks

Calculate the total duration, displacement and whole-journey mean velocity for both. Determine which mean is the arithmetic average $1.25\,\mathrm{m/s}$.



Source: Manuscript Ch. 2 Example 2.7

P16 — Solution 1 of 2: model and derivation

1. Represent and choose a method

Average velocity is duration-weighted: $\bar{v} = \sum_j v_j \Delta t_j / \sum_j \Delta t_j$. Equal distance legs generally have unequal time weights.

2. Work through the governing relations

For D: $\Delta t_1 = 2/2 = 1\,\mathrm{s}$, $\Delta t_2 = 2/0.5 = 4\,\mathrm{s}$, so $\Delta x = 4\,\mathrm{m}$ and $\Delta t = 5\,\mathrm{s}$.

$$\bar{v}_D = 4/5 = 0.800\,\mathrm{m/s}.$$

For T, each duration is $2\,\mathrm{s}$: displacements are 4 and $1\,\mathrm{m}$.

Source: Manuscript Ch. 2 Example 2.7

P16 — Solution 2 of 2: results and checks

3. Evaluate each requested quantity

$$\Delta x_T = 5.00\,\mathrm{m}, \quad \Delta t_T = 4.00\,\mathrm{s}, \quad \bar{v}_T = 1.25\,\mathrm{m/s}.$$

For equal distances moving forward, $\bar{v} = 2v_1v_2/(v_1 + v_2) = 0.800\,\mathrm{m/s}$.

4. Interpret, verify, and avoid a common error

The arithmetic mean is correct for equal time weights, not equal distances. A bar graph of each leg's interval-average speed uses area $v_j\Delta t_j$ as displacement; it does not imply uniform motion within the legs.

Source: Manuscript Ch. 2 Example 2.7

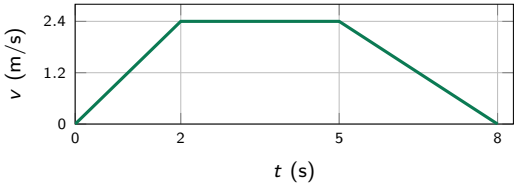
P17 — Lift: three phases must join continuously

Student exercise — Chapter 2

An ideal lift starts from rest at $x = 0$. It accelerates upward at $+1.20\,\mathrm{m/s^2}$ for $2.00\,\mathrm{s}$, cruises at constant speed for $3.00\,\mathrm{s}$, and then decelerates at $-0.80\,\mathrm{m/s^2}$ until rest. Its prescribed speed cap is $2.40\,\mathrm{m/s}$.

Tasks

Find total time and displacement. Construct a phase-resolved $v(t)$ and check its continuity and cap.



Source: Manuscript Ch. 2 Example 2.9

P17 — Solution 1 of 2: model and derivation

1. Represent and choose a method

Write a fresh local time $\tau = t - t_{\text{phase,start}}$ for each phase; pass endpoint x and v forward. The total displacement is also the positive area under the velocity graph.

2. Work through the governing relations

Phase I: $v = 1.20t$, so $v(2) = 2.40\,\text{m/s}$ and $x(2) = \frac{1}{2}(1.20)(2)^2 = 2.40\,\text{m}$. Phase II: $v = 2.40$ for $2 \le t \le 5$, giving $\Delta x_2 = 7.20\,\text{m}$.

Source: Manuscript Ch. 2 Example 2.9

P17 — Solution 2 of 2: results and checks

3. Evaluate each requested quantity

Phase III: $v = 2.40 - 0.80(t - 5)$; stopping occurs after $3.00\,\mathrm{s}$ at $t = 8.00\,\mathrm{s}$.

$$\Delta x_3 = \tfrac{1}{2}(2.40)(3) = 3.60\,\mathrm{m},$$

$$\Delta x_{\mathrm{tot}} = 2.40 + 7.20 + 3.60 = 13.20\,\mathrm{m}.$$

4. Interpret, verify, and avoid a common error

Peak velocity is exactly $2.40\,\mathrm{m/s}$. Position and velocity are continuous at $t = 2$ and $5\,\mathrm{s}$ even though idealised acceleration changes abruptly; extending braking past $t = 8$ would model an unintended reverse motion.

Source: Manuscript Ch. 2 Example 2.9

P18 — Delayed launch: when do two markers meet?

Guided worked example — Chapter 2

Two markers on parallel rails share a laboratory x axis. Marker A leaves $x = 0$ at $t = 0$ with $u = 2.00\,\mathrm{m/s}$. Marker B remains at the origin until $t = \delta = 2.00\,\mathrm{s}$, then accelerates from rest at $a = 1.00\,\mathrm{m/s^2}$. The model is valid through $8.00\,\mathrm{s}$.

Tasks

Find the later longitudinal encounter and both velocities there. Find the maximum separation before the encounter.

Sketch the system or its graph before revealing the solution.

Source: Manuscript Ch. 2 Example 2.14

P18 — Solution 1 of 2: model and derivation

1. Represent and choose a method

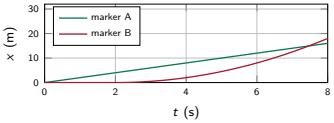
Use one global clock: $x_A = ut$, $x_B = 0$ for $t < \delta$ and $x_B = \frac{1}{2}a(t-\delta)^2$ afterwards. Equal positions, not equal velocities, define the encounter.

2. Work through the governing relations

Let $\tau = t - 2 \geq 0$. The later encounter obeys $\frac{1}{2}\tau^2 = 2(\tau + 2)$, hence $\tau^2 - 4\tau - 8 = 0$.

$$\tau = 2 + \sqrt{12} \simeq 5.464\,\mathrm{s}, \quad t = 7.464\,\mathrm{s}.$$

The negative root is not in B's accelerating phase.



Source: Manuscript Ch. 2 Example 2.14

P18 — Solution 2 of 2: results and checks

3. Evaluate each requested quantity

$$x_A = x_B = 2(7.464) = 14.928\,\mathrm{m}.$$

$$v_A = 2.00, \quad v_B = \tau \simeq 5.464\,\mathrm{m/s}.$$

Separation $s = x_A - x_B = 2(\tau + 2) - \tau^2/2$ is greatest when $\mathrm{d}s/\mathrm{d}\tau = 2 - \tau = 0$, at global $t = 4\,\mathrm{s}$, giving $s_{\max} = 6.00\,\mathrm{m}$.

4. Interpret, verify, and avoid a common error

Equal velocities occur at maximum separation, not at the meeting. A mathematically valid negative local-time root must be rejected using the specified piecewise model. Parallel rails give a longitudinal encounter, not an assumed collision.

Source: Manuscript Ch. 2 Example 2.14

P19 — A vertical launch passes one height twice

Student exercise — Chapter 2

From a platform $1.80\,\mathrm{m}$ above the ground, a ball is thrown vertically upward at $14.7\,\mathrm{m/s}$ in the ideal no-drag model. Take upward positive and $g = 9.81\,\mathrm{m/s^2}$. An observation plane lies at height $10.0\,\mathrm{m}$ above the ground.

Tasks

At what two times does the ball cross that plane, and what is the velocity at each crossing? Does it reach the plane before the apex?

Sketch the system or its graph before revealing the solution.

Source: Original on Ch. 2 free-fall model

P19 — Solution 1 of 2: model and derivation

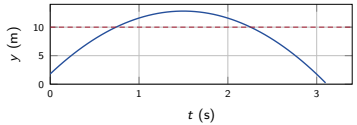
1. Represent and choose a method

Use $y(t) = 1.80 + 14.7t - 4.905t^2$. An event $y = 10.0$ produces a quadratic with two physical roots if the plane lies below the trajectory's maximum height.

2. Work through the governing relations

$$4.905t^2 - 14.7t + 8.20 = 0, \quad t_{\pm} = \frac{14.7 \pm \sqrt{14.7^2 - 4(4.905)(8.20)}}{9.81}.$$

Discriminant $\Delta = 55.206\,\mathrm{m^2/s^2} > 0$; both times occur after launch.



Source: Original on Ch. 2 free-fall model

P19 — Solution 2 of 2: results and checks

3. Evaluate each requested quantity

$$t_- \simeq 0.741\,\mathrm{s}, \qquad t_+ \simeq 2.256\,\mathrm{s},$$

$$v_y(t_\mp) = 14.7 - 9.81 t_\mp = \pm\sqrt{55.206} \simeq \pm 7.430\,\mathrm{m/s}.$$

The apex is at $t_a = 14.7/9.81 \simeq 1.498\,\mathrm{s}$ between them.

4. Interpret, verify, and avoid a common error

The same location is crossed once upward and once downward. The magnitudes of the two velocities agree in the constant-g model. At the apex $v = 0$ but $a_y = -9.81\,\mathrm{m/s^2}$.

Source: Original on Ch. 2 free-fall model

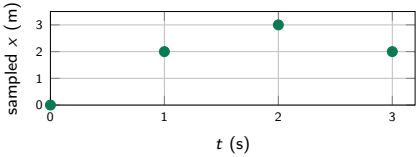
P20 — Sampled positions and what the data cannot show

Student exercise — Chapter 2

A marker is recorded at $x = 0, 2, 3, 2\,\mathrm{m}$ at times $t = 0, 1, 2, 3\,\mathrm{s}$. Time stamps are reliable but the motion between recordings is not observed; no monotonicity information is supplied.

Tasks

Find displacement, average velocity, a lower bound for distance, and the central-difference velocity estimate at $t = 1\,\mathrm{s}$. Does the data determine exact travelled distance or instantaneous velocity?



Source: Original on manuscript Ch. 2 §§2.1, 2.7

P20 — Solution 1 of 2: model and derivation

1. Represent and choose a method

Endpoint displacement is independent of interpolation. Sum of absolute sample-to-sample displacements bounds true path length below; a centred finite difference approximates $v(1)$ only if the sample spacing and local smoothness assumptions justify it.

2. Work through the governing relations

$$\Delta x = x(3) - x(0) = +2\,\mathrm{m}, \quad \bar{v} = 2/3 = +0.667\,\mathrm{m/s}.$$

$$\ell \geq |2-0| + |3-2| + |2-3| = 4\,\mathrm{m}.$$

A central numerical slope at $1\,\mathrm{s}$ gives $[x(2) - x(0)]/(2\,\mathrm{s}) = 1.50\,\mathrm{m/s}$.

Source: Original on manuscript Ch. 2 §§2.1, 2.7

3. Evaluate each requested quantity

The data do not specify the true instantaneous $v(1)$; an arbitrarily short unrecorded excursion can increase ℓ while preserving every recorded position. If a monotone-leg assumption were supplied, then and only then would $\ell = 4\,\mathrm{m}$ follow.

4. Interpret, verify, and avoid a common error

A smooth curve through discrete points is a proposed model, not an extra observation. Differentiating such a curve cannot remove ambiguity about unobserved motion.

Source: Original on manuscript Ch. 2 §§2.1, 2.7

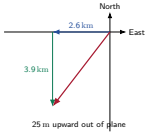
P21 — Spelunking: the cave route versus displacement

Guided worked example — Chapter 3

For two decades spelunking teams crawled, climbed, and squirmed through 200 km of Mammoth Cave and the Flint Ridge cave system, seeking a connection. The team that finally found the connection "caved" for 12 hours to go from Austin Entrance in the Flint Ridge system to Echo River in Mammoth Cave (Fig. 3.1.9a), traveling a net 2.6 km westward, 3.9 km southward, and 25 m upward. That established the system as the longest cave system in the world.

Tasks

What were the magnitude and angle of the team's displacement from start to finish?



Source: Halliday, Resnick and Walker, *Fundamentals of Physics*, Vol. 1, 12th ed.; Ch. 3, Sample Problem 3.1.1, p. 48.

P21 — Solution 1 of 2: model and derivation

1. Represent and choose a method

Take $+x$ east, $+y$ north and $+z$ up. Convert $25\,\mathrm{m} = 0.025\,\mathrm{km}$, then calculate horizontal magnitude and elevation angle using genuine right triangles.

2. Work through the governing relations

$$\boldsymbol{d} = (-2.6\,\hat{\imath} - 3.9\,\hat{\jmath} + 0.025\,\hat{k})\,\mathrm{km}.$$

$$d_h = \sqrt{2.6^2 + 3.9^2} = 4.687\,\mathrm{km}.$$

The horizontal bearing is $\theta_h = \arctan(3.9/2.6) = 56.31^\circ$ south of west.

Source: Halliday, Resnick and Walker, *Fundamentals of Physics*, Vol. 1, 12th ed.; Ch. 3, Sample Problem 3.1.1, p. 48.

P21 — Solution 2 of 2: results and checks

3. Evaluate each requested quantity

$$d = \sqrt{d_h^2 + 0.025^2} = 4.687\,\mathrm{km} \simeq 4.7\,\mathrm{km},$$

$$\theta_v = \arctan(0.025/d_h) = 0.306^\circ \quad \text{above horizontal.}$$

4. Interpret, verify, and avoid a common error

The survey's vector connects endpoints without encoding a cave path. The actual travelled route can be far longer than $4.7\,\mathrm{km}$. The small vertical component still fixes a nonzero elevation angle.

Source: Halliday, Resnick and Walker, *Fundamentals of Physics*, Vol. 1, 12th ed.; Ch. 3, Sample Problem 3.1.1, p. 48.

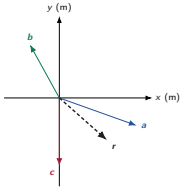
P22 — Adding three vectors by their components

Guided worked example — Chapter 3

Figure 3.2.4a shows the following three vectors: $\boldsymbol{a} = (4.2\,\mathrm{m})\hat{\imath} - (1.5\,\mathrm{m})\hat{\jmath}$, $\boldsymbol{b} = -(1.6\,\mathrm{m})\hat{\imath} + (2.9\,\mathrm{m})\hat{\jmath}$, and $\boldsymbol{c} = -(3.7\,\mathrm{m})\hat{\jmath}$.

Tasks

What is their vector sum $\boldsymbol{r}$, which is also shown?



Source: Halliday, Resnick and Walker, *Fundamentals of Physics*, Vol. 1, 12th ed.; Ch. 3, Sample Problem 3.2.1, p. 52.

AGH

1. Represent and choose a method

Vectors add componentwise only when all components use the same physical axes and units. The sum's direction follows its two signed components.

2. Work through the governing relations

$$r_x = 4.2 - 1.6 + 0 = +2.6\,\mathrm{m},$$

$$r_y = -1.5 + 2.9 - 3.7 = -2.3\,\mathrm{m}.$$

Consequently $\boldsymbol{r} = (2.6\hat{\imath} - 2.3\hat{\jmath})\,\mathrm{m}$.

Source: Halliday, Resnick and Walker, *Fundamentals of Physics*, Vol. 1, 12th ed.; Ch. 3, Sample Problem 3.2.1, p. 52.

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P22 — Solution 2 of 2: results and checks

3. Evaluate each requested quantity

$$\|\boldsymbol{r}\| = \sqrt{2.6^2 + (-2.3)^2} = 3.47\,\mathrm{m},$$

$$\theta = \operatorname{atan2}(-2.3, 2.6) = -41.5^\circ.$$

Equivalently 318.5° counterclockwise from $+x$.

4. Interpret, verify, and avoid a common error

The positive x and negative y components place the resultant in quadrant IV, consistent with the clockwise negative angle. Magnitudes $|a| + |b| + |c|$ must not be added as scalars.

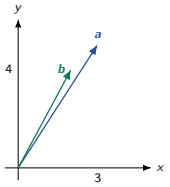
Source: Halliday, Resnick and Walker, *Fundamentals of Physics*, Vol. 1, 12th ed.; Ch. 3, Sample Problem 3.2.1, p. 52.

Student exercise — Chapter 3

Two vectors are presented as $\boldsymbol{a} = 3.0\hat{\imath} + 5.0\hat{\jmath}$ and $\boldsymbol{b} = 2.0\hat{\imath} + 4.0\hat{\jmath}$.

Tasks

Find (a) $\boldsymbol{a} \times \boldsymbol{b}$, (b) $\boldsymbol{a} \cdot \boldsymbol{b}$, (c) $(\boldsymbol{a} + \boldsymbol{b}) \cdot \boldsymbol{b}$, and (d) the component of $\boldsymbol{a}$ along the direction of $\boldsymbol{b}$. (Hint: For (d), consider Eq. 3.3.1 and Fig. 3.3.1.)



Source: Halliday, Resnick and Walker, *Fundamentals of Physics*, Vol. 1, 12th ed.; Ch. 3, Problem 34, p. 62.

P23 — Solution 1 of 2: model and derivation

1. Represent and choose a method

Embed the planar vectors into a right-handed 3D basis with zero z components. Dot products produce a scalar; cross products produce a normal vector; projection along $\hat{b}$ uses $\boldsymbol{a} \cdot \boldsymbol{b}/\|\boldsymbol{b}\|$.

2. Work through the governing relations

$$\boldsymbol{a} \times \boldsymbol{b} = (a_x b_y - a_y b_x)\hat{k} = (12 - 10)\hat{k} = 2\hat{k}.$$

$$\boldsymbol{a} \cdot \boldsymbol{b} = 3(2) + 5(4) = 26.$$

Source: Halliday, Resnick and Walker, *Fundamentals of Physics*, Vol. 1, 12th ed.; Ch. 3, Problem 34, p. 62.

AGH

3. Evaluate each requested quantity

$$(\boldsymbol{a}+\boldsymbol{b})\cdot\boldsymbol{b} = (5,9)\cdot(2,4) = 46,$$

$$a_{\parallel b} = \frac{26}{\sqrt{2^2+4^2}} = \frac{13}{\sqrt{5}} \simeq 5.81.$$

The vector projection, if desired, is $\boldsymbol{a}_{\parallel} = (2.6, 5.2)$.

4. Interpret, verify, and avoid a common error

The cross-product sign is $+z$ because rotating $\boldsymbol{a}$ toward $\boldsymbol{b}$ is counterclockwise. The projection magnitude cannot exceed $\|\boldsymbol{a}\| = \sqrt{34}$; $5.81 < 5.83$ is a useful check.

Source: Halliday, Resnick and Walker, *Fundamentals of Physics*, Vol. 1, 12th ed.; Ch. 3, Problem 34, p. 62.

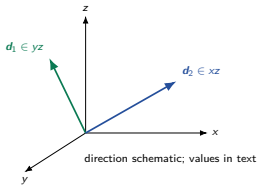
P24 — Vectors specified in perpendicular coordinate planes

Student exercise — Chapter 3

Displacement $\boldsymbol{d}_1$ is in the yz plane 63.0° from the positive direction of the y axis, has a positive z component, and has a magnitude of $4.50\,\mathrm{m}$. Displacement $\boldsymbol{d}_2$ is in the xz plane 30.0° from the positive direction of the x axis, has a positive z component, and has magnitude $1.40\,\mathrm{m}$.

Tasks

What are (a) $\boldsymbol{d}_1 \cdot \boldsymbol{d}_2$, (b) $\boldsymbol{d}_1 \times \boldsymbol{d}_2$, and (c) the angle between $\boldsymbol{d}_1$ and $\boldsymbol{d}_2$?



Source: Halliday, Resnick and Walker, *Fundamentals of Physics*, Vol. 1, 12th ed.; Ch. 3, Problem 40, p. 62.

P24 — Solution 1 of 2: model and derivation

1. Represent and choose a method

Resolve first, using $d_{1x} = 0$ and $d_{2y} = 0$. State the right-handed basis. The mixed-plane geometry simplifies the dot product but not every cross component.

2. Work through the governing relations

$$\boldsymbol{d}_1 = (0, 4.50\cos 63^\circ, 4.50\sin 63^\circ)\,\mathrm{m},$$

$$\boldsymbol{d}_2 = (1.40\cos 30^\circ, 0, 1.40\sin 30^\circ)\,\mathrm{m}.$$

Consequently $\boldsymbol{d}_1 \cdot \boldsymbol{d}_2 = (4.50\sin 63^\circ)(0.700) \simeq 2.807\,\mathrm{m}^2$.

Source: Halliday, Resnick and Walker, *Fundamentals of Physics*, Vol. 1, 12th ed.; Ch. 3, Problem 40, p. 62.

AGH

3. Evaluate each requested quantity

$$\boldsymbol{d}_1 \times \boldsymbol{d}_2 = (d_{1y}d_{2z},\ d_{1z}d_{2x},\ -d_{1y}d_{2x})$$
$$\simeq (1.430,\ 4.861,\ -2.477)\,\mathrm{m}^2.$$
$$\theta = \arccos\frac{2.807}{(4.50)(1.40)} \simeq 63.54^\circ.$$

4. Interpret, verify, and avoid a common error

Both input directions have positive z, yet their cross product can have a negative z component. Verify $\boldsymbol{d}_1 \cdot (\boldsymbol{d}_1 \times \boldsymbol{d}_2) = \boldsymbol{d}_2 \cdot (\boldsymbol{d}_1 \times \boldsymbol{d}_2) = 0$.

Source: Halliday, Resnick and Walker, *Fundamentals of Physics*, Vol. 1, 12th ed.; Ch. 3, Problem 40, p. 62.

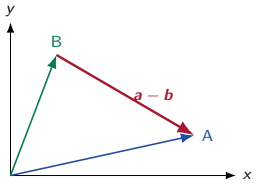
P25 — A displacement between two recorded endpoints

Student exercise — Chapter 3

Two inspection records in common east–north axes give position vectors $\boldsymbol{a} = (4, 1)\,\mathrm{m}$ and $\boldsymbol{b} = (1, 3)\,\mathrm{m}$ from the same origin O.

Tasks

Find displacement from the endpoint of $\boldsymbol{b}$ to the endpoint of $\boldsymbol{a}$, its magnitude and bearing, and the command from the endpoint of $\boldsymbol{b}$ back to O.



Source: Manuscript Ch. 3 Example 3.1

P25 — Solution 1 of 2: model and derivation

1. Represent and choose a method

An arrow from point B to point A is endpoint minus start, $\boldsymbol{d} = \boldsymbol{a} - \boldsymbol{b}$. The order controls the orientation; do not subtract only magnitudes.

2. Work through the governing relations

$$\boldsymbol{d} = (4-1, 1-3)\,\mathrm{m} = (3, -2)\,\mathrm{m}.$$

$$d = \sqrt{3^2 + (-2)^2} = \sqrt{13}\,\mathrm{m} = 3.606\,\mathrm{m}.$$

Bearings from $+x$ use a quadrant-aware inverse tangent.

Source: Manuscript Ch. 3 Example 3.1

P25 — Solution 2 of 2: results and checks

3. Evaluate each requested quantity

$$\theta_d = \text{atan2}(-2, 3) = -33.69^\circ,$$

or 326.31° counterclockwise from east. Return from B to origin is $-\boldsymbol{b} = (-1, -3)\,\text{m}$.

4. Interpret, verify, and avoid a common error

Check closure: $\boldsymbol{b} + \boldsymbol{d} - \boldsymbol{a} = \boldsymbol{0}$. The arrow $\boldsymbol{a} - \boldsymbol{b}$ is from B to A; reversing subtraction would give the opposite bearing.

Source: Manuscript Ch. 3 Example 3.1

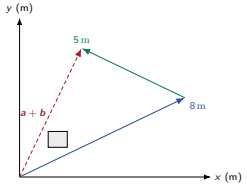
P26 — A two-leg survey: path length and resultant

Guided worked example — Chapter 3

A surveyor moves 8.0 m at 30° counterclockwise from east, then 5.0 m at 150° from east in the same plane. The route bends around a stipulated obstacle. Bearings are measured from positive x (east).

Tasks

Find the final displacement components, magnitude and bearing, and the total walked path length. Check the resultant geometrically.



Source: Manuscript Ch. 3 Example 3.3

P26 — Solution 1 of 2: model and derivation

1. Represent and choose a method

Resolve each leg with $a_x = a\cos\theta$, $a_y = a\sin\theta$ in the declared frame. The straight displacement may cross an obstacle even though the physical walked route does not.

2. Work through the governing relations

$$\boldsymbol{a} = (8\cos 30^\circ, 8\sin 30^\circ) = (4\sqrt{3}, 4)\,\mathrm{m},$$

$$\boldsymbol{b} = (5\cos 150^\circ, 5\sin 150^\circ) = (-5\sqrt{3}/2, 2.5)\,\mathrm{m}.$$

$$\boldsymbol{d} = \boldsymbol{a} + \boldsymbol{b} = (3\sqrt{3}/2, 6.5)\,\mathrm{m}.$$

Source: Manuscript Ch. 3 Example 3.3

P26 — Solution 2 of 2: results and checks

3. Evaluate each requested quantity

$$d = \sqrt{(3\sqrt{3}/2)^2 + 6.5^2} = 7.00\,\mathrm{m}, \quad \theta = 68.21^\circ.$$

The path length is $8 + 5 = 13\,\mathrm{m}$. Using the 120° common-tail angle independently gives $d^2 = 8^2 + 5^2 + 2(8)(5)\cos 120^\circ = 49$.

4. Interpret, verify, and avoid a common error

The net displacement is only $7.00\,\mathrm{m}$ despite $13.0\,\mathrm{m}$ walked. Components and geometric cosine rule agree. A path blocked by an obstacle is not automatically a valid route along the displacement arrow.

Source: Manuscript Ch. 3 Example 3.3

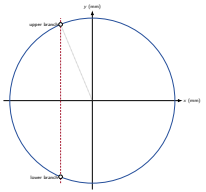
P27 — Reconstruction: two branches or no real answer

Student exercise — Chapter 3

A displacement record gives magnitude $L = 13\,\mathrm{mm}$ and signed horizontal component $d_x = -5\,\mathrm{mm}$. An independent observation places the endpoint above the starting horizontal. Another record claims $L = 10\,\mathrm{mm}$ with $d_x = -11\,\mathrm{mm}$.

Tasks

Find the first vector and its counterclockwise bearing. Decide whether the second exact record is geometrically possible.



Source: Manuscript Ch. 3 Example 3.4

P27 — Solution 1 of 2: model and derivation

1. Represent and choose a method

In a Cartesian plane, $d_y = \pm\sqrt{L^2 - d_x^2}$; the inequality $|d_x| \leq L$ is required. A verbal location condition selects one of the two branches.

2. Work through the governing relations

$$d_y = \pm\sqrt{13^2 - (-5)^2} = \pm 12\,\mathrm{mm}.$$

The first endpoint is above the horizontal, so $d_y = +12\,\mathrm{mm}$ and $\boldsymbol{d} = (-5, 12)\,\mathrm{mm}$.

Source: Manuscript Ch. 3 Example 3.4

P27 — Solution 2 of 2: results and checks

3. Evaluate each requested quantity

$$\theta = \operatorname{atan2}(12, -5) = 112.62^\circ.$$

For the second record, $L^2 - d_x^2 = 100 - 121 = -21\,\mathrm{mm}^2 < 0$, so no real displacement meets those exact constraints.

4. Interpret, verify, and avoid a common error

A calculator's inverse tangent alone cannot select the correct quadrant. When exact constraints contradict geometry, do not hide the contradiction by changing a sign or taking the absolute value of a negative radicand.

Source: Manuscript Ch. 3 Example 3.4

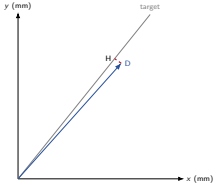
P28 — A positioning stage's perpendicular correction

Student exercise — Chapter 3

A stage executes two planar translation commands $\boldsymbol{p} = (40, 10)\,\mathrm{mm}$ and $\boldsymbol{q} = (-15, 20)\,\mathrm{mm}$. The desired infinite target line through the starting point has direction $(3, 4)$.

Tasks

Find net displacement, signed along-line component, projected vector, perpendicular error and shortest correcting command to the line.



Source: Manuscript Ch. 3 Example 3.7

P28 — Solution 1 of 2: model and derivation

1. Represent and choose a method

Add translations first. Normalise the line direction $\hat{u} = (3/5, 4/5)$; then the scalar projection is $d_{\parallel} = \boldsymbol{d} \cdot \hat{u}$ and the error is $\boldsymbol{d}_{\perp} = \boldsymbol{d} - d_{\parallel}\hat{u}$.

2. Work through the governing relations

$$\boldsymbol{d} = (40 - 15, 10 + 20) = (25, 30)\,\mathrm{mm},$$

$$d_{\parallel} = 25(3/5) + 30(4/5) = 39\,\mathrm{mm}.$$

$$\boldsymbol{d}_{\parallel} = 39(3/5, 4/5) = (23.4, 31.2)\,\mathrm{mm}.$$

Source: Manuscript Ch. 3 Example 3.7

P28 — Solution 2 of 2: results and checks

3. Evaluate each requested quantity

$$\boldsymbol{d}_{\perp} = (25, 30) - (23.4, 31.2) = (1.6, -1.2)\,\mathrm{mm}.$$

Its length is $2.00\,\mathrm{mm}$; the shortest correction is $(-1.6, +1.2)\,\mathrm{mm}$.

4. Interpret, verify, and avoid a common error

The residual is perpendicular to the target: $(1.6, -1.2) \cdot (0.6, 0.8) = 0$. Pythagoras checks $\|\boldsymbol{d}\|^2 = 39^2 + 2^2 = 1525\,\mathrm{mm}^2$.

Source: Manuscript Ch. 3 Example 3.7

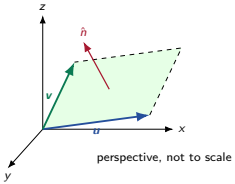
P29 — The area and normal of a mounting face

Guided worked example — Chapter 3

A hypothetical planar mounting face is a parallelogram with adjacent three-dimensional edge vectors $\boldsymbol{u} = (40, 0, 20)\,\mathrm{mm}$ and $\boldsymbol{v} = (0, 30, 10)\,\mathrm{mm}$ in a right-handed Cartesian basis.

Tasks

Find its oriented area vector, nonnegative face area, unit normal for the ordered pair $(\boldsymbol{u}, \boldsymbol{v})$, and the triangle area on the same edges.



Source: Manuscript Ch. 3 Example 3.10

P29 — Solution 1 of 2: model and derivation

1. Represent and choose a method

Use $\boldsymbol{C} = \boldsymbol{u} \times \boldsymbol{v}$. Its magnitude is the parallelogram area; the normal is $\hat{n} = \boldsymbol{C}/\|\boldsymbol{C}\|$. Check both orthogonality products.

2. Work through the governing relations

$$\begin{aligned}\boldsymbol{C} &= (0(10) - 20(30),\ 20(0) - 40(10),\ 40(30) - 0(0)) \\ &= (-600, -400, 1200)\,\mathrm{mm}^2 = 200(-3, -2, 6)\,\mathrm{mm}^2.\end{aligned}$$

Source: Manuscript Ch. 3 Example 3.10

P29 — Solution 2 of 2: results and checks

3. Evaluate each requested quantity

$$A = 200\sqrt{9+4+36} = 1400\,\mathrm{mm}^2, \quad \hat{n} = \tfrac{1}{7}(-3,-2,6).$$

$$A_{\triangle} = \tfrac{1}{2}A = 700\,\mathrm{mm}^2.$$

4. Interpret, verify, and avoid a common error

Both $\boldsymbol{u}\cdot\boldsymbol{C} = 0$ and $\boldsymbol{v}\cdot\boldsymbol{C} = 0$. Reversing the edge order reverses $\boldsymbol{C}$ but not A. The picture is an oblique projection: projected paper angles cannot be used to measure the physical area.

Source: Manuscript Ch. 3 Example 3.10

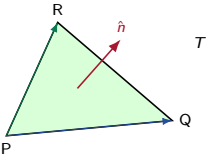
P30 — A triangular sensor: orientation and signed side

Student exercise — Chapter 3

An oriented triangular sensor has vertices $P = (1, -1, 2)\,\mathrm{cm}$, $Q = (4, 0, 3)\,\mathrm{cm}$ and $R = (2, 2, 1)\,\mathrm{cm}$. The normal convention follows the ordered boundary $P \to Q \to R$. A test point is $T = (1, -1, 3)\,\mathrm{cm}$.

Tasks

Find triangle area, oriented unit normal and whether T lies on the positive or negative normal side. Check using two methods.



oriented face, projected geometry

Source: Manuscript Ch. 3 Example 3.11

P30 — Solution 1 of 2: model and derivation

1. Represent and choose a method

Construct edges from the same base vertex, $\boldsymbol{u} = Q - P$ and $\boldsymbol{v} = R - P$, then $\boldsymbol{C} = \boldsymbol{u} \times \boldsymbol{v}$. The signed point-to-plane distance is $\hat{n} \cdot (T - P)$.

2. Work through the governing relations

$$\boldsymbol{u} = (3, 1, 1)\,\mathrm{cm}, \quad \boldsymbol{v} = (1, 3, -1)\,\mathrm{cm},$$

$$\boldsymbol{C} = (-4, 4, 8)\,\mathrm{cm}^2, \quad \|\boldsymbol{C}\| = 4\sqrt{6}\,\mathrm{cm}^2.$$

Source: Manuscript Ch. 3 Example 3.11

P30 — Solution 2 of 2: results and checks

3. Evaluate each requested quantity

$$A_{\triangle} = \tfrac{1}{2}\|\boldsymbol{C}\| = 2\sqrt{6}\,\mathrm{cm}^2, \quad \hat{n} = \frac{(-1,1,2)}{\sqrt{6}}.$$

$$d_s = \hat{n}\cdot(T-P) = \frac{2}{\sqrt{6}}\,\mathrm{cm} > 0.$$

T lies on the positive ordered-normal side.

4. Interpret, verify, and avoid a common error

Verify $\boldsymbol{u}\cdot\boldsymbol{C} = 0$, $\boldsymbol{v}\cdot\boldsymbol{C} = 0$ and $\|\boldsymbol{C}\|^2 = \|\boldsymbol{u}\|^2\|\boldsymbol{v}\|^2 - (\boldsymbol{u}\cdot\boldsymbol{v})^2 = 96$. Reversing vertices flips the side sign only.

Source: Manuscript Ch. 3 Example 3.11

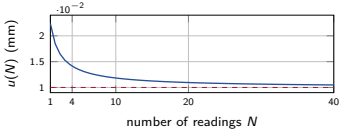
P31 — OPTIONAL: uncertainty with a shared correction

Optional extension — Chapter 1

For a constructed measurement program, the repeatability standard deviation of individual readings is $s = 0.020\,\mathrm{mm}$ and a common calibration-correction uncertainty is $u_b = 0.010\,\mathrm{mm}$. Assume independent, zero-mean repeatability contributions and a correction uncertainty shared by all readings.

Tasks

Find the standard uncertainty of the corrected mean for $N = 1, 4, 16$ and the large-N limit. Explain why treating the shared correction as independent for each reading fails.



Source: Manuscript Ch. 1 §1.8

P31 — Solution 1 of 2: model and derivation

1. Represent and choose a method

Under the stated independence model, averaging reduces the repeatability variance by N but does not reduce the single common-correction variance: $u^2(N) = s^2/N + u_b^2$.

2. Work through the governing relations

$$u(1) = \sqrt{0.020^2 + 0.010^2} = 0.02236\,\mathrm{mm};$$

$$u(4) = \sqrt{0.020^2/4 + 0.010^2} = 0.01414\,\mathrm{mm}.$$

The square root combines standard uncertainties, not worst-case absolute bounds.

Source: Manuscript Ch. 1 § 1.8

P31 — Solution 2 of 2: results and checks

3. Evaluate each requested quantity

$$u(16) = \sqrt{0.020^2/16 + 0.010^2} = 0.01118\,\mathrm{mm};$$

$$\lim_{N\to\infty} u(N) = u_b = 0.010\,\mathrm{mm}.$$

4. Interpret, verify, and avoid a common error

A calibration term common to every reading does not become $u_b/\sqrt{N}$. This is explicitly an OPTIONAL model with stated statistical assumptions, not a deduction from a digital display's increment.

Source: Manuscript Ch. 1 §1.8

P32 — OPTIONAL: doubling a spacer at fixed wall thickness

Optional extension — Chapter 1

A through-bored spacer has $R_o = 12.0\,\mathrm{mm}$, $R_i = 8.0\,\mathrm{mm}$ and length L. The redesign doubles outer radius and length but keeps radial wall thickness $t = R_o - R_i = 4.0\,\mathrm{mm}$. Density is unchanged.

Tasks

Find the mass ratio m'/m. Compare with full geometric similarity and derive a general expression for scaling factor s.

Sketch the system or its graph before revealing the solution.

Source: Manuscript Ch. 1 Example 1.7

AGH

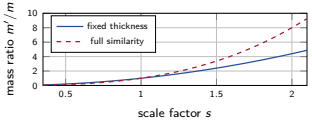
1. Represent and choose a method

Under the fixed-wall condition $R_o' = sR_o$, $R_i' = sR_o - t$, $L' = sL$. Unlike geometric similarity, the wall thickness does not scale with s.

2. Work through the governing relations

$$\frac{m'}{m} = s\frac{(sR_o)^2 - (sR_o - t)^2}{R_o^2 - (R_o - t)^2}$$

$$= s\frac{2sR_o t - t^2}{2R_o t - t^2} = 1.2s^2 - 0.2s, \quad s \geq \tfrac{1}{3}.$$



Source: Manuscript Ch. 1 Example 1.7

3. Evaluate each requested quantity

For $s = 2$, $R'_o = 24\,\mathrm{mm}$, $R'_i = 20\,\mathrm{mm}$ and

$$\frac{m'}{m} = 1.2(4) - 0.2(2) = 4.40.$$

With all dimensions truly doubled (including t), the ratio would instead be $2^3 = 8$.

4. Interpret, verify, and avoid a common error

The apparent contradiction disappears when the modelling constraint is stated: fixed thickness and complete similarity are different designs. Check $s = 1$ returns unity and $R'_i \geq 0$ holds.

Source: Manuscript Ch. 1 Example 1.7

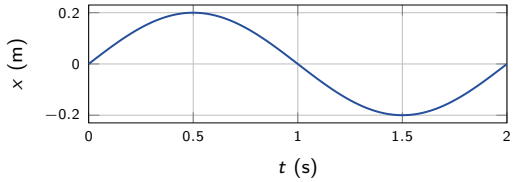
P33 — OPTIONAL: one sinusoidal motion, three aligned graphs

Optional extension — Chapter 2

An ideal prescribed one-dimensional trajectory is $x(t) = A\sin(\omega t)$ with $A = 0.20\,\mathrm{m}$, $\omega = \pi\,\mathrm{s}^{-1}$ and $0 \le t \le 2\,\mathrm{s}$. No model of the force generating the motion is required.

Tasks

Derive $v(t)$ and $a(t)$, classify $t = 0.5\,\mathrm{s}$ and $t = 1.0\,\mathrm{s}$, and find total displacement and distance.



Source: Manuscript Ch. 2 Example 2.8

P33 — Solution 1 of 2: model and derivation

1. Represent and choose a method

Differentiate with the chain rule; the trigonometric arguments are in radians. For distance split at every zero of $v(t)$, rather than taking the absolute value of net displacement.

2. Work through the governing relations

$$v(t) = A\omega\cos(\omega t) = 0.20\pi\cos(\pi t)\,\mathrm{m/s},$$

$$a(t) = -A\omega^2\sin(\omega t) = -0.20\pi^2\sin(\pi t)\,\mathrm{m/s^2}.$$

At $t = 0.5$, $v = 0$ and $a = -0.20\pi^2$; at $t = 1$, $x = 0$, $v = -0.20\pi$ and $a = 0$.

Source: Manuscript Ch. 2 Example 2.8

P33 — Solution 2 of 2: results and checks

3. Evaluate each requested quantity

At $t = 0, 0.5, 1.5, 2$ positions are $0, +0.20, -0.20, 0\,\mathrm{m}$.

$$\Delta x = 0 - 0 = 0, \qquad \ell = 0.20 + 0.40 + 0.20 = 0.80\,\mathrm{m}.$$

The average speed is $0.40\,\mathrm{m/s}$.

4. Interpret, verify, and avoid a common error

A zero acceleration at $t = 1$ does not mean the object rests. All three curves share event times but have different ordinate units and amplitudes.

Source: Manuscript Ch. 2 Example 2.8

P34 — OPTIONAL: numerical differentiation amplifies reading errors

Optional extension — Chapter 2

The exact constructed position model is $x(t) = (0.20\,\mathrm{m}) + (0.40\,\mathrm{m/s})t + (0.30\,\mathrm{m/s^2})t^2$. Estimate the velocity at $t_0 = 1.00\,\mathrm{s}$ by a central difference with half-interval h. The reading at $t_0 + h$ contains $+0.010\,\mathrm{m}$ additive error and the one at $t_0 - h$ contains $-0.010\,\mathrm{m}$ error.

Tasks

Calculate the estimated velocity and error at $h = 1.00\,\mathrm{s}$ and $h = 0.10\,\mathrm{s}$. Explain why a smaller interval need not yield a better result.

Sketch the system or its graph before revealing the solution.

Source: Original optional Ch. 2 § 2.9 concepts

P34 — Solution 1 of 3: model and derivation

1. Represent and choose a method

The centred estimator is $\hat{v} = [x_{\mathrm{meas}}(t_0+h) - x_{\mathrm{meas}}(t_0-h)]/(2h)$. For this quadratic exact model, truncation error vanishes; the stipulated opposite errors create a noise term $0.020/(2h)$.

2. Work through the governing relations

$$v(t) = 0.40 + 0.60t, \qquad v(1.00) = 1.00\,\mathrm{m/s}.$$

$$\hat{v} = \frac{x(1+h) - x(1-h)}{2h} + \frac{(+0.010) - (-0.010)}{2h}$$

$$= 1.00 + \frac{0.010\,\mathrm{m}}{h}.$$

Source: Original optional Ch. 2 §2.9 concepts

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3. Evaluate each requested quantity

$$h = 1.00\,\mathrm{s}: \quad \hat{v} = 1.010\,\mathrm{m/s};$$

$$h = 0.10\,\mathrm{s}: \quad \hat{v} = 1.100\,\mathrm{m/s}.$$

The error increases from 0.010 to $0.100\,\mathrm{m/s}$ as the interval shrinks by ten.

4. Interpret, verify, and avoid a common error

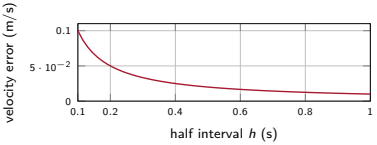
Exact differencing of this quadratic does not cause the problem; measurement error divided by a small time does. Higher-order smooth models introduce an additional truncation term, so choosing h is a balance between competing errors.

Source: Original optional Ch. 2 § 2.9 concepts

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A quantitative diagnosis of noise amplification

The exact quadratic model has no central-difference truncation bias at the selected event. The imposed opposite-sign measurement errors nevertheless produce $\delta v(h) = 0.010\,\mathrm{m}/h$. As h shrinks, the denominator shrinks while the error difference remains fixed.



Transferable lesson

Estimate both discretisation error and measurement noise before choosing a numerical derivative interval. Finer sampling alone cannot replace a measurement uncertainty model.

Source: Original optional Ch. 2 §2.9 concepts

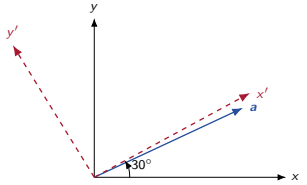
P35 — OPTIONAL: rotating Cartesian axes without moving a vector

Optional extension — Chapter 3

A fixed planar displacement has Cartesian components $\boldsymbol{a} = (4.0, 2.0)\,\mathrm{m}$. The coordinate axes are passively rotated counterclockwise through $\phi = 30°$; the physical displacement remains unchanged. A second vector is $\boldsymbol{b} = (0, 3.0)\,\mathrm{m}$.

Tasks

Find both vectors' new components. Check invariance of $\|\boldsymbol{a}\|$ and $\boldsymbol{a} \cdot \boldsymbol{b}$.



Source: Manuscript Ch. 3 § 3.6

P35 — Solution 1 of 2: model and derivation

1. Represent and choose a method

For a passive rotation by ϕ, use $\begin{pmatrix} a_{x'} \\ a_{y'} \end{pmatrix} = \begin{pmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{pmatrix} \begin{pmatrix} a_x \\ a_y \end{pmatrix}$. Do not use the active rotation matrix for a fixed physical vector.

2. Work through the governing relations

$$a_{x'} = 4\cos 30^\circ + 2\sin 30^\circ = 4.4641\,\mathrm{m},$$

$$a_{y'} = -4\sin 30^\circ + 2\cos 30^\circ = -0.26795\,\mathrm{m}.$$

$$\boldsymbol{b}' = (3\sin 30^\circ, 3\cos 30^\circ) = (1.500, 2.5981)\,\mathrm{m}.$$

Source: Manuscript Ch. 3 § 3.6

P35 — Solution 2 of 2: results and checks

3. Evaluate each requested quantity

$$\|\boldsymbol{a}\|^2 = 4^2 + 2^2 = 20\,\mathrm{m}^2 = a_{x'}^2 + a_{y'}^2,$$

$$\boldsymbol{a} \cdot \boldsymbol{b} = 6.00\,\mathrm{m}^2 = \boldsymbol{a}' \cdot \boldsymbol{b}'.$$

4. Interpret, verify, and avoid a common error

The numerical components depend on axes; the physical length and the angle between two vectors do not. This is an OPTIONAL coordinate-transformation tool, not a prerequisite for ordinary vector addition.

Source: Manuscript Ch. 3 § 3.6

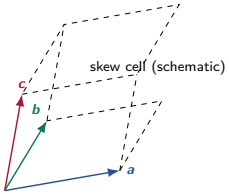
P36 — OPTIONAL: a skew cell and scalar triple product

Optional extension — Chapter 3

A geometric calibration cell is described by edge vectors $\boldsymbol{a} = (2, 0, 1)\,\mathrm{cm}$, $\boldsymbol{b} = (0, 3, 1)\,\mathrm{cm}$ and $\boldsymbol{c} = (1, 1, 2)\,\mathrm{cm}$ from the same corner. Treat the data as a right-handed Euclidean model.

Tasks

Find its nonnegative parallelepiped volume, signed height over the ordered $(\boldsymbol{a}, \boldsymbol{b})$ base, the in-plane component of $\boldsymbol{c}$, and the associated tetrahedron volume.



Source: Manuscript Ch. 3 Example 3.12

P36 — Solution 1 of 2: model and derivation

1. Represent and choose a method

The base's oriented area is $\boldsymbol{A} = \boldsymbol{a} \times \boldsymbol{b}$; signed volume $V_s = \boldsymbol{A} \cdot \boldsymbol{c}$ and signed height $h_s = V_s/\|\boldsymbol{A}\|$. The in-plane component is $\boldsymbol{c} - h_s\hat{n}$.

2. Work through the governing relations

$$\boldsymbol{A} = (-3, -2, 6)\,\mathrm{cm}^2, \quad \|\boldsymbol{A}\| = 7\,\mathrm{cm}^2;$$

$$V_s = (-3, -2, 6) \cdot (1, 1, 2) = 7\,\mathrm{cm}^3, \quad h_s = 1\,\mathrm{cm}.$$

Source: Manuscript Ch. 3 Example 3.12

P36 — Solution 2 of 2: results and checks

3. Evaluate each requested quantity

$$\hat{n} = (-3, -2, 6)/7, \quad \boldsymbol{c}_{\parallel} = \boldsymbol{c} - 1\,\mathrm{cm}\,\hat{n} = \left(\frac{10}{7}, \frac{9}{7}, \frac{8}{7}\right)\mathrm{cm}.$$

$$V_{\mathrm{tetrahedron}} = \frac{|V_s|}{6} = \frac{7}{6}\,\mathrm{cm}^3.$$

4. Interpret, verify, and avoid a common error

Check $\boldsymbol{c}_{\parallel} \cdot \boldsymbol{A} = 0$, and $\boldsymbol{c}_{\parallel} + (1\,\mathrm{cm})\hat{n} = \boldsymbol{c}$. The sign of the triple product reflects orientation; physical volume is its absolute value.

Source: Manuscript Ch. 3 Example 3.12

References, source conventions and teaching use

Primary chapter materials

Hamzeh Khanpour, *Physics for Technical and Engineering Students, Vol. I*, revised Chapters 1–3: Measurement, Units, and Physical Modelling; Motion Along a Straight Line; Vectors.

Selected textbook items

Halliday, Resnick and Walker, *Fundamentals of Physics*, Volume 1, 12th edition (Wiley, 2022), Chapters 1–3. Each textbook problem is individually attributed on its statement and solution slides. Detailed item identifiers and source PDF page numbers are preserved in the accompanying register.

Classroom note

Solve the displayed problem before advancing to the next solution slide. The 30 core problems and 6 optional extensions form a reusable teaching bank, not a claim that every item can fit one short tutorial meeting.


THANK
YOU!