Akademia Górniczo-Hutnicza
im. Stanisława Staszica w Krakowie

AGH University of Krakow

# Physics I Tutorial

## Chapter 4: Motion in Two and Three Dimensions

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# Lecturer information and contact details


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# R1 — Vector position, velocity and acceleration

## One clock and one fixed Cartesian basis

$$\mathbf{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}, \qquad \Delta\mathbf{r} = \mathbf{r}_f - \mathbf{r}_i.$$

$$\mathbf{v} = \frac{\mathrm{d}\mathbf{r}}{\mathrm{d}t}, \qquad \mathbf{a} = \frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t}, \qquad v = \sqrt{v_x^2 + v_y^2 + v_z^2}.$$

At nonzero speed, velocity is tangent to the three-dimensional trajectory.

## Displacement is not route length

$$s = \int_{t_i}^{t_f} \|\mathbf{v}(t)\|\,\mathrm{d}t \geq \|\Delta\mathbf{r}\|, \qquad \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{v_y}{v_x}\ (v_x \neq 0).$$

One view can conceal out-of-plane motion; a projected trajectory is not a full 3D reconstruction.

# R2 — Constant vector acceleration and projectile components

## Constant vector acceleration over a single phase

$$\mathbf{v}(t) = \mathbf{v}_0 + \mathbf{a}\tau, \qquad \mathbf{r}(t) = \mathbf{r}_0 + \mathbf{v}_0\tau + \tfrac{1}{2}\mathbf{a}\tau^2, \quad \tau = t - t_0.$$

**Condition:** the acceleration *vector* is constant, not just its magnitude.

## No-drag flight with upward-positive y

$$x = x_0 + v_0\cos\theta\, t, \quad y = y_0 + v_0\sin\theta\, t - \tfrac{1}{2}gt^2,$$

$$v_x = v_0\cos\theta, \quad v_y = v_0\sin\theta - gt, \quad \mathbf{a} = -g\hat{\jmath}.$$

Gravity is a positive magnitude, $g = 9.81\,\mathrm{m/s^2}$ for original manuscript exercises. Flight stops at first contact.



### Event conditions are simultaneous

$$t_{\pm} = \frac{v_{0y} \pm \sqrt{v_{0y}^2 + 2g(y_0 - y_f)}}{g}, \qquad t_A = \frac{v_{0y}}{g}.$$

Check the real discriminant, time domain, ascent/descent branch and *horizontal* target condition. A height crossing is not necessarily a collision.

### Equal-level, unobstructed landing only

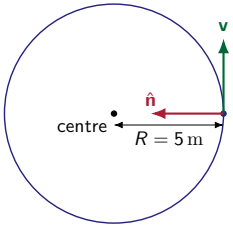
$$T = \frac{2v_0 \sin\theta}{g}, \qquad R = \frac{v_0^2 \sin 2\theta}{g}.$$

For a launch above the landing plane, the general height root must be used. $45^\circ$ maximizes range only under its special equal-height, fixed-speed assumptions.

## Tangent velocity and inward acceleration

$$v = R|\omega|, \qquad a_n = \frac{v^2}{R} = R\omega^2, \qquad T = \frac{2\pi}{|\omega|} = \frac{2\pi R}{v}.$$

With stationary centre $\mathbf{r}_C$, $\mathbf{a} = -\omega^2(\mathbf{r} - \mathbf{r}_C)$. Reversing rotation reverses velocity, *not* inward acceleration at a fixed position.




$v(t) = 1 + 0.4t$, evaluate at $t = 5\,\mathrm{s}$

# R5 — Turning, tangential acceleration and relative motion

## Speed change and curvature

$$\mathbf{a} = a_t\hat{\mathbf{t}} + a_n\hat{\mathbf{n}}, \quad a_t = \frac{\mathrm{d}v}{\mathrm{d}t} = \frac{\mathbf{v}\cdot\mathbf{a}}{v}, \quad a_n = \frac{v^2}{R_c}, \quad \|\mathbf{a}\| = \sqrt{a_t^2 + a_n^2}.$$

$a_t$ is signed; $a_n \geq 0$. The tangent $\mathbf{v}/v$ is undefined at a stop.

## Nonrotating translating observers; common clock

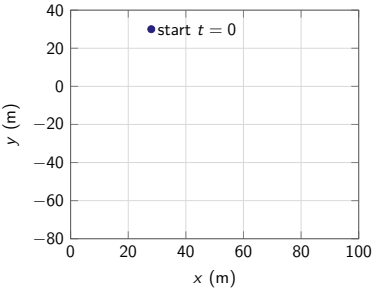
$$\mathbf{v}_{A/C} = \mathbf{v}_{A/B} + \mathbf{v}_{B/C}, \qquad \mathbf{a}_{A/C} = \mathbf{a}_{A/B} + \mathbf{a}_{B/C}.$$

Speeds cannot generally be added as scalars. Even when track and heading differ, both event coordinates must agree at one time.

# P01 — Rabbit trajectory: position and plotted path

## Problem statement

A rabbit crosses a parking lot with coordinates (metres; time in seconds) $x(t) = -0.31t^2 + 7.2t + 28$, $y(t) = 0.22t^2 - 9.1t + 30$. (a) At $t = 15\,\mathrm{s}$, give its position vector in Cartesian and magnitude–angle forms. (b) Plot its path for $0 \le t \le 25\,\mathrm{s}$, marking the positions at 0, 5, 10, 15, 20 and 25 s.




**Source:** Halliday, Resnick & Walker, *Fundamentals of Physics*, 12th ed., Vol. 1, Ch. 4, Sample Problem 4.1.1, p. 69.

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## Set-up and governing equations

**Coordinate method.** The laboratory position is $\mathbf{r}(t) = x(t)\hat{\imath} + y(t)\hat{\jmath}$. Evaluate both components at the *same* clock time: $x(15) = -0.31(225) + 7.2(15) + 28 = 66.25\,\mathrm{m}$, $y(15) = 0.22(225) - 9.1(15) + 30 = -57.0\,\mathrm{m}$.

## Derivation and method

**Polar form.** $r = \sqrt{x^2 + y^2} = \sqrt{66.25^2 + 57^2} = 87.4\,\mathrm{m}$. $\theta = \mathrm{atan2}(-57, 66.25) \approx -40.7^\circ$: the arrow lies in quadrant IV. A plain $\arctan(y/x)$ must be checked against signs.

**Source:** Halliday, Resnick & Walker, *Fundamentals of Physics*, 12th ed., Vol. 1, Ch. 4, Sample Problem 4.1.1, p. 69.

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## Evaluation and physical checks

**Plot by the parameter.** Evaluate $(x(t), y(t))$ for each chosen $t$; connect the time-ordered points along *one* smooth parametric curve. It is not generally the graph of $y(t)$ against $x = t$, and a line through just the endpoints would miss the path.

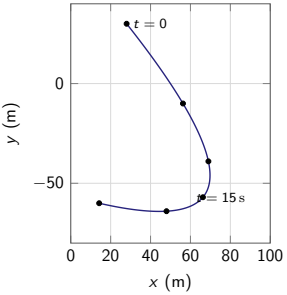
## Interpretation and verification

**Check.** The graph should pass through $(28, 30)\,\mathrm{m}$ at $t = 0$ and $(66.25, -57.0)\,\mathrm{m}$ at $t = 15\,\mathrm{s}$. The position vector starts at the chosen origin, not at the previous sample. The rounded source result is $r \simeq 87\,\mathrm{m}$, $\theta \simeq -41^\circ$.

**Source:** Halliday, Resnick & Walker, *Fundamentals of Physics*, 12th ed., Vol. 1, Ch. 4, Sample Problem 4.1.1, p. 69.

## Scientific interpretation of the completed figure

The time-labelled curve is obtained by evaluating both coordinates at a common time, not by drawing the endpoint chord. The point at $t = 15\,\mathrm{s}$ is in quadrant IV; its displacement from the origin is the calculated position vector.



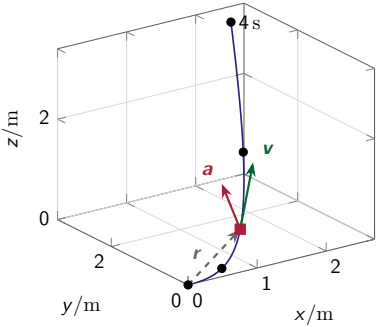

**Source:** Halliday, Resnick & Walker, *Fundamentals of Physics*, 12th ed., Vol. 1, Ch. 4, Sample Problem 4.1.1, p. 69.

# P02 — Three-dimensional inspection probe

## Problem statement

For $0 \le t \le 4\,\mathrm{s}$, a probe follows $x = (0.60\,\mathrm{m/s})t$, $y = (0.20\,\mathrm{m/s^2})t^2$, $z = (0.050\,\mathrm{m/s^3})t^3$. Find $\mathbf{r}, \mathbf{v}, \|\mathbf{v}\|, \mathbf{a}$ at $t = 2.00\,\mathrm{s}$. Interpret the spatial trajectory and the position, velocity and acceleration arrows.

(a) Laboratory trajectory




**Source:** Khanpour, *Physics I, Chapter 4, Final*, Example 4.1, printed p. 2–3.

## Set-up and governing equations

**Differentiate before evaluating.** In fixed Cartesian axes, $\mathbf{v} = (0.60, 0.40t, 0.150t^2)$ and $\mathbf{a} = (0, 0.40, 0.300t)$, with units inherited from the explicitly dimensional coefficients.

## Derivation and method

**Same-time state.** At $t = 2\,\mathrm{s}$, $\mathbf{r} = (1.20, 0.80, 0.40)\,\mathrm{m}$, $\mathbf{v} = (0.60, 0.80, 0.60)\,\mathrm{m/s}$, and $\mathbf{a} = (0, 0.40, 0.60)\,\mathrm{m/s^2}$.

**Source:** Khanpour, *Physics I, Chapter 4, Final*, Example 4.1, printed p. 2–3.

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## Evaluation and physical checks

**Magnitudes and directions.** $v = \sqrt{0.6^2 + 0.8^2 + 0.6^2} = \sqrt{1.36} = 1.166\,\mathrm{m/s}$. Likewise $\|\mathbf{a}\| = \sqrt{0.4^2 + 0.6^2} = 0.721\,\mathrm{m/s^2}$. The velocity is tangent to the curve, whereas acceleration is not generally tangent.

## Interpretation and verification

**Checks.** The path tangent has slopes $dy/dx = v_y/v_x = 4/3$, $dz/dx = v_z/v_x = 1$. Position starts at the origin; velocity and acceleration attach to the event and need their own distinct drawing scales. Dimensional derivatives and direct substitution agree.

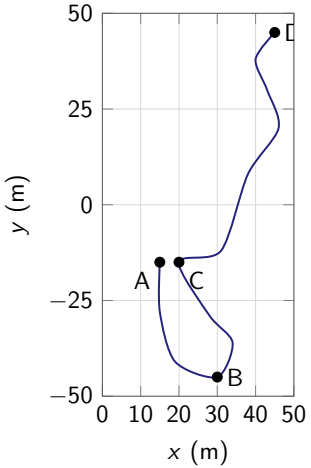
**Source:** Khanpour, *Physics I, Chapter 4, Final*, Example 4.1, printed p. 2–3.

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# P03 — Squirrel path: which average velocity is smallest?

## Problem statement

Figure 4.10 gives a squirrel's successive locations on level ground: A at 0 min, B at 5.00 min, C at 10.0 min and D at 15.0 min. Compare the average velocities from A to each later point. Find the (a) magnitude and (b) direction of the smallest, and the (c) magnitude and (d) direction of the largest. Use the graph coordinates, not the curved route length.




**Source:** Halliday, Resnick & Walker, *Fundamentals of Physics*, 12th ed., Vol. 1, Ch. 4, Problem 9, p. 91.

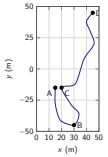
# P03 — Worked solution 1 of 2

## Set-up and governing equations

**Extract the graph.** From its 5 m grid, $A = (15, -15)$, $B = (30, -45)$, $C = (20, -15)$, $D = (45, 45)$, all in metres. Averages are $\bar{\mathbf{v}}_{AP} = (\mathbf{r}_P - \mathbf{r}_A)/(t_P - t_A)$.

## Derivation and method

**Compute all three.** $\bar{\mathbf{v}}_{AB} = (15, -30)/300 = (0.050, -0.100)\,\mathrm{m/s}$. $\bar{\mathbf{v}}_{AC} = (5, 0)/600 = (0.00833, 0)\,\mathrm{m/s}$. $\bar{\mathbf{v}}_{AD} = (30, 60)/900 = (0.0333, 0.0667)\,\mathrm{m/s}$.




**Source:** Halliday, Resnick & Walker, *Fundamentals of Physics*, 12th ed., Vol. 1, Ch. 4, Problem 9, p. 91.

## Evaluation and physical checks

**Rank and report.** Their magnitudes are 0.1118, 0.00833, 0.0745 $\mathrm{m/s}$, respectively. $AC$ is smallest, direction $0^\circ$ (east); $AB$ is largest, $\mathrm{atan2}(-30, 15) = -63.4^\circ$ (south of east).

## Interpretation and verification

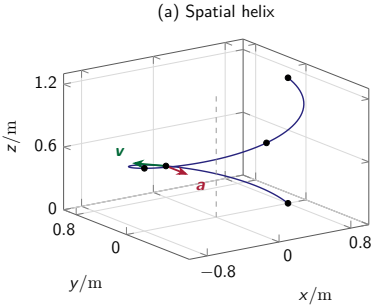
**Check and lesson.** The curved route through B does *not* enter the endpoint average. Different denominators matter: the longest displacement need not give the largest time-averaged velocity. Directions follow signed endpoint differences.

**Source:** Halliday, Resnick & Walker, *Fundamentals of Physics*, 12th ed., Vol. 1, Ch. 4, Problem 9, p. 91.

# P04 — Helical motion: displacement versus path length

## Problem statement

An inspection marker follows $\mathbf{r}(t) = (R\cos\omega t, R\sin\omega t, ut)$, with $R = 0.800\,\mathrm{m}$, $\omega = \pi/2\,\mathrm{s}^{-1}$, $u = 0.300\,\mathrm{m/s}$. (a) Find its state at 1.00 s; (b) compare vector displacement and travelled distance after one projected full turn.




**Source:** Khanpour, *Physics I, Chapter 4, Final*, Example 4.4, printed p. 5–6.

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## Set-up and governing equations

**Differentiate the 3D trajectory.** $\mathbf{v} = (-R\omega\sin\omega t, R\omega\cos\omega t, u)$, $\mathbf{a} = (-R\omega^2\cos\omega t, -R\omega^2\sin\omega t, 0)$. The fixed vertical component changes position, but produces no vertical acceleration.

## Derivation and method

**At one second.** $\omega t = \pi/2$ gives $\mathbf{r} = (0, 0.800, 0.300)\,\mathrm{m}$; $\mathbf{v} = (-1.2566, 0, 0.300)\,\mathrm{m/s}$; $\mathbf{a} = (0, -1.9739, 0)\,\mathrm{m/s^2}$. Speed is $\sqrt{(R\omega)^2 + u^2} = 1.292\,\mathrm{m/s}$.

**Source:** Khanpour, *Physics I, Chapter 4, Final*, Example 4.4, printed p. 5–6.

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## Evaluation and physical checks

**Full turn.** $T = 2\pi/\omega = 4.00\,\mathrm{s}$. After this interval the horizontal coordinates return, but $z$ increases by $uT = 1.20\,\mathrm{m}$: $\Delta\mathbf{r} = (0, 0, 1.20)\,\mathrm{m}$.

## Interpretation and verification

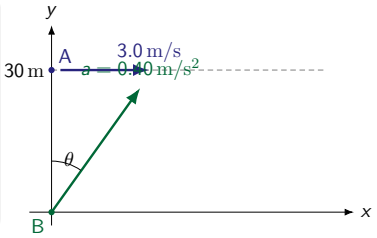
**Distance and check.** Constant 3D speed yields $s = vT = 5.168\,\mathrm{m}$, much larger than $\|\Delta\mathbf{r}\| = 1.20\,\mathrm{m}$. $\mathbf{v}\cdot\mathbf{a} = 0$ independently confirms constant speed. The circular top view alone misses the ascent.

**Source:** Khanpour, *Physics I, Chapter 4, Final*, Example 4.4, printed p. 5–6.

# P05 — Collision between a particle and an accelerating target

## Problem statement

Particle A passes the $y$-axis at $(0, 30\,\mathrm{m})$ at $t = 0$ and then travels parallel to $+x$ at $3.0\,\mathrm{m/s}$. Particle B starts from the origin at rest at the same time, with constant acceleration of magnitude $0.40\,\mathrm{m/s^2}$ directed at angle $\theta$ from $+y$ toward $+x$. Find $\theta$ for the particles to collide.



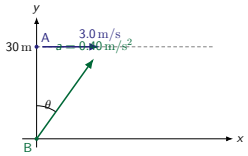

**Source:** Halliday, Resnick & Walker, *Fundamentals of Physics*, 12th ed., Vol. 1, Ch. 4, Problem 20, p. 91.

## Set-up and governing equations

**Describe both particles on one clock.** $\mathbf{r}_A = (3t, 30)$. For B, $\mathbf{a}_B = 0.40(\sin\theta, \cos\theta)$ and $\mathbf{r}_B = (0.20t^2 \sin\theta, 0.20t^2 \cos\theta)$, with numerical SI values.

## Derivation and method

**Demand simultaneous coordinates.** Set $3t = 0.20t^2 \sin\theta$ and $30 = 0.20t^2 \cos\theta$. The nonzero meeting time obeys $t = 15/\sin\theta$ and $t^2 = 150/\cos\theta$.




**Source:** Halliday, Resnick & Walker, *Fundamentals of Physics*, 12th ed., Vol. 1, Ch. 4, Problem 20, p. 91.

## Evaluation and physical checks

**Eliminate time.** $225\cos\theta = 150\sin^2\theta$, so $3c = 2(1 - c^2)$ with $c = \cos\theta$. This gives $2c^2 + 3c - 2 = 0$, hence $c = 1/2$ or $-2$; only $1/2$ is feasible.

## Interpretation and verification

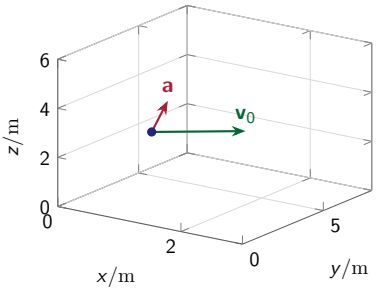
**Result and check.** $\boxed{\theta = 60^\circ}$, $t = 15/\sin 60^\circ = 17.32\,\mathrm{s}$. Both x coordinates are $51.96\,\mathrm{m}$, and both y coordinates are $30\,\mathrm{m}$. Equal speeds would *not* be an alternative collision condition.

**Source:** Halliday, Resnick & Walker, *Fundamentals of Physics*, 12th ed., Vol. 1, Ch. 4, Problem 20, p. 91.

# P06 — A spatial trajectory confined to a plane

## Problem statement

A point starts at $\mathbf{r}_0 = (1, 2, 3)\,\mathrm{m}$, with $\mathbf{v}_0 = (1, 2, 0)\,\mathrm{m/s}$ and a constant $\mathbf{a} = (0, 1, 1)\,\mathrm{m/s^2}$. (a) Find $\mathbf{r}(2\,\mathrm{s})$; (b) compute a normal to its plane of motion using $\mathbf{v}_0 \times \mathbf{a}$; (c) give a Cartesian plane equation and check the endpoint.




**Source:** Khanpour, *Physics I, Chapter 4, Final*, Problem 34, printed p. 55; solution 71.

## Set-up and governing equations

**Integrate constant vector acceleration.** $\mathbf{r}(t) = \mathbf{r}_0 + \mathbf{v}_0 t + \frac{1}{2}\mathbf{a}t^2$. At $t = 2\,\mathrm{s}$, add $(1,2,3) + (2,4,0) + (0,2,2) = (3,8,5)\,\mathrm{m}$.

## Derivation and method

**Find a spatial normal.** $\mathbf{n} \propto \mathbf{v}_0 \times \mathbf{a} = (2,-1,1)$ in component order. Both $\mathbf{v}_0$ and $\mathbf{a}$ are orthogonal to $\mathbf{n}$; therefore any displacement $\mathbf{v}_0 t + \frac{1}{2}\mathbf{a}t^2$ remains in the plane.

**Source:** Khanpour, *Physics I, Chapter 4, Final*, Problem 34, printed p. 55; solution 71.

## Evaluation and physical checks

**Plane equation.** A plane through $\mathbf{r}_0$ with normal $(2,-1,1)$ obeys $2(x-1)-(y-2)+(z-3)=0$, equivalently $2x-y+z=3\,\mathrm{m}$ for numerical coordinates in metres.

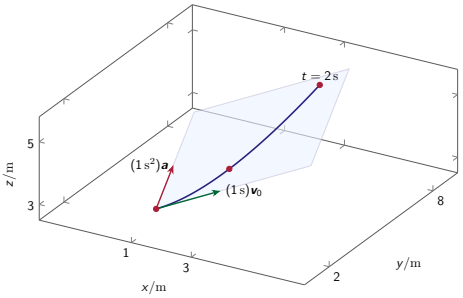
## Interpretation and verification

**Checks.** Endpoint $(3,8,5)$ gives $6-8+5=3$. The cross product is nonzero, so the span is a plane rather than a line. Three changing coordinate values do not require a nonplanar spatial curve.

**Source:** Khanpour, *Physics I, Chapter 4, Final*, Problem 34, printed p. 55; solution 71.

## Scientific interpretation of the completed figure

The blue trajectory remains in one affine plane because its displacement is generated by $\mathbf{v}_0$ and $\mathbf{a}$. The construction shows the independent spanning directions, the endpoint at $t = 2\,\mathrm{s}$ and the plane normal $\mathbf{v}_0 \times \mathbf{a}$.

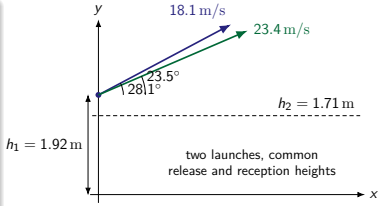



**Source:** Khanpour, *Physics I, Chapter 4, Final*, Problem 34, printed p. 55; solution 71.

# P07 — Soccer conventional vs handspring throw-in

## Problem statement

Compare soccer throw-ins launched at $h_1 = 1.92\,\mathrm{m}$, received at a teammate's forehead $h_2 = 1.71\,\mathrm{m}$. Conventional: $v_0 = 18.1\,\mathrm{m/s}$, $\theta_0 = 28.1°$. Handspring: $v_0 = 23.4\,\mathrm{m/s}$, $\theta_0 = 23.5°$. Find (a,b) conventional flight time and horizontal reach, (c,d) handspring flight time and reach, and (e) the practical advantage. Use $g = 9.8\,\mathrm{m/s^2}$.




**Source:** Halliday, Resnick & Walker, *Fundamentals of Physics*, 12th ed., Vol. 1, Ch. 4, Sample Problem 4.4.1, p. 80.

# P07 — Worked solution 1 of 2

## Set-up and governing equations

**Vertical event is below launch.** For each throw, $h_2 - h_1 = -0.21\,\mathrm{m} = v_0 \sin\theta\, t - \frac{1}{2} g t^2$. The admissible positive root is $t = [v_0 \sin\theta + \sqrt{(v_0 \sin\theta)^2 + 2g(0.21)}]/g$.

## Derivation and method

**Conventional launch (a,b).** $t_c = 1.7642\,\mathrm{s}$. Horizontal acceleration is zero, so $d_c = (18.1 \cos 28.1^\circ) t_c = 28.17\,\mathrm{m}$. Round to source precision: $1.76\,\mathrm{s}$ and $28.2\,\mathrm{m}$.

**Source:** Halliday, Resnick & Walker, *Fundamentals of Physics*, 12th ed., Vol. 1, Ch. 4, Sample Problem 4.4.1, p. 80.

## Evaluation and physical checks

**Handspring launch (c,d).** Replace the parameters: $t_{hs} = 1.9265\,\mathrm{s}$, $d_{hs} = (23.4\cos 23.5^\circ)t_{hs} = 41.34\,\mathrm{m}$; approximately $1.93\,\mathrm{s}$ and $41.3\,\mathrm{m}$. Both are descending intercepts at the lower reception level.

## Interpretation and verification

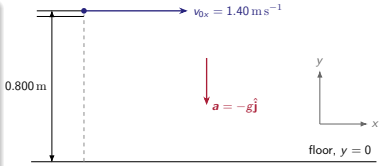
**Interpretation (e).** The handspring provides much greater horizontal reach (approximately $13.2\,\mathrm{m}$ more), mainly because of its higher launch speed. Verify both flights satisfy the given reception height and occur after their apices. This is a no-drag trajectory comparison, not a guarantee of actual game outcomes.

**Source:** Halliday, Resnick & Walker, *Fundamentals of Physics*, 12th ed., Vol. 1, Ch. 4, Sample Problem 4.4.1, p. 80.

# P08 — Horizontal launch: first floor contact

## Problem statement

A point-like bead leaves a horizontal guide at height $0.800\,\mathrm{m}$ above a level floor with horizontal speed $1.40\,\mathrm{m/s}$. Neglect air drag; use $g = 9.81\,\mathrm{m/s^2}$. Find the first floor-contact time, horizontal displacement, and velocity immediately before contact, including the velocity angle from $+x$.



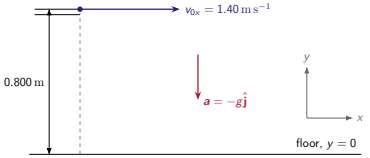

**Source:** Khanpour, *Physics I, Chapter 4, Final*, Problem 12, printed p. 51–52.

## Set-up and governing equations

**Separate coordinates.** With origin below release, $x(t) = 1.40t$, $y(t) = 0.800 - \frac{1}{2}(9.81)t^2$; $v_x = 1.40$, $v_y = -9.81t$ in SI units. The projectile equations apply only until first floor contact.

## Derivation and method

**Solve the event.** Set $y = 0$: $t_f = \sqrt{1.600/9.81} = 0.403855\,\mathrm{s}$. Select the positive root because release is $t = 0$ and no pre-launch root represents a later collision.




**Source:** Khanpour, *Physics I, Chapter 4, Final*, Problem 12, printed p. 51–52.

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## Evaluation and physical checks

**Range and velocity.** $x_f = (1.40)t_f = 0.565397\,\mathrm{m}$. Just before contact $\mathbf{v}_f = (1.40, -3.96182)\,\mathrm{m/s}$, whose speed is $\sqrt{1.40^2 + 3.96182^2} = 4.20190\,\mathrm{m/s}$.

## Interpretation and verification

**Impact direction.** $\theta_f = \mathrm{atan2}(-3.96182, 1.40) = -70.54^\circ$ below $+x$. A single positive angle without its downward sense is ambiguous. No post-impact velocity is predicted by this free-flight model.

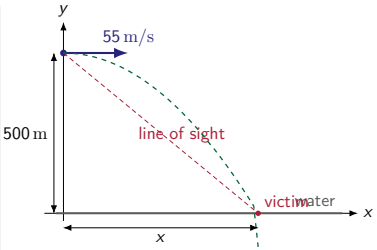
**Source:** Khanpour, *Physics I, Chapter 4, Final*, Problem 12, printed p. 51–52.

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# P09 — Airplane rescue capsule: line of sight

## Problem statement

A rescue aircraft flies level at $198\,\mathrm{km/h} = 55.0\,\mathrm{m/s}$ and altitude $h = 500\,\mathrm{m}$. It releases a capsule so that it falls toward a person in the water, with negligible air drag. (a) Find the angle $\phi$ of the pilot's downward line of sight to the person at release. (b) Find the capsule's velocity immediately upon reaching the water. Use $g = 9.8\,\mathrm{m/s^2}$.




**Source:** Halliday, Resnick & Walker, *Fundamentals of Physics*, 12th ed., Vol. 1, Ch. 4, Sample Problem 4.4.2, p. 81.

# P09 — Worked solution 1 of 2

## Set-up and governing equations

**State the release model.** The capsule initially shares the plane's horizontal velocity: $v_{0x} = 55.0\,\mathrm{m/s}$, $v_{0y} = 0$. Take the vertical displacement $\Delta y = -500\,\mathrm{m}$. No horizontal acceleration acts in this model.

## Derivation and method

**Arrival time and sight line (a).** $-500 = -\frac{1}{2}(9.8)t^2$ gives $t = \sqrt{1000/9.8} = 10.1015\,\mathrm{s}$. Horizontal separation on release is $x = 55t = 555.58\,\mathrm{m}$. Thus $\phi = \tan^{-1}(x/500) = 48.0^\circ$ from the downward vertical.

**Source:** Halliday, Resnick & Walker, *Fundamentals of Physics*, 12th ed., Vol. 1, Ch. 4, Sample Problem 4.4.2, p. 81.

## Evaluation and physical checks

**Velocity at water (b).** $\mathbf{v}_f = (55.0, -9.8t) = (55.0, -98.995)\,\mathrm{m/s}$. The speed is $113.25\,\mathrm{m/s}$; the velocity points $\mathrm{atan2}(-98.995, 55.0) = -60.94^\circ$ from $+\mathrm{x}$.

## Interpretation and verification

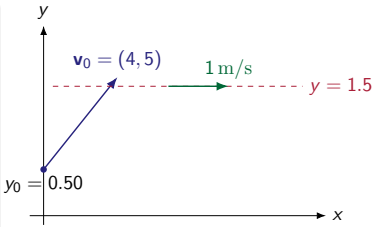
**Checks.** At the arrival time the capsule lies $555.6\,\mathrm{m}$ forward and $500\,\mathrm{m}$ lower than its release point. Horizontal velocity is unchanged, but vertical velocity has grown in magnitude. A passenger in the aircraft sees the capsule initially drop vertically relative to the moving plane.

**Source:** Halliday, Resnick & Walker, *Fundamentals of Physics*, 12th ed., Vol. 1, Ch. 4, Sample Problem 4.4.2, p. 81.

# P10 — Descending interception of a moving target

## Problem statement

A bead launches from $(0, 0.50)\,\mathrm{m}$ with $\mathbf{v}_0 = (4.00, 5.00)\,\mathrm{m/s}$. A target moves right at $1.00\,\mathrm{m/s}$ along $y = 1.50\,\mathrm{m}$. (a) Find the target's initial x coordinate for a descending interception, and the bead's impact velocity. (b) Decide whether a target starting at $x_{T0} = 2.00\,\mathrm{m}$ is hit. Neglect drag; no early obstacles.




**Source:** Khanpour, *Physics I, Chapter 4, Final*, Example 4.6, printed p. 10–11.

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## Set-up and governing equations

**Find height-event candidates.** $1.50 = 0.50 + 5t - 4.905t^2$ gives $t_- = 0.273243\,\mathrm{s}$, $t_+ = 0.746125\,\mathrm{s}$. Apex time is $t_A = 5/9.81 = 0.509684\,\mathrm{s}$, so $t_+$ is the descending event.

## Derivation and method

**Match horizontal positions (a).** At $t_+$, bead $x_B = 4t_+ = 2.98450\,\mathrm{m}$; target $x_T = x_{T0} + t_+$. Therefore $x_{T0} = 3t_+ = 2.23837\,\mathrm{m}$. This is a same-clock condition.

**Source:** Khanpour, *Physics I, Chapter 4, Final*, Example 4.6, printed p. 10–11.

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## Evaluation and physical checks

**Velocity at contact.** $\mathbf{v}_B(t_+) = (4.00, 5.00 - 9.81 t_+) = (4.00, -2.31948)\,\mathrm{m/s}$. Its speed is $4.62\,\mathrm{m/s}$ and flight angle $-30.1^\circ$.

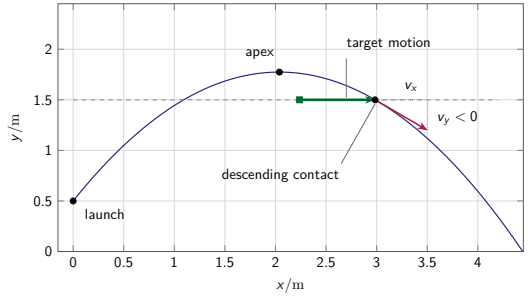
## Interpretation and verification

**Test the alternative target (b).** Equality of x would require $4t = 2 + t$, so $t = 2/3\,\mathrm{s}$. At that time $y_B = 1.65333\,\mathrm{m} \neq 1.50\,\mathrm{m}$: the bead passes $0.15333\,\mathrm{m}$ above. Crossing a height and intercepting a target are different events.

**Source:** Khanpour, *Physics I, Chapter 4, Final*, Example 4.6, printed p. 10–11.

## Scientific interpretation of the completed figure

The green target displacement and the blue projectile curve meet at the same position and time only for the descending height root. The red arrow shows the projectileś downward-tilted contact velocity, not an applied force.

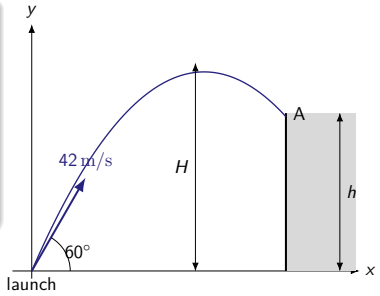



**Source:** Khanpour, *Physics I, Chapter 4, Final*, Example 4.6, printed p. 10–11.

# P11 — Stone launched toward a high cliff

## Problem statement

A stone is fired at a cliff with initial speed $42.0\,\mathrm{m/s}$ at $60.0^\circ$ above horizontal, striking point A at $t = 5.50\,\mathrm{s}$. The cliff top is at height $h$, and the trajectory's apex is at height $H$ above ground. Find (a) $h$, (b) impact speed at A, (c) $H$. Use $g = 9.8\,\mathrm{m/s^2}$.




**Source:** Halliday, Resnick & Walker, *Fundamentals of Physics*, 12th ed., Vol. 1, Ch. 4, Problem 28, p. 92.

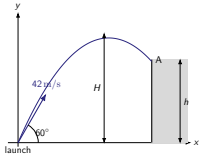
# P11 — Worked solution 1 of 2

## Set-up and governing equations

**Decompose the launch.** $v_{0x} = 42\cos 60^\circ = 21.0\,\mathrm{m/s}$, $v_{0y} = 42\sin 60^\circ = 36.373\,\mathrm{m/s}$. Choose the launch point as $(0, 0)$.

## Derivation and method

**Height at impact (a).** $h = v_{0y}(5.50) - \frac{1}{2}(9.8)(5.50)^2 = 51.827\,\mathrm{m}$. It is the height of the point where the stone hits the cliff, not automatically the global apex.




**Source:** Halliday, Resnick & Walker, *Fundamentals of Physics*, 12th ed., Vol. 1, Ch. 4, Problem 28, p. 92.

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## Evaluation and physical checks

**Impact speed (b).** $v_x = 21.0\,\mathrm{m/s}$, $v_y = 36.373 - 9.8(5.50) = -17.527\,\mathrm{m/s}$. The speed is $\sqrt{21^2 + (-17.527)^2} = 27.35\,\mathrm{m/s}$, and the negative y component confirms descent.

## Interpretation and verification

**Highest point (c).** $t_A = v_{0y}/g = 3.7115\,\mathrm{s} < 5.50\,\mathrm{s}$, so apex occurs before contact. $H = v_{0y}^2/(2g) = 67.5\,\mathrm{m}$. The curve intersects the cliff below its highest position, consistent with the figure.

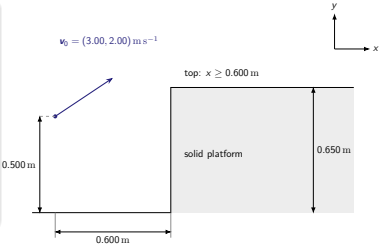
**Source:** Halliday, Resnick & Walker, *Fundamentals of Physics*, 12th ed., Vol. 1, Ch. 4, Problem 28, p. 92.

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# P12 — Platform edge or riser: first contact

## Problem statement

A bead launches from $(0, 0.500)\,\mathrm{m}$ at $\mathbf{v}_0 = (3.00, 2.00)\,\mathrm{m/s}$. Ground at $y = 0$ extends to $x = 0.600\,\mathrm{m}$; a vertical riser there reaches $0.650\,\mathrm{m}$, and the platform top for $x \geq 0.600\,\mathrm{m}$ is at $y = 0.650\,\mathrm{m}$. Find the first contact, checking the riser and the platform's domain.



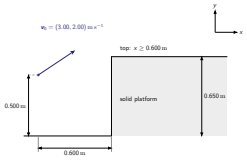

**Source:** Khanpour, *Physics I, Chapter 4, Final*, Problem 16, printed p. 52–53.

## Set-up and governing equations

**Flight before contact.** $x = 3.00t$ and $y = 0.500 + 2.00t - 4.905t^2$. The first possible riser event is fixed by its x-coordinate, $t_r = 0.600/3.00 = 0.200\,\mathrm{s}$.

## Derivation and method

**Check the riser.** At that time $y(t_r) = 0.500 + 0.400 - 4.905(0.0400) = 0.7038\,\mathrm{m} > 0.650\,\mathrm{m}$. The bead clears the vertical face; before it, the point stays above the floor.




**Source:** Khanpour, *Physics I, Chapter 4, Final*, Problem 16, printed p. 52–53.

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## Evaluation and physical checks

**Check platform-height roots.** Solve $0.650 = 0.500 + 2t - 4.905t^2$. The two roots are 0.0990719 and 0.3086753 s. At the earlier root $x = 0.2972\,\mathrm{m} < 0.600\,\mathrm{m}$, where the platform does not exist.

## Interpretation and verification

**First real contact.** The later root occurs at $x = 3(0.3086753) = 0.926026\,\mathrm{m}$. Therefore $\boxed{(t_f, x_f, y_f) = (0.308675\,\mathrm{s}, 0.926026\,\mathrm{m}, 0.650\,\mathrm{m})}$. Checking a height without checking the surface domain leads to a false earlier collision.

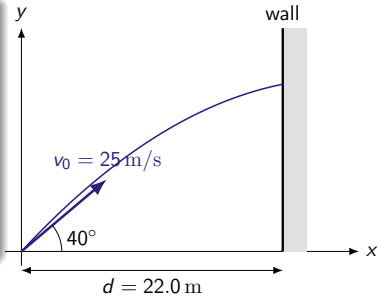
**Source:** Khanpour, *Physics I, Chapter 4, Final*, Problem 16, printed p. 52–53.

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# P13 — Throwing a ball toward a wall

## Problem statement

A ball leaves the thrower at speed $25.0\,\mathrm{m/s}$, launched $40.0^\circ$ above horizontal toward a wall $d = 22.0\,\mathrm{m}$ away. Find (a) how far above its release level it strikes the wall, (b) its horizontal velocity, (c) its vertical velocity at the wall, and (d) whether the ball has already passed the apex. Use $g = 9.8\,\mathrm{m/s^2}$.




**Source:** Halliday, Resnick & Walker, *Fundamentals of Physics*, 12th ed., Vol. 1, Ch. 4, Problem 32, p. 92.

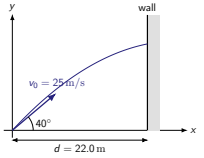
# P13 — Worked solution 1 of 2

## Set-up and governing equations

**Find the wall-plane event.** The given horizontal location fixes $t_w = d/(v_0 \cos\theta) = 22/[25\cos 40^\circ] = 1.14876\,\mathrm{s}$. This is not a height-root calculation.

## Derivation and method

**Vertical rise (a).** $\Delta y = (25\sin 40^\circ)t_w - \frac{1}{2}(9.8)t_w^2 = 11.994\,\mathrm{m}$. The actual contact is near $12.0\,\mathrm{m}$ above release, assuming the wall is tall enough.




**Source:** Halliday, Resnick & Walker, *Fundamentals of Physics*, 12th ed., Vol. 1, Ch. 4, Problem 32, p. 92.

## Evaluation and physical checks

**Velocity components (b,c).** $v_x = 25\cos 40^\circ = 19.151\,\mathrm{m/s}$ stays constant; $v_y = 25\sin 40^\circ - 9.8t_w = +4.812\,\mathrm{m/s}$ at contact.

## Interpretation and verification

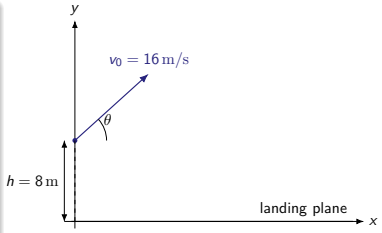
**Apex test (d).** $t_A = (25\sin 40^\circ)/9.8 = 1.640\,\mathrm{s}$. Since $t_w < t_A$ and $v_y > 0$, the ball is *still rising*. Substitution into $x(t_w)$ returns 22.0 m and checks the same event.

**Source:** Halliday, Resnick & Walker, *Fundamentals of Physics*, 12th ed., Vol. 1, Ch. 4, Problem 32, p. 92.

# P14 — Optimal range for an elevated launch

## Problem statement

A projectile leaves height $h = 8.00\,\mathrm{m}$ above a level horizontal landing plane with fixed launch speed $v_0 = 16.0\,\mathrm{m/s}$. In the uniform-gravity, no-drag model, find (a) launch angle maximizing range, (b) maximum range, and (c) compare with a 45.0° launch. Explain why the familiar equal-height optimum does not apply.




**Source:** Khanpour, *Physics I, Chapter 4, Final*, Example 4.9, printed p. 15–16.

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## Set-up and governing equations

**Event and objective.** The floor time is the positive root of $0 = h + v_0 \sin\theta\, t - \frac{1}{2} g t^2$. Range is $R(\theta) = \frac{v_0 \cos\theta}{g}[v_0 \sin\theta + \sqrt{v_0^2 \sin^2\theta + 2gh}]$.

## Derivation and method

**Optimize with a fixed speed.** The geometric envelope or differentiation gives $\tan\theta_{\text{opt}} = v_0/\sqrt{v_0^2 + 2gh}$. Thus $\theta_{\text{opt}} = \tan^{-1}[16/\sqrt{256 + 2(9.81)(8)}] = 38.215°$.

**Source:** Khanpour, *Physics I, Chapter 4, Final*, Example 4.9, printed p. 15–16.

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## Evaluation and physical checks

**Maximum range.** $R_{\max} = \frac{v_0}{g}\sqrt{v_0^2 + 2gh} = 33.1440\,\mathrm{m}$. Inserting the optimal angle in the floor event gives $t_f \approx 2.6365\,\mathrm{s}$.

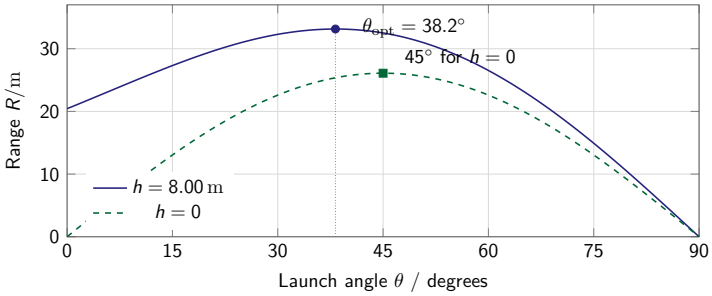
## Interpretation and verification

**Compare** 45°**.** Directly substituting 45° into the unequal-height range yields $R_{45} = 32.5162\,\mathrm{m}$, $0.628\,\mathrm{m}$ shorter. The familiar formula $v_0^2/g$ refers instead to landing at the launch height; no energy or force law is required.

**Source:** Khanpour, *Physics I, Chapter 4, Final*, Example 4.9, printed p. 15–16.

## Scientific interpretation of the completed figure

The optimum landing range for an elevated release occurs below 45°. The plotted angular dependence must use the floor-contact root; the familiar equal-height curve represents a different landing event.

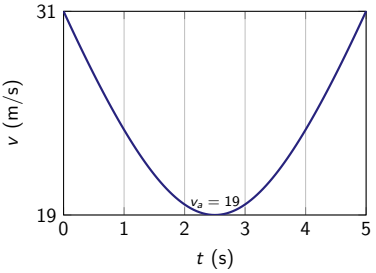



**Source:** Khanpour, *Physics I, Chapter 4, Final*, Example 4.9, printed p. 15–16.

# P15 — Golf ball speed graph: range and apex

## Problem statement

A golf ball is struck from level ground. The plotted speed $v(t)$ decreases from $v_b = 31\,\mathrm{m/s}$ to $v_a = 19\,\mathrm{m/s}$ at the middle of flight and returns to $v_b$ at $t \simeq 5\,\mathrm{s}$. From the speed-versus-time graph, find (a) horizontal distance before returning to the ground and (b) maximum height. Assume ideal no-drag projectile motion and $g = 9.8\,\mathrm{m/s^2}$.




**Source:** Halliday, Resnick & Walker, *Fundamentals of Physics*, 12th ed., Vol. 1, Ch. 4, Problem 38, p. 93.

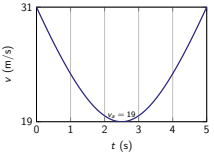
# P15 — Worked solution 1 of 2

## Set-up and governing equations

**Interpret the minimum.** In no-drag flight, the horizontal component is constant. At the apex $v_y = 0$, so the speed minimum is exactly $v_x = v_a = 19\,\mathrm{m/s}$. The launch speed is $v_b = 31\,\mathrm{m/s}$.

## Derivation and method

**Recover launch vertical component.** $v_{0y} = \sqrt{v_b^2 - v_a^2} = \sqrt{31^2 - 19^2} = \sqrt{600} = 24.495\,\mathrm{m/s}$. The return-to-height time is $T = 2v_{0y}/g = 4.999\,\mathrm{s}$, agreeing with graph endpoints.




**Source:** Halliday, Resnick & Walker, *Fundamentals of Physics*, 12th ed., Vol. 1, Ch. 4, Problem 38, p. 93.

AGH

## Evaluation and physical checks

**Horizontal range (a).** $R = v_x T = (19)(4.999) = 94.98\,\mathrm{m}$, about $95\,\mathrm{m}$. The area under the speed graph is *distance*, not horizontal range; use the constant horizontal component.

## Interpretation and verification

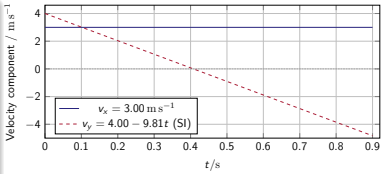
**Apex height (b).** $H = v_{0y}^2/(2g) = 600/19.6 = 30.61\,\mathrm{m}$. Check: the speed curve is symmetric around the apex; it does not cross zero because $v_x$ remains positive.

**Source:** Halliday, Resnick & Walker, *Fundamentals of Physics*, 12th ed., Vol. 1, Ch. 4, Problem 38, p. 93.

# P16 — Read a projectile velocity-component graph

## Problem statement

A projectile begins at height $y_0 = 0.200\,\mathrm{m}$. Its component graph gives $v_x = 3.00\,\mathrm{m/s}$ and $v_y(t) = 4.00 - 9.81t$ (SI). (a) Find height at $t = 0.400\,\mathrm{s}$ from signed area. (b) Find apex time and speed. (c) Find first contact with a level floor $y = 0$, stating the actual free-flight graph domain.



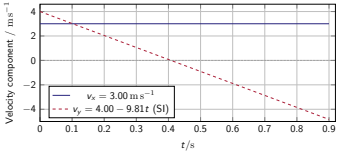

**Source:** Khanpour, *Physics I, Chapter 4, Final*, Problem 35, printed p. 55.

## Set-up and governing equations

**Signed graph area (a).** $\Delta y = \int_0^{.400} (4 - 9.81t)dt = 4(.400) - \frac{1}{2}(9.81)(.400)^2 = 0.8152\,\mathrm{m}$. Add initial height: $y(.400) = 1.0152\,\mathrm{m}$.

## Derivation and method

**Apex (b).** Set $v_y = 0$: $t_A = 4/9.81 = 0.407747\,\mathrm{s}$. Horizontal speed stays $3.00\,\mathrm{m/s}$, so total apex speed is $3.00\,\mathrm{m/s}$, even though vertical speed vanishes.




**Source:** Khanpour, *Physics I, Chapter 4, Final*, Problem 35, printed p. 55.

## Evaluation and physical checks

**Floor event (c).** $0 = 0.200 + 4t - 4.905t^2$, so $t_f = [4 + \sqrt{16 + 3.924}]/9.81 = 0.862755\,\mathrm{s}$. Discard the negative prelaunch root.

## Interpretation and verification

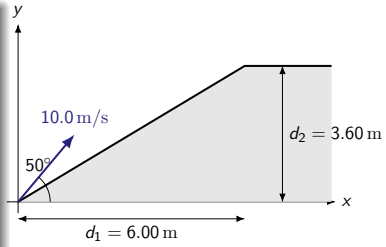
**Physical graph domain.** Ideal projectile flight occupies $0 \leq t \leq 0.862755\,\mathrm{s}$. The plotted straight line for $v_y$ may be extrapolated after that moment only as a hypothetical no-contact continuation. Integrating the signed area is not the same as integrating absolute speed.

**Source:** Khanpour, *Physics I, Chapter 4, Final*, Problem 35, printed p. 55.

# P17 — Ball approaching a ramp and plateau

## Problem statement

A ball launches from the foot of a ramp at speed $10.0\,\mathrm{m/s}$, $50.0^\circ$ above horizontal. The ramp rises linearly from $(0, 0)$ to $(d_1, d_2) = (6.00, 3.60)\,\mathrm{m}$; a horizontal plateau continues to the right. Determine (a) whether the ball lands on the ramp or plateau and (b,c) the magnitude and angle of landing displacement from launch. Use $g = 9.8\,\mathrm{m/s^2}$.



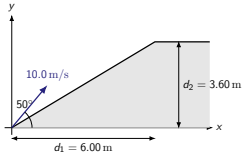

**Source:** Halliday, Resnick & Walker, *Fundamentals of Physics*, 12th ed., Vol. 1, Ch. 4, Problem 45, p. 93.

## Set-up and governing equations

**Projectile and surface equations.** $y_p(x) = x\tan 50^\circ - \frac{9.8x^2}{2(10\cos 50^\circ)^2}$. For $0 \le x \le 6$, ramp height is $y_r(x) = (3.6/6)x = 0.600x$; for $x \ge 6$, plateau height is $3.60\,\mathrm{m}$.

## Derivation and method

**Test the riser/ramp first.** At $x = 6$, the projectile has $y_p(6) \approx 2.88\,\mathrm{m}$, *below* the ramp top $3.60\,\mathrm{m}$. Therefore it must intersect the sloping ramp before reaching its crest.




**Source:** Halliday, Resnick & Walker, *Fundamentals of Physics*, 12th ed., Vol. 1, Ch. 4, Problem 45, p. 93.

AGH

## Evaluation and physical checks

**Solve intersection.** Set $y_p(x) = 0.600x$ for nonzero x: $x = (\tan 50^\circ - 0.600)\,2(10\cos 50^\circ)^2/9.8 \approx 4.99\,\mathrm{m}$. Ramp impact height is $y = 0.600x \approx 2.99\,\mathrm{m}$, both inside the ramp domain.

## Interpretation and verification

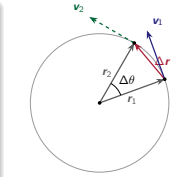
**Displacement (b,c).** $d = \sqrt{x^2+y^2} = x\sqrt{1+0.6^2} \approx 5.81\,\mathrm{m}$. Direction is $\tan^{-1}(0.600) = 30.96^\circ$ above horizontal. A mathematically later plateau intersection would not be physical because the ramp was struck first.

**Source:** Halliday, Resnick & Walker, *Fundamentals of Physics*, 12th ed., Vol. 1, Ch. 4, Problem 45, p. 93.

# P18 — Uniform circular motion on a disk

## Problem statement

A marker on a horizontal disk has $R = 0.300\,\mathrm{m}$, constant counterclockwise angular rate $\omega = 4.00\,\mathrm{rad/s}$, and starts on $+x$. Find its position, velocity and acceleration at $t = \pi/12\,\mathrm{s}$; compare vector directions at $\theta = 0$ and $120°$. Find speed, acceleration magnitude and period.




(a) Position chord and local tangents




(b) Velocity-space difference $\boldsymbol{v}_2 = \boldsymbol{v}_1 + \Delta\boldsymbol{v}$

**Source:** Khanpour, *Physics I, Chapter 4, Final*, Example 4.11, printed p. 19–20.

# P18 — Worked solution 1 of 2

## Set-up and governing equations

**Fixed-centre model.** $\theta = \omega t$; $\mathbf{r} = R(\cos\theta, \sin\theta)$, $\mathbf{v} = R\omega(-\sin\theta, \cos\theta)$, $\mathbf{a} = -R\omega^2(\cos\theta, \sin\theta)$. At the requested event $\theta = \pi/3$.

## Derivation and method

**At $60°$.** $\mathbf{r} = (0.150, 0.25981)\,\mathrm{m}$, $\mathbf{v} = (-1.03923, 0.600)\,\mathrm{m/s}$, $\mathbf{a} = (-2.400, -4.15692)\,\mathrm{m/s^2}$. Speed $R\omega = 1.20\,\mathrm{m/s}$, acceleration magnitude $R\omega^2 = 4.80\,\mathrm{m/s^2}$.

**Source:** Khanpour, *Physics I, Chapter 4, Final*, Example 4.11, printed p. 19–20.

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## Evaluation and physical checks

**Other locations.** At $0^\circ$: $\mathbf{v} = (0, +1.20)$, $\mathbf{a} = (-4.80, 0)$. At $120^\circ$: $\mathbf{v} = (-1.0392, -0.600)$, $\mathbf{a} = (+2.400, -4.1569)$, in their velocity/acceleration units.

## Interpretation and verification

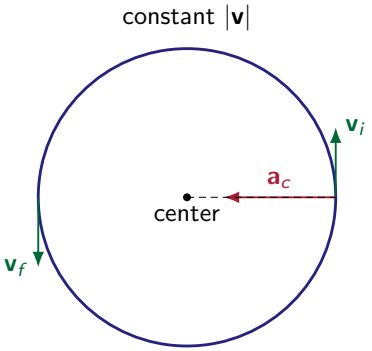
**Period and checks.** $T = 2\pi/|\omega| = \pi/2 = 1.5708\,\mathrm{s}$. At every point $\mathbf{r} \cdot \mathbf{v} = 0$, $\mathbf{v} \cdot \mathbf{a} = 0$ and $\mathbf{a} = -\omega^2 \mathbf{r}$. Constant scalar speed *does not* imply constant velocity vector.

**Source:** Khanpour, *Physics I, Chapter 4, Final*, Example 4.11, printed p. 19–20.

# P19 — Pilot's acceleration in a half-circle

## Problem statement

A fighter pilot enters a level circular turn with velocity $\mathbf{v}_i = (400, 500)\,\mathrm{m/s}$ and 24.0 s later leaves with opposite velocity $\mathbf{v}_f = (-400, -500)\,\mathrm{m/s}$. Assume constant-speed circular motion over a half-turn. What is the centripetal acceleration magnitude in units of $g = 9.8\,\mathrm{m/s^2}$?




**Source:** Halliday, Resnick & Walker, *Fundamentals of Physics*, 12th ed., Vol. 1, Ch. 4, Sample Problem 4.5.1, p. 83.

AGH

## Set-up and governing equations

**Use direction change correctly.** Opposite entry/exit velocities specify half a circular revolution, not complete flight or constant vector acceleration. The speed is $v = \sqrt{400^2 + 500^2} = 640.312\,\mathrm{m/s}$.

## Derivation and method

**Determine angular rate.** A half-turn is $\pi$ radians in $\Delta t = 24.0\,\mathrm{s}$, so $|\omega| = \pi/(24.0\,\mathrm{s})$, and full period would be $T = 48.0\,\mathrm{s}$. The radius is $R = v/|\omega| = v(24)/\pi$.

**Source:** Halliday, Resnick & Walker, *Fundamentals of Physics*, 12th ed., Vol. 1, Ch. 4, Sample Problem 4.5.1, p. 83.

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## Evaluation and physical checks

**Centripetal magnitude.** $a_c = v^2/R = v|\omega| = (640.312)\pi/24 = 83.82\,\mathrm{m/s^2}$. In gravitational units $a_c/g = 83.82/9.8 = 8.55$, about $8.6g$.

## Interpretation and verification

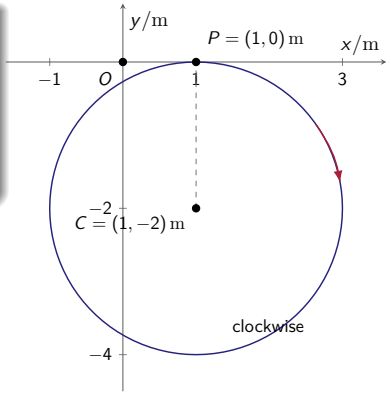
**Interpretation.** $a_c$ points inward and continuously changes direction. Dividing the total vector change $\|\Delta\mathbf{v}\| = 2v$ by 24 s yields the interval-average acceleration magnitude, not the instantaneous centripetal acceleration. The physiological discussion is illustrative, not a personalized medical assessment.

**Source:** Halliday, Resnick & Walker, *Fundamentals of Physics*, 12th ed., Vol. 1, Ch. 4, Sample Problem 4.5.1, p. 83.

# P20 — Uniform circle centred away from origin

## Problem statement

A circle is centred at $\mathbf{r}_C = (1.00, -2.00)\,\mathrm{m}$. At a certain event the point is at $\mathbf{r}_P = (1.00, 0)\,\mathrm{m}$, moving clockwise at constant speed $3.00\,\mathrm{m/s}$ (viewed from $+z$). Find $\mathbf{v}$ and $\mathbf{a}$. Explain why $\mathbf{a} = -\omega^2 \mathbf{r}_P$ would be wrong.




**Source:** Khanpour, *Physics I, Chapter 4, Final*, Problem 18, printed p. 53.

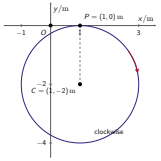
# P20 — Worked solution 1 of 2

## Set-up and governing equations

**Use the actual circle centre.** $\mathbf{r}_P - \mathbf{r}_C = (0, 2.00)\,\mathrm{m}$, so $R = 2.00\,\mathrm{m}$. The point is physically at the top of the circle, even though its laboratory y-coordinate is zero.

## Derivation and method

**Velocity orientation.** Viewed along $-z$ from $+z$, clockwise motion at the top has a tangent pointing to $+x$. Hence $\mathbf{v} = (3.00, 0)\,\mathrm{m/s}$.




**Source:** Khanpour, *Physics I, Chapter 4, Final*, Problem 18, printed p. 53.

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## Evaluation and physical checks

**Acceleration magnitude and direction.** $a_c = v^2/R = 9/2 = 4.50\,\mathrm{m/s^2}$ toward the centre, so $\mathbf{a} = (0, -4.50)\,\mathrm{m/s^2}$.

## Interpretation and verification

**Find the error.** $|\omega| = v/R = 1.50\,\mathrm{s^{-1}}$, and the correct formula is $\mathbf{a} = -\omega^2(\mathbf{r}_P - \mathbf{r}_C)$. Using $-\omega^2\mathbf{r}_P$ would point along x toward the laboratory origin, giving $(-2.25, 0)$, physically wrong.

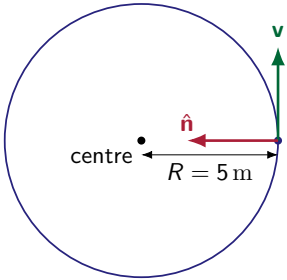
**Source:** Khanpour, *Physics I, Chapter 4, Final*, Problem 18, printed p. 53.

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# P21 — Speeding up while following a circular bend

## Problem statement

A point carriage follows a horizontal circular bend of radius $R_c = 5.00\,\mathrm{m}$. Over $0 \le t \le 5.00\,\mathrm{s}$, speed is $v(t) = 1.00 + 0.400t$ (SI units). At $t = 5.00\,\mathrm{s}$, it is at the rightmost point moving upward. Find tangential, inward-normal and vector accelerations, their magnitude and direction.




$v(t) = 1 + 0.4t$, evaluate at $t = 5\,\mathrm{s}$

**Source:** Khanpour, *Physics I, Chapter 4, Final*, Example 4.12, printed p. 21.

## Set-up and governing equations

**Geometry of the moving event.** On the circle's right side, forward tangent $\hat{\mathbf{t}} = +\hat{\jmath}$, inward normal $\hat{\mathbf{n}} = -\hat{\imath}$. These are orthogonal, so components must be combined as vectors.

## Derivation and method

**Separate acceleration effects.** Speed at the event is $v = 1.00 + 0.400(5) = 3.00\,\mathrm{m/s}$. Tangential component $a_t = dv/dt = +0.400\,\mathrm{m/s^2}$. Inward component $a_n = v^2/R_c = 9/5 = 1.80\,\mathrm{m/s^2}$.

**Source:** Khanpour, *Physics I, Chapter 4, Final*, Example 4.12, printed p. 21.

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## Evaluation and physical checks

**Vector result.** $\mathbf{a} = a_t\hat{\mathbf{t}} + a_n\hat{\mathbf{n}} = (-1.80\hat{\imath} + 0.400\hat{\jmath})\,\mathrm{m/s^2}$. Magnitude $\sqrt{1.80^2 + 0.400^2} = 1.8439\,\mathrm{m/s^2}$.

## Interpretation and verification

**Angle and checks.** Direction is $\tan^{-1}(1.80/0.400) = 77.5^\circ$ from the forward tangent toward the inward normal (or $12.5^\circ$ forward of inward). Scalar addition $1.80 + 0.400$ would be wrong. Directly differentiating $R(\cos\theta, \sin\theta)$ reproduces the two components.

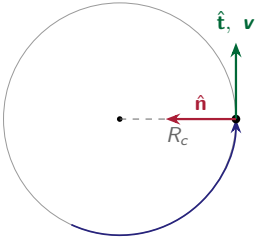
**Source:** Khanpour, *Physics I, Chapter 4, Final*, Example 4.12, printed p. 21.

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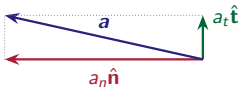
# P21 — Source diagram and completed construction

## Scientific interpretation of the completed figure

Tangential acceleration follows the direction of motion as speed increases, while the normal component points toward the bend centre. Their perpendicular vector sum gives the total acceleration rather than the sum of their magnitudes.




(a) Geometry at $t = 5\,\mathrm{s}$
direction arrows not to a common scale



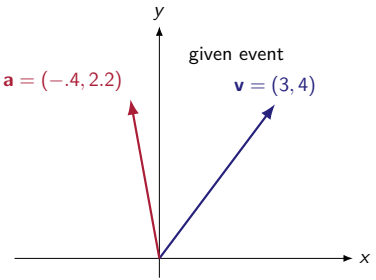

(b) Acceleration vectors
1.80 inward, 0.400 forward
in $\mathrm{m\,s^{-2}}$

**Source:** Khanpour, *Physics I, Chapter 4, Final*, Example 4.12, printed p. 21.

# P22 — Extract tangent, normal and curvature from vectors

## Problem statement

At a moving event $\mathbf{v} = (3.00, 4.00)\,\mathrm{m/s}$, $\mathbf{a} = (-0.400, 2.20)\,\mathrm{m/s^2}$. Find (a) signed tangential acceleration, (b) perpendicular acceleration vector, (c) normal magnitude and unit normal, (d) curvature radius. Check orthogonality and magnitude. What changes if $\mathbf{a}$ is reversed but $\mathbf{v}$ is not?




**Source:** Khanpour, *Physics I, Chapter 4, Final*, Problem 20, printed p. 54; solution 63–64.

## Set-up and governing equations

**Find forward tangent.** $v = \sqrt{3^2 + 4^2} = 5.00\,\mathrm{m/s}$; $\hat{\mathbf{t}} = \mathbf{v}/v = (0.6, 0.8)$. Project $a_t = \mathbf{a} \cdot \hat{\mathbf{t}} = (-0.4)(0.6) + (2.2)(0.8) = +1.52\,\mathrm{m/s^2}$.

## Derivation and method

**Isolate normal component.** $a_t\hat{\mathbf{t}} = (0.912, 1.216)\,\mathrm{m/s^2}$; hence $\mathbf{a}_\perp = (-0.400, 2.20) - (0.912, 1.216) = (-1.312, 0.984)\,\mathrm{m/s^2}$.

**Source:** Khanpour, *Physics I, Chapter 4, Final*, Problem 20, printed p. 54; solution 63–64.

## Evaluation and physical checks

**Calculate radius.** $a_n = \|\mathbf{a}_\perp\| = 1.64\,\mathrm{m/s^2}$, $\hat{\mathbf{n}} = (-0.8, 0.6)$, $R_c = v^2/a_n = 25/1.64 = 15.244\,\mathrm{m}$. The unit normal is toward local curvature, not generally toward a coordinate origin.

## Interpretation and verification

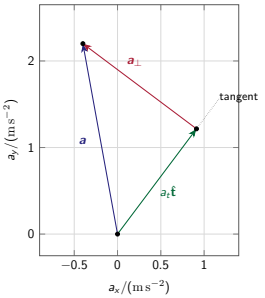
**Checks and reversal.** $\mathbf{v} \cdot \mathbf{a}_\perp = 0$ and $a_t^2 + a_n^2 = 1.52^2 + 1.64^2 = 5 = \|\mathbf{a}\|^2$. With only acceleration reversed, $a_t \to -1.52$, $\mathbf{a}_\perp \to -\mathbf{a}_\perp$, $\hat{\mathbf{n}} \to -\hat{\mathbf{n}}$; magnitudes and $R_c$ stay unchanged.

**Source:** Khanpour, *Physics I, Chapter 4, Final*, Problem 20, printed p. 54; solution 63–64.

# P22 — Source diagram and completed construction

## Scientific interpretation of the completed figure

The acceleration triangle resolves the measured Cartesian acceleration into a component along the unit velocity and a perpendicular part. Projection gives the signed speed-change rate; the remaining part sets the local radius of curvature.

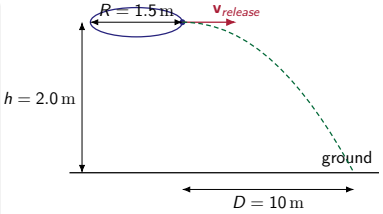



**Source:** Khanpour, *Physics I, Chapter 4, Final*, Problem 20, printed p. 54; solution 63–64.

# P23 — Released from circular motion: stone on a string

## Problem statement

A boy swings a stone in a horizontal circle of radius $1.5\,\mathrm{m}$, at a height of $2.0\,\mathrm{m}$ above level ground. The string breaks, so the stone leaves horizontally and lands $10\,\mathrm{m}$ away horizontally. Find the centripetal acceleration magnitude during the previous uniform circular motion. Use $g = 9.8\,\mathrm{m/s^2}$.



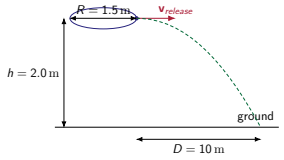

**Source:** Halliday, Resnick & Walker, *Fundamentals of Physics*, 12th ed., Vol. 1, Ch. 4, Problem 67, p. 95.

## Set-up and governing equations

**Two phases, one initial speed.** At the release instant, the stone has horizontal *tangential* speed $v$; it is not shot radially outward. After release the vertical motion starts with zero vertical velocity.

## Derivation and method

**Find fall time.** From height $h = 2.0\,\mathrm{m}$, $0 = h - \frac{1}{2}gt_f^2$, giving $t_f = \sqrt{2h/g} = \sqrt{4/9.8} = 0.63888\,\mathrm{s}$.




**Source:** Halliday, Resnick & Walker, *Fundamentals of Physics*, 12th ed., Vol. 1, Ch. 4, Problem 67, p. 95.

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## Evaluation and physical checks

**Recover release speed.** The horizontal displacement during the fall is $D = vt_f$, so $v = D/t_f = 10/0.63888 = 15.65\,\mathrm{m/s}$. This speed is the same magnitude as during circular motion immediately before release.

## Interpretation and verification

**Centripetal acceleration.** $a_c = v^2/R = 15.65^2/1.5 = 163.33\,\mathrm{m/s^2}$ (approximately $1.6 \times 10^2\,\mathrm{m/s^2}$). Check direct elimination: $a_c = D^2 g/(2hR) = 100(9.8)/(2 \cdot 2 \cdot 1.5)$.

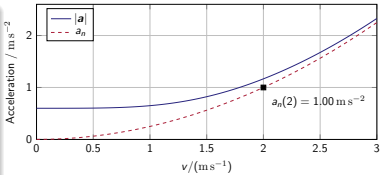
**Source:** Halliday, Resnick & Walker, *Fundamentals of Physics*, 12th ed., Vol. 1, Ch. 4, Problem 67, p. 95.

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# P24 — Acceleration limit on a prescribed bend

## Problem statement

A point travels a bend of constant curvature radius, with signed tangential acceleration $a_t = -0.600\,\mathrm{m/s^2}$. The supplied acceleration graph specifies $a_n = 1.00\,\mathrm{m/s^2}$ at $v = 2.00\,\mathrm{m/s}$. (a) Infer radius. (b) Find maximum speed if total acceleration cannot exceed $1.50\,\mathrm{m/s^2}$. (c) Explain why the acceleration magnitudes do not add linearly.



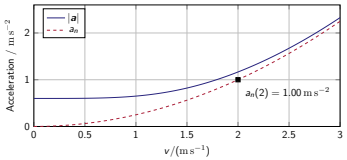

**Source:** Khanpour, *Physics I, Chapter 4, Final*, Problem 36, printed p. 55–56.

## Set-up and governing equations

**Decode the graph (a).** For a known circular bend $a_n = v^2/R_c$. Thus at $v = 2.00\,\mathrm{m/s}$, $R_c = v^2/a_n = 4.00/1.00 = 4.00\,\mathrm{m}$. The normal component depends quadratically on speed.

## Derivation and method

**Apply the total bound (b).** Tangent and inward normal are perpendicular: $\|\mathbf{a}\|^2 = a_t^2 + (v^2/R_c)^2 \le (1.50)^2$, yielding $0.600^2 + v^4/4.00^2 \le 1.50^2$.




**Source:** Khanpour, *Physics I, Chapter 4, Final*, Problem 36, printed p. 55–56.

## Evaluation and physical checks

**Solve for speed.** $v^4 \le 16(1.50^2 - 0.600^2) = 30.24$. Hence $v \le (30.24)^{1/4} = 2.345\,\mathrm{m/s}$. At equality, $a_n = \sqrt{2.25 - 0.36} = 1.3748\,\mathrm{m/s^2}$.

## Interpretation and verification

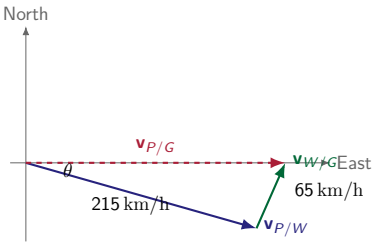
**Interpret (c).** The $-0.600$ tangential component points backward while the normal component points inward; their geometric sum has magnitude 1.50, not $0.600 + 1.3748$. This is a kinematic constraint, not proof that a real motor or tire can meet it.

**Source:** Khanpour, *Physics I, Chapter 4, Final*, Problem 36, printed p. 55–56.

# P25 — Airplane heading through a steady wind

## Problem statement

A plane travels due east relative to the ground. Airspeed is 215 km/h at heading $\theta$ south of east. A steady wind blows at 65.0 km/h, direction 20.0° east of north. Determine (a) the heading angle $\theta$ and (b) the plane's ground speed.




**Source:** Halliday, Resnick & Walker, *Fundamentals of Physics*, 12th ed., Vol. 1, Ch. 4, Sample Problem 4.7.1, p. 87.

# P25 — Worked solution 1 of 2

## Set-up and governing equations

**Name the relative velocities.** $\mathbf{v}_{P/G} = \mathbf{v}_{P/W} + \mathbf{v}_{W/G}$. Take x east, y north. Wind has components $(65\sin 20^\circ, 65\cos 20^\circ)$. Aircraft through air has components $(215\cos\theta, -215\sin\theta)$.

## Derivation and method

**Ground y-component must vanish (a).** $0 = -215\sin\theta + 65\cos 20^\circ$ implies $\theta = \arcsin[(65/215)\cos 20^\circ] = 16.505^\circ$ south of east.

**Source:** Halliday, Resnick & Walker, *Fundamentals of Physics*, 12th ed., Vol. 1, Ch. 4, Sample Problem 4.7.1, p. 87.

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## Evaluation and physical checks

**Ground speed (b).** Due-east ground motion has $v_G = (215\cos 16.505^\circ) + (65\sin 20^\circ) = 228.373\,\mathrm{km/h}$. Since its north component cancels, its magnitude equals this x-component.

## Interpretation and verification

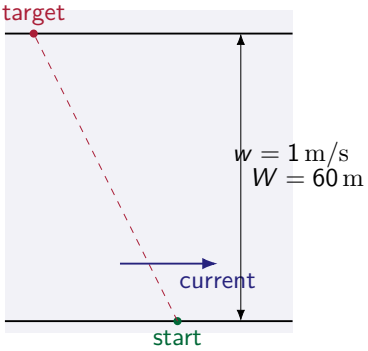
**Consistency.** $215\sin 16.505^\circ = 65\cos 20^\circ$ verifies no north/south drift. Ground velocity is not the sum of the two scalar speeds 215+65; the vectors have different bearings. The headings and ground track are distinct arrows.

**Source:** Halliday, Resnick & Walker, *Fundamentals of Physics*, 12th ed., Vol. 1, Ch. 4, Sample Problem 4.7.1, p. 87.

# P26 — Cross a channel to a point upstream

## Problem statement

An ideal craft crosses a 60.0 m wide channel to a point 30.0 m upstream of its start. Uniform current 1.00 m/s flows downstream, and craft speed relative to water is 2.00 m/s. Find a constant water-relative heading and arrival time. Use x downstream and y across; assume no course changes.




**Source:** Khanpour, *Physics I, Chapter 4, Final*, Example 4.16, printed p. 28–29.

AGH

## Set-up and governing equations

**Vector composition.** Destination is $\Delta\mathbf{r} = (-30, 60)\,\mathrm{m}$. If travel time is $T > 0$, ground velocity is $(-30/T, 60/T)$, and boat-through-water velocity is $(-30/T - 1, 60/T)$, all in m/s.

## Derivation and method

**Impose craft speed.** $(-30/T-1)^2+(60/T)^2 = 2^2$. Multiply by $T^2$ and simplify: $3T^2 - 60T - 4500 = 0 = 3(T - 50)(T + 30)$. The negative time is inadmissible.

**Source:** Khanpour, *Physics I, Chapter 4, Final*, Example 4.16, printed p. 28–29.

AGH

## Evaluation and physical checks

**Arrival and headings.** $T = 50.0\,\mathrm{s}$. Ground velocity $(-0.600, 1.200)\,\mathrm{m/s}$, through-water velocity $(-1.600, 1.200)\,\mathrm{m/s}$, whose speed is 2.00. Heading is $\arctan(1.600/1.200) = 53.13^\circ$ upstream of straight across.

## Interpretation and verification

**Check.** $50(-0.6, 1.2) = (-30, 60)\,\mathrm{m}$. Ground track is $26.57^\circ$ upstream of across, not identical to the boat's water-relative heading. The current adds to vector velocity, not to the desired displacement without multiplying by time.

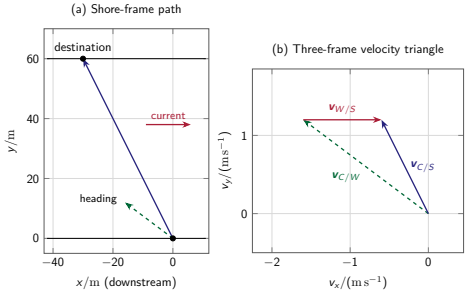
**Source:** Khanpour, *Physics I, Chapter 4, Final*, Example 4.16, printed p. 28–29.

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# P26 — Source diagram and completed construction

## Scientific interpretation of the completed figure

The shore-frame heading is the vector sum of the craft velocity relative to water and the downstream current. The destination fixes both displacement components at one common arrival time.

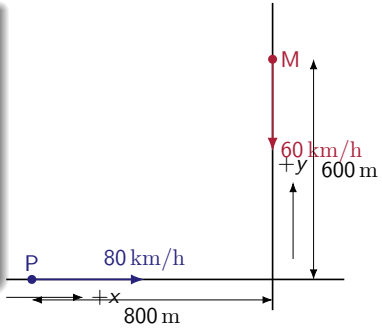



**Source:** Khanpour, *Physics I, Chapter 4, Final*, Example 4.16, printed p. 28–29.

# P27 — Two cars approach the same intersection

## Problem statement

A police car P is 800 m west of an intersection, moving east at 80 km/h. A motorist M is 600 m north of the intersection, moving south at 60 km/h (as in Fig. 4.26). (a) Find velocity of M relative to P as a vector. (b) Find its angle with the line of sight from P to M at that instant. (c) Do those answers change as they approach if both retain constant velocities?




**Source:** Halliday, Resnick & Walker, *Fundamentals of Physics*, 12th ed., Vol. 1, Ch. 4, Problem 73, p. 95.

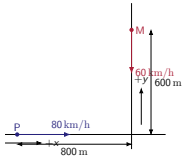
# P27 — Worked solution 1 of 2

## Set-up and governing equations

**Place both on east–north coordinates.** $\mathbf{r}_P = (-800, 0)\,\mathrm{m}$, $\mathbf{r}_M = (0, 600)\,\mathrm{m}$. Their velocities are $\mathbf{v}_P = (80, 0)$, $\mathbf{v}_M = (0, -60)$ in km/h.

## Derivation and method

**Relative velocity (a).** $\mathbf{v}_{M/P} = \mathbf{v}_M - \mathbf{v}_P = (-80, -60)\,\mathrm{km/h}$, magnitude 100 km/h. The relative position $\mathbf{r}_{M/P} = (800, 600)\,\mathrm{m}$.




**Source:** Halliday, Resnick & Walker, *Fundamentals of Physics*, 12th ed., Vol. 1, Ch. 4, Problem 73, p. 95.

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## Evaluation and physical checks

**Line-of-sight angle (b).** The relative velocity is exactly antiparallel to the line-of-sight vector: $(-80, -60) \propto -(800, 600)$, so the angle is $180°$. The separation decreases along its line of sight.

## Interpretation and verification

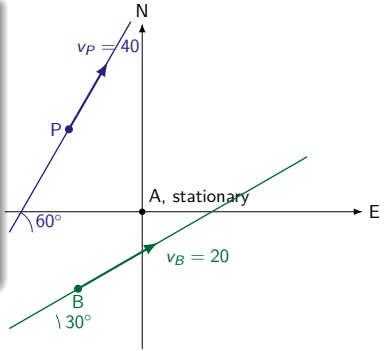
**Invariance (c).** Both velocity vectors remain constant, so their difference remains $(-80, -60)$. Before reaching the intersection, relative position remains collinear with that velocity. Both travel times to the intersection are 36.0 s; at their exact meeting the line-of-sight direction is undefined because separation vanishes.

**Source:** Halliday, Resnick & Walker, *Fundamentals of Physics*, 12th ed., Vol. 1, Ch. 4, Problem 73, p. 95.

# P28 — Two accelerating and uniform-motion jeeps

## Problem statement

In the overhead diagram, east is $+x$, north is $+y$. Jeep B moves at constant $20.0\,\mathrm{m/s}$, $30.0^\circ$ north of east. Jeep P, starting from rest, accelerates at constant $0.400\,\mathrm{m/s^2}$, $60.0^\circ$ north of east. At the instant P reaches speed $40.0\,\mathrm{m/s}$, find (a,b) magnitude and direction of P's velocity relative to B and (c,d) relative acceleration magnitude and direction.



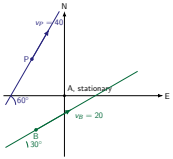

**Source:** Halliday, Resnick & Walker, *Fundamentals of Physics*, 12th ed., Vol. 1, Ch. 4, Problem 78, p. 96.

## Set-up and governing equations

**Resolve headings.** At the specified instant $\mathbf{v}_P = 40(\cos 60^\circ, \sin 60^\circ) = (20.0, 34.641)\,\mathrm{m/s}$; $\mathbf{v}_B = 20(\cos 30^\circ, \sin 30^\circ) = (17.321, 10.0)\,\mathrm{m/s}$.

## Derivation and method

**Relative velocity (a,b).** $\mathbf{v}_{P/B} = \mathbf{v}_P - \mathbf{v}_B = (2.6795, 24.6410)\,\mathrm{m/s}$. Speed $24.786\,\mathrm{m/s}$. Direction $\mathrm{atan2}(24.641, 2.6795) = 83.79^\circ$ north of east.




**Source:** Halliday, Resnick & Walker, *Fundamentals of Physics*, 12th ed., Vol. 1, Ch. 4, Problem 78, p. 96.

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## Evaluation and physical checks

**Relative acceleration (c,d).** Since Jeep B has constant velocity, $\mathbf{a}_B = 0$. Therefore $\mathbf{a}_{P/B} = \mathbf{a}_P = 0.400(\cos 60^\circ, \sin 60^\circ) = (0.200, 0.3464)\,\mathrm{m/s^2}$, magnitude 0.400 at 60° north of east.

## Interpretation and verification

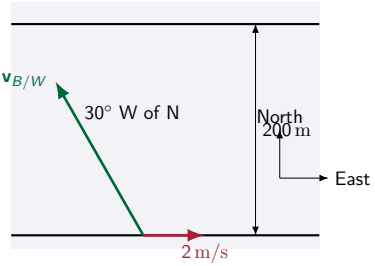
**Checks.** Relative acceleration cannot be obtained by subtracting speeds, and its direction need not align with relative velocity. The velocities share units of m/s, whereas acceleration units are $\mathrm{m/s^2}$. The source angles are measured above the east–west axis, not from north.

**Source:** Halliday, Resnick & Walker, *Fundamentals of Physics*, 12th ed., Vol. 1, Ch. 4, Problem 78, p. 96.

# P29 — River crossing: heading versus ground track

## Problem statement

A 200-m-wide river flows due east at $2.0\,\mathrm{m/s}$. A boat with water-relative speed $8.0\,\mathrm{m/s}$ leaves the southern bank pointed $30^\circ$ west of north. Find (a) ground-speed magnitude, (b) ground-track direction, and (c) crossing time. Treat river flow as uniform and the heading constant.



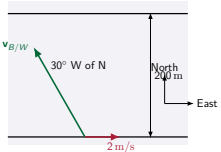

**Source:** Halliday, Resnick & Walker, *Fundamentals of Physics*, 12th ed., Vol. 1, Ch. 4, Problem 80, p. 96.

## Set-up and governing equations

**Set east–north axes.** Water velocity relative to ground is $\mathbf{v}_{W/G} = (2.0, 0)\,\mathrm{m/s}$. Boat through water has $\mathbf{v}_{B/W} = (-8\sin 30^\circ, 8\cos 30^\circ) = (-4.0, 6.928)\,\mathrm{m/s}$.

## Derivation and method

**Add velocities (a).** $\mathbf{v}_{B/G} = (-2.0, 6.928)\,\mathrm{m/s}$. Its magnitude is $\sqrt{(-2)^2 + (8\cos 30^\circ)^2} = \sqrt{52} = 7.211\,\mathrm{m/s}$.




**Source:** Halliday, Resnick & Walker, *Fundamentals of Physics*, 12th ed., Vol. 1, Ch. 4, Problem 80, p. 96.

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## Evaluation and physical checks

**Ground bearing (b).** The north component is positive and east component negative, so the track points $\tan^{-1}(2/6.928) = 16.10^\circ$ west of north. It differs from the $30^\circ$ water-relative heading because the eastward current partly cancels the upstream component.

## Interpretation and verification

**Crossing time (c).** The northward ground velocity is $v_y = 6.928\,\mathrm{m/s}$, so $T = 200/6.928 = 28.87\,\mathrm{s}$. Check vertical displacement $v_y T = 200\,\mathrm{m}$ and westward drift $v_x T \approx -57.7\,\mathrm{m}$.

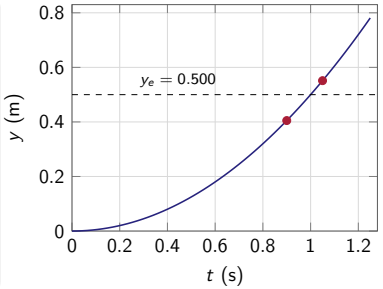
**Source:** Halliday, Resnick & Walker, *Fundamentals of Physics*, 12th ed., Vol. 1, Ch. 4, Problem 80, p. 96.

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# P30 — Midpoint motion algorithm and interpolated event

## Problem statement

For the prescribed motion with $\mathbf{a}(t) = ((0.600\,\mathrm{m/s^3})t, 1.00\,\mathrm{m/s^2})$, $\mathbf{r}_0 = 0$, $\mathbf{v}_0 = (1.00, 0)\,\mathrm{m/s}$: (a) calculate midpoint-scheme $x(1.20\,\mathrm{s})$ for $h = 0.150\,\mathrm{s}$; (b) find the upward crossing of $y = 0.500\,\mathrm{m}$ by linear interpolation; (c) derive forward-Euler position error for any constant vector acceleration; (d) compare with midpoint and discuss event accuracy.




**Source:** Khanpour, *Physics I, Chapter 4, Final*, Problem 25, printed p. 54; related Example 4.21, 35–36.

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## Set-up and governing equations

**Analytic reference and nodes (a).** Exact path has $x = t + 0.1t^3$, $y = 0.5t^2$ in SI numerical units. Midpoint nodes obey $x_n = t_n + 0.1t_n^3 - 0.1t_n h^2$. At 1.20 s with $h = .150$, $x_n = 1.370100\,\mathrm{m}$ versus exact $1.372800\,\mathrm{m}$.

## Derivation and method

**Find bracket (b).** At $t = .900, 1.050\,\mathrm{s}$, $y = .405000, .551250\,\mathrm{m}$. Interpolation fraction $\lambda = (.500 - .405)/(.55125 - .405) = .64957265$. Then $t_e^{(h)} = .900 + .150\lambda = .99743590\,\mathrm{s}$.

**Source:** Khanpour, *Physics I, Chapter 4, Final*, Problem 25, printed p. 54; related Example 4.21, 35–36.

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## Evaluation and physical checks

**Interpolate x consistently.** Midpoint x nodes are $x_L = .970875\,\mathrm{m}$, $x_U = 1.163400\,\mathrm{m}$; thus $x_e^{(h)} = x_L + \lambda(x_U - x_L) = 1.095934\,\mathrm{m}$. Exact event at 1.000 s has $x = 1.100000\,\mathrm{m}$. The two interpolated errors are nonzero.

## Interpretation and verification

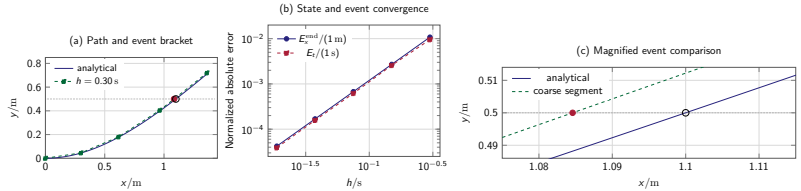
**Euler error and comparison (c,d).** For constant $\mathbf{a}$, forward Euler uses $\mathbf{r}_{n+1} = \mathbf{r}_n + h\mathbf{v}_n$ and $\mathbf{v}_{n+1} = \mathbf{v}_n + h\mathbf{a}$. Summing $\sum_{n=0}^{N-1} n = N(N-1)/2$, with $T = Nh$, yields $\mathbf{r}_N^E - \mathbf{r}(T) = -\frac{1}{2}\mathbf{a}Th$. Midpoint is exact at nodes for constant acceleration, yet linear interpolation of curved events is still approximate.

**Source:** Khanpour, *Physics I, Chapter 4, Final*, Problem 25, printed p. 54; related Example 4.21, 35–36.

# P30 — Source diagram and completed construction

## Scientific interpretation of the completed figure

The midpoint numerical path samples bracket the requested upward height crossing. Linear interpolation estimates an event between nodes; improving the state-grid accuracy does not by itself make the interpolated event exact.

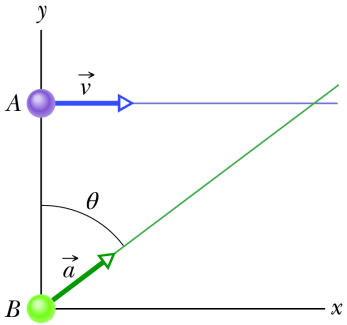



**Source:** Khanpour, *Physics I, Chapter 4, Final*, Problem 25, printed p. 54; related Example 4.21, 35–36.

# P31 — Halliday Problem 20

**Problem statement.** In this figure particle $A$ moves along the line $y = 30\,\mathrm{m}$ with a constant velocity $\vec{v}$ of magnitude $3.0\,\mathrm{m/s}$ and parallel to the $x$ axis. At the instant particle $A$ passes the $y$ axis, particle $B$ leaves the origin with a zero initial speed and a constant acceleration $\vec{a}$ of magnitude $0.40\,\mathrm{m/s^2}$. What angle $\theta$ between $\vec{a}$ and the positive direction of the $y$ axis would result in a collision?

*Source:* Halliday, Resnick, Walker, *Fundamentals of Physics*, Vol. 1, 12th ed., Ch. 4, Problem 20.



# P31 — Solution 1

Let $t = 0$ when particle $A$ passes the $y$ axis and particle $B$ starts from the origin.

$$A: \qquad x_A = vt = (3.0)t, \qquad y_A = 30.$$

If the acceleration of $B$ makes angle $\theta$ with the positive $y$ axis, then

$$a_x = a\sin\theta, \qquad\qquad a_y = a\cos\theta,$$

with $a = 0.40\,\mathrm{m/s^2}$. Since particle $B$ starts from rest,

$$x_B = \tfrac{1}{2}a_x t^2 = \tfrac{1}{2}(0.40\sin\theta)t^2 = 0.20\,t^2\sin\theta,$$
$$y_B = \tfrac{1}{2}a_y t^2 = \tfrac{1}{2}(0.40\cos\theta)t^2 = 0.20\,t^2\cos\theta.$$

For a collision, both coordinates must match at the same time:

$$0.20\,t^2\cos\theta = 30, \tag{1}$$
$$0.20\,t^2\sin\theta = 3.0t. \tag{2}$$

# P31 — Solution 2

From Eq. (2), with $t > 0$,

$$0.20\, t \sin\theta = 3.0 \quad \Rightarrow \quad t = \frac{15}{\sin\theta}.$$

From Eq. (1),

$$t^2 = \frac{150}{\cos\theta}.$$

Substitute $t = 15/\sin\theta$ into the second relation:

$$\frac{225}{\sin^2\theta} = \frac{150}{\cos\theta} \quad \Rightarrow \quad 225\cos\theta = 150\sin^2\theta.$$

Using $\sin^2\theta = 1 - \cos^2\theta$,

$$225\cos\theta = 150(1 - \cos^2\theta),$$

$$2\cos^2\theta + 3\cos\theta - 2 = 0.$$

This gives $\cos\theta = \frac{1}{2}$ (the other root is unphysical), so

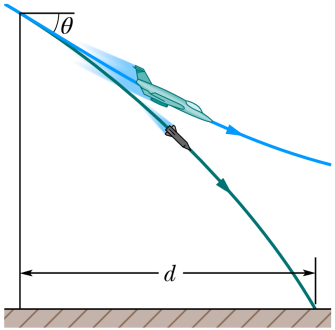
$$\boxed{\theta = 60.0^\circ}.$$

The corresponding collision time is $t = 15/\sin 60^\circ \approx 17.3\,\mathrm{s}$.

# P32 — Halliday Problem 27

**Problem statement.** A certain airplane has a speed of 290.0 km/h and is diving at an angle of $\theta = 30.0^\circ$ below the horizontal when the pilot releases a radar decoy. The horizontal distance between the release point and the point where the decoy strikes the ground is $d = 700\,\mathrm{m}$. (a) How long is the decoy in the air? (b) How high was the release point?

*Source:* Halliday, Resnick, Walker, *Fundamentals of Physics*, Vol. 1, 12th ed., Ch. 4, Problem 27.



# P32 — Solution 1

Convert the speed to SI units:

$$v_0 = 290.0\,\frac{\text{km}}{\text{h}} = 80.6\,\text{m/s}.$$

Take $+x$ horizontal and $+y$ upward. Then

$$v_{0x} = v_0 \cos 30^\circ = 69.8\,\text{m/s},$$
$$v_{0y} = -v_0 \sin 30^\circ = -40.3\,\text{m/s}.$$

From the horizontal motion,

$$d = v_{0x} t \quad \Rightarrow \quad t = \frac{700}{69.8} = 10.03\,\text{s}.$$

Thus,

$$\boxed{t \approx 10.0\,\text{s}}.$$

# P32 — Solution 2

Let the release point be at height $h$ above the ground. The vertical motion obeys

$$y = h + v_{0y}t - \tfrac{1}{2}gt^2.$$

At impact, $y = 0$, so

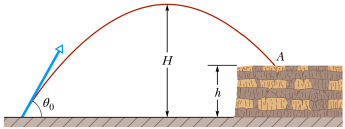
$$\begin{aligned} h &= -v_{0y}t + \tfrac{1}{2}gt^2 \\ &= (40.3)(10.03) + 4.90(10.03)^2 \\ &\approx 8.97 \times 10^2\,\mathrm{m}. \end{aligned}$$

Therefore,

$$\boxed{h \approx 8.97 \times 10^2\,\mathrm{m} \approx 897\,\mathrm{m}}.$$

# P33 — Halliday Problem 28

**Problem statement.** In this figure, a stone is projected at a cliff of height $h$ with an initial speed of $42.0\,\mathrm{m/s}$ directed at angle $\theta_0 = 60.0^\circ$ above the horizontal. The stone strikes at $A$, $5.50\,\mathrm{s}$ after launching. Find (a) the height $h$ of the cliff, (b) the speed of the stone just before impact at $A$, and (c) the maximum height $H$ reached above the ground.




*Source:* Halliday, Resnick, Walker, *Fundamentals of Physics*, Vol. 1, 12th ed., Ch. 4, Problem 28.

# P33 — Solution 1

Resolve the launch speed into components:

$$v_{0x} = 42.0\cos 60^\circ = 21.0\,\mathrm{m/s},$$
$$v_{0y} = 42.0\sin 60^\circ = 36.4\,\mathrm{m/s}.$$

At $t = 5.50\,\mathrm{s}$, the vertical coordinate of the stone is

$$\begin{aligned} y &= v_{0y}t - \tfrac{1}{2}gt^2 \\ &= (36.4)(5.50) - 4.90(5.50)^2 \\ &= 51.8\,\mathrm{m}. \end{aligned}$$

Thus the height of the cliff is

$$\boxed{h = 51.8\,\mathrm{m}}.$$

# P33 — Solution 2

The horizontal velocity remains constant:

$$v_x = 21.0\,\mathrm{m/s}.$$

The vertical velocity at impact is

$$\begin{aligned} v_y &= v_{0y} - gt \\ &= 36.4 - 9.80(5.50) \\ &= -17.5\,\mathrm{m/s}. \end{aligned}$$

Therefore the speed just before impact is

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(21.0)^2 + (-17.5)^2} = 27.4\,\mathrm{m/s}.$$

So,

$$\boxed{v \approx 27.4\,\mathrm{m/s}}.$$

# P33 — Solution 3

At the top of the trajectory the vertical velocity is zero. Hence

$$H = \frac{v_{0y}^2}{2g} = \frac{(36.4)^2}{2(9.80)} = 67.5\,\mathrm{m}.$$

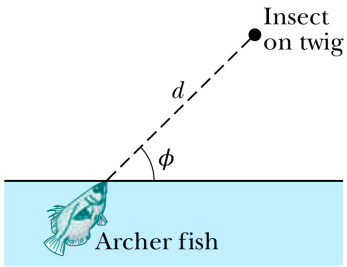
Therefore,

$$\boxed{H = 67.5\,\mathrm{m}}.$$

# P34 — Halliday Problem 41

**Problem statement.** Upon spotting an insect on a twig overhanging water, an archer fish squirts water drops at the insect to knock it into the water. Although the insect is located along a straight-line path at angle $\phi$ and distance $d$, a drop must be launched at a different angle $\theta_0$ if its parabolic path is to intersect the insect. If $\phi = 36.0^\circ$ and $d = 0.900\,\mathrm{m}$, what launch angle $\theta_0$ is required for the drop to be at the top of the parabolic path when it reaches the insect?

*Source:* Halliday, Resnick, Walker, *Fundamentals of Physics*, Vol. 1, 12th ed., Ch. 4, Problem 41.



# P34 — Solution

Let the insect coordinates be

$$x = d\cos\phi, \qquad y = d\sin\phi.$$

If the insect is reached at the *top* of the drop's trajectory, then the insect is at the projectile apex. For a projectile launched at angle $\theta_0$,

$$x_{\text{top}} = \frac{v_0^2 \sin 2\theta_0}{2g},$$

$$y_{\text{top}} = \frac{v_0^2 \sin^2\theta_0}{2g}.$$

Taking the ratio eliminates $v_0$:

$$\frac{y}{x} = \frac{\sin^2\theta_0}{\sin 2\theta_0} = \frac{\tan\theta_0}{2}.$$

Since $y/x = \tan\phi$,

$$\tan\phi = \frac{\tan\theta_0}{2} \quad \Rightarrow \quad \tan\theta_0 = 2\tan\phi.$$

With $\phi = 36.0^\circ$,

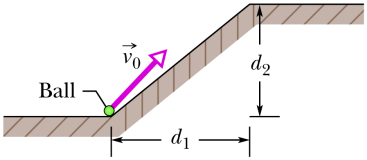
$$\tan\theta_0 = 2\tan 36.0^\circ = 1.45,$$

$$\theta_0 = \tan^{-1}(1.45) = 55.5^\circ.$$

# P35 — Halliday Problem 45

**Problem statement.** In this figure, a ball is launched with a velocity of magnitude $10.0\,\mathrm{m/s}$, at an angle of $50.0^\circ$ to the horizontal. The launch point is at the base of a ramp of horizontal length $d_1 = 6.00\,\mathrm{m}$ and height $d_2 = 3.60\,\mathrm{m}$. A plateau is located at the top of the ramp. (a) Does the ball land on the ramp or the plateau? When it lands, what are the (b) magnitude and (c) angle of its displacement from the launch point?

*Source:* Halliday, Resnick, Walker, *Fundamentals of Physics*, Vol. 1, 12th ed., Ch. 4, Problem 45.



# P35 — Solution 1

The ramp has slope

$$m = \frac{d_2}{d_1} = \frac{3.60}{6.00} = 0.600,$$

so its equation is $y_{\text{ramp}} = 0.600x$ for $0 \le x \le 6.00\,\text{m}$. The projectile path is

$$y(x) = x\tan 50.0^\circ - \frac{gx^2}{2v_0^2\cos^2 50.0^\circ}.$$

With $v_0 = 10.0\,\text{m/s}$, the landing point on the ramp must satisfy

$$x\tan 50.0^\circ - \frac{9.80x^2}{2(10.0)^2\cos^2 50.0^\circ} = 0.600x.$$

One solution is $x = 0$ (launch point); the nonzero solution is

$$\boxed{x = 4.99\,\text{m}}.$$

Since $4.99\,\text{m} < 6.00\,\text{m}$, the ball lands on the **ramp**, not on the plateau.

At the landing point,

$$y = 0.600x = 0.600(4.99) = 2.99\,\mathrm{m}.$$

Thus the displacement from the launch point is

$$\Delta\vec{r} = (4.99\,\hat{\imath} + 2.99\,\hat{\jmath})\,\mathrm{m}.$$

Its magnitude is

$$|\Delta\vec{r}| = \sqrt{(4.99)^2 + (2.99)^2} = 5.82\,\mathrm{m},$$

and its direction above the horizontal is

$$\alpha = \tan^{-1}\left(\frac{2.99}{4.99}\right) = 31.0^\circ.$$

Therefore,

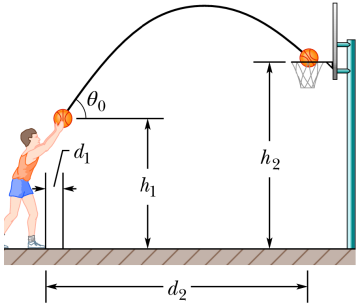
$$\boxed{|\Delta\vec{r}| = 5.82\,\mathrm{m}}, \qquad \boxed{\alpha = 31.0^\circ \text{ above the horizontal}}.$$

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# P36 — Halliday Problem 90

**Problem statement.** At what initial speed must the basketball player in figure throw the ball, at angle $\theta_0 = 55^\circ$ above the horizontal, to make the foul shot? The horizontal distances are $d_1 = 1.0\,\text{ft}$ and $d_2 = 14\,\text{ft}$, and the heights are $h_1 = 7.0\,\text{ft}$ and $h_2 = 10\,\text{ft}$.

*Source:* Halliday, Resnick, Walker, *Fundamentals of Physics*, Vol. 1, 12th ed., Ch. 4, Problem 90.



# P36 — Solution

Use feet and seconds throughout. The horizontal distance from release to hoop is

$$x = d_1 + d_2 = 1.0 + 14.0 = 15.0\,\text{ft},$$

and the vertical rise is

$$y = h_2 - h_1 = 10.0 - 7.0 = 3.0\,\text{ft}.$$

For projectile motion,

$$y = x\tan\theta_0 - \frac{gx^2}{2v_0^2\cos^2\theta_0},$$

with $g = 32.0\,\text{ft/s}^2$. Solve for $v_0$:

$$\begin{aligned} v_0 &= \sqrt{\frac{gx^2}{2\cos^2\theta_0\,(x\tan\theta_0 - y)}} \\ &= \sqrt{\frac{(32.0)(15.0)^2}{2\cos^2 55^\circ\,[15.0\tan 55^\circ - 3.0]}} \\ &= 24.4\,\text{ft/s}. \end{aligned}$$

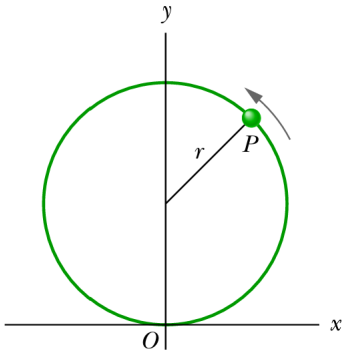
Therefore,

$$\boxed{v_0 \approx 24.4\,\text{ft/s}}$$

# P37 — Halliday Problem 107

**Problem statement.** A particle $P$ travels with constant speed on a circle of radius $r = 3.00\,\mathrm{m}$ and completes one revolution in $20.0\,\mathrm{s}$. The particle passes through $O$ at time $t = 0$. State the following vectors in magnitude-angle notation (angle relative to the positive direction of $x$). With respect to $O$, find the particle's position vector at the times of (a) $5.00\,\mathrm{s}$, (b) $7.50\,\mathrm{s}$, and (c) $10.0\,\mathrm{s}$. (d) For the $5.00\,\mathrm{s}$ interval from the end of the fifth second to the end of the tenth second, find the particle's displacement. For that interval, find (e) its average velocity and its velocity at the (f) beginning and (g) end. Next, find the acceleration at the (h) beginning and (i) end of that interval.

*Source:* Halliday, Resnick, Walker, *Fundamentals of Physics*, Vol. 1, 12th ed., Ch. 4, Problem 107.



# P37 — Solution 1

The motion is uniform circular motion with

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{20.0} = \frac{\pi}{10}\,\mathrm{rad/s},$$
$$v = \omega r = \frac{\pi}{10}(3.00) = 0.942\,\mathrm{m/s},$$
$$a_c = \omega^2 r = \left(\frac{\pi}{10}\right)^2 (3.00) = 0.296\,\mathrm{m/s^2}.$$

With respect to $O$, the position is

$$x(t) = r\sin(\omega t),$$
$$y(t) = r - r\cos(\omega t).$$

Hence:

$$\vec{r}(5.00\,\mathrm{s}) = (3.00\,\hat{\imath} + 3.00\,\hat{\jmath})\,\mathrm{m},$$
$$\vec{r}(7.50\,\mathrm{s}) = (2.12\,\hat{\imath} + 5.12\,\hat{\jmath})\,\mathrm{m},$$
$$\vec{r}(10.0\,\mathrm{s}) = (0\,\hat{\imath} + 6.00\,\hat{\jmath})\,\mathrm{m}.$$

In magnitude-angle form,

$$\boxed{\vec{r}(5.00\,\mathrm{s}) = 4.24\,\mathrm{m}\ \text{at}\ 45.0^\circ},$$

$$\boxed{\vec{r}(7.50\,\mathrm{s}) = 5.54\,\mathrm{m}\ \text{at}\ 67.5^\circ},$$

# P37 — Solution 2

From $t = 5.00\,\mathrm{s}$ to $t = 10.0\,\mathrm{s}$,

$$\Delta\vec{r} = \vec{r}(10.0) - \vec{r}(5.00) = (-3.00\,\hat{\imath} + 3.00\,\hat{\jmath})\,\mathrm{m}.$$

Thus,

$$\boxed{\Delta\vec{r} = 4.24\,\mathrm{m} \text{ at } 135^\circ}.$$

The average velocity over this $5.00\,\mathrm{s}$ interval is

$$\vec{v}_{\text{avg}} = \frac{\Delta\vec{r}}{\Delta t} = (-0.600\,\hat{\imath} + 0.600\,\hat{\jmath})\,\mathrm{m/s},$$

so

$$\boxed{\vec{v}_{\text{avg}} = 0.849\,\mathrm{m/s} \text{ at } 135^\circ}.$$

The instantaneous velocity is tangent to the circle, with constant magnitude $0.942\,\mathrm{m/s}$:

$$\boxed{\vec{v}(5.00\,\mathrm{s}) = 0.942\,\mathrm{m/s} \text{ at } 90.0^\circ},$$

$$\boxed{\vec{v}(10.0\,\mathrm{s}) = 0.942\,\mathrm{m/s} \text{ at } 180^\circ}.$$

# P37 — Solution 3

The acceleration is always centripetal, directed toward the circle center, with magnitude $a_c = 0.296\,\mathrm{m/s^2}$.

$$\boxed{\vec{a}(5.00\,\mathrm{s}) = 0.296\,\mathrm{m/s^2} \text{ at } 180^\circ},$$

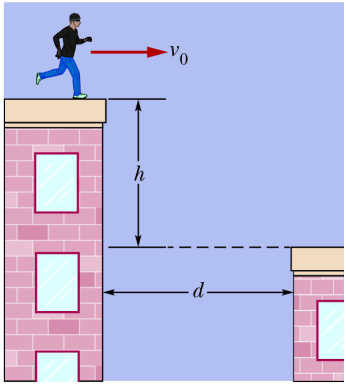
$$\boxed{\vec{a}(10.0\,\mathrm{s}) = 0.296\,\mathrm{m/s^2} \text{ at } 270^\circ}.$$

These directions are leftward at $t = 5.00\,\mathrm{s}$ and downward at $t = 10.0\,\mathrm{s}$, exactly as required for motion toward the center.

# P38 — Halliday Problem 125

**Problem statement.** *Stuntman jump.* A movie stuntman is to run across a rooftop and jump horizontally off it, to land on the roof of the next building. The rooftops are separated by $h = 4.8\,\mathrm{m}$ vertically and $d = 6.2\,\mathrm{m}$ horizontally. Before he attempts the jump, he wisely calculates if the jump is possible. Can he make the jump if his maximum rooftop speed is $v_0 = 4.5\,\mathrm{m/s}$?

*Source:* Halliday, Resnick, Walker, *Fundamentals of Physics*, Vol. 1, 12th ed., Ch. 4, Problem 125.



# P38 — Solution

The stuntman jumps horizontally, so the initial vertical velocity is zero. The time required to fall a vertical distance $h = 4.8\,\mathrm{m}$ is

$$h = \tfrac{1}{2}gt^2 \quad \Rightarrow \quad t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2(4.8)}{9.80}} = 0.990\,\mathrm{s}.$$

In that time, the greatest horizontal distance he can cover is

$$x_{\max} = v_0 t = (4.5)(0.990) = 4.45\,\mathrm{m}.$$

But the required horizontal gap is $d = 6.2\,\mathrm{m}$, so $x_{\max} < d$. Equivalently, the required launch speed would be

$$v_{0,\mathrm{req}} = \frac{d}{t} = \frac{6.2}{0.990} = 6.26\,\mathrm{m/s},$$

which is greater than $4.5\,\mathrm{m/s}$.

No, he cannot make the jump.

# References and scientific figure provenance

## Course and textbook sources

H. Khanpour, *Physics I, Chapter 4: Motion in Two and Three Dimensions*, First Edition, final manuscript, Sections 4.1–4.11.

D. Halliday, R. Resnick and J. Walker, *Fundamentals of Physics*, 12th edition, Volume 1, Wiley (2022), Chapter 4.

## Diagram and task conventions

The Halliday diagrams are new editable scientific redrawings of the given geometry and data, not reproductions of publisher artwork. Original manuscript TikZ figures are included as editable source; when an original figure disclosed the answer, its question version is givens-only. The complete figure/source map and methods are provided with the ZIP.


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