

Measurement, Units, and Physical Modelling

Physics I - Chapter 1

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Learning goals



Chapter question

How do we turn a physical question into a numerical result that can actually be tested?

By the end of the lecture, you should be able to:

- define a measurand and state the system/model boundary;
- convert compound units correctly, including powers;
- use dimensional homogeneity as a necessary-not sufficient-check;
- make defensible estimates and physically admissible bounds;
- distinguish resolution, repeatability, calibration, accuracy and uncertainty;
- use graphs, residuals and held-out comparisons to test a model;
- state what an observation does and does not establish.

SI quantities, units, and standards



For a chosen unit u , write

$$q = \{q\}_u u.$$

Changing the unit changes the numerical value, not the physical quantity.

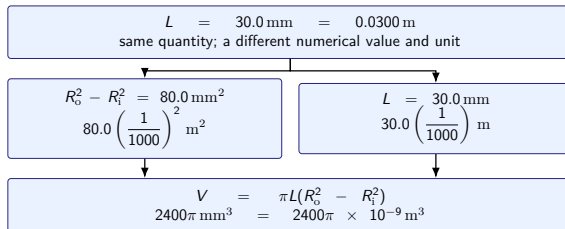
Exact definitions and relations

- $1 \text{ mm} = 10^{-3} \text{ m}$ is an exact unit relation.
- $c = 299\,792\,458 \text{ m s}^{-1}$ is an exact SI defining value.
- Exactness of the definition does *not* make a ruler reading exact.

Measured or stipulated

- $G = 6.674\,30(15) \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$ is measured.
- $\rho = 7.80 \text{ g cm}^{-3}$ in the spacer examples is a stipulated teaching input.
- A laboratory realisation can improve without changing the SI definition.

Prefixes under powers and unit conversion



$$\underbrace{\text{kg m}^{-3}}_{\text{density}} \underbrace{\text{m}^3}_{\text{volume}} = \text{kg}$$

the powers of the units cancel; the numerical factors remain

Power rule and density conversion

$$(\text{mm})^2 = (10^{-3}\text{m})^2 = 10^{-6}\text{m}^2, \quad 1 \text{ mm}^3 = 10^{-9} \text{ m}^3,$$

$$1 \text{ g cm}^{-3} = 10^3 \text{ kg m}^{-3}.$$

The exponent acts on the *whole prefixed unit*; the physical quantity is unchanged by the representation.

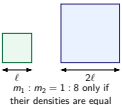
Dimensions, homogeneity, and scaling

Exponent ledger for $C\rho Av^q$

	target	proposed
M	1	1
L	1	$q-1$
T	-2	$-q$

Both nontrivial rows require $q = 2$.

Corresponding face areas: 1 : 4
Similar three-dimensional bodies: $V_1 : V_2 = 1 : 8$



l $2l$
 $m_1 : m_2 = 1 : 8$ only if
their densities are equal

Restricted dimensional model

For $F = C\rho Av^q$ with dimensionless C ,

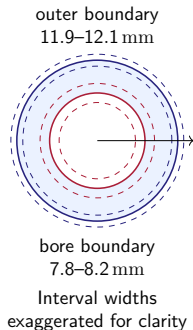
$$[\rho Av^q] = \text{M L}^{q-1} \text{T}^{-q},$$

so matching $[F] = \text{M L T}^{-2}$ requires
 $q = 2$.

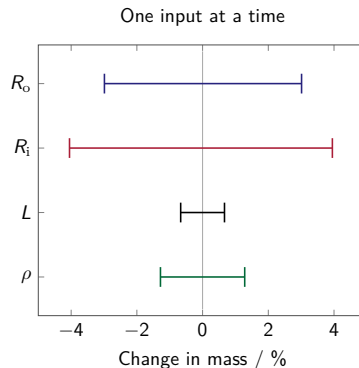
What dimensions do *not* prove

They do not determine C , prove the variable list is complete, fix direction, or establish the physical regime. Geometric similarity also requires *all* corresponding lengths to scale together.

Estimation, admissible bounds, and sensitivity to inputs



Estimate/admissibility. Match precision to the information available; orders of magnitude expose gross conversion failures. Require $0 \leq R_i < R_o$.

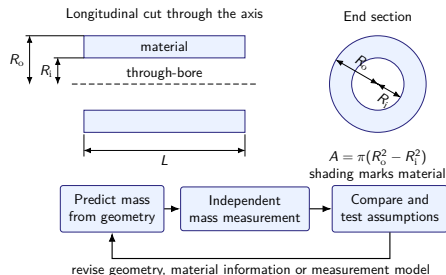


Sensitivity here. Increasing ρ, L, R_o raises mass; increasing R_i lowers it. Segments are exact one-input interval effects—*not* error bars or variances to add.

From a physical question to a model

A physical model connects a requested measurand to available information. For a uniform, concentric through-bored spacer,

$$A = \pi(R_o^2 - R_i^2), \quad V = \pi L(R_o^2 - R_i^2), \quad m = \rho V.$$



Model test. A calculator evaluates the model; it cannot discover an omitted bore. Test the prediction against *independent* evidence and revise assumptions when evidence requires it.

Worked Example 1.1 - complete statement and representation

A machine spacer with an unrecorded bore

An incomplete drawing is used to model a spacer as solid. A subsequent inspection finds

$$R_i = \frac{2R_o}{3}.$$

Assume that the bore extends through the full length and that the density is uniform. **What fraction of the solid-model mass remains? What would an independent weighing test?**

Represent before calculating

The outer cylinder and removed cylinder have the same length and density, so form the mass ratio before inserting numerical dimensions:

$$\begin{aligned}\frac{m_{\text{hollow}}}{m_{\text{solid}}} &= \frac{\rho\pi L(R_o^2 - R_i^2)}{\rho\pi LR_o^2} \\ &= 1 - \left(\frac{R_i}{R_o}\right)^2.\end{aligned}$$

What is being tested? A separately calibrated weighing challenges the geometry-plus-density prediction only when it is independent of the model inputs.

Worked Example 1.1 - solution, checks, interpretation



$$\frac{m_{\text{hollow}}}{m_{\text{solid}}} = 1 - \left(\frac{2}{3}\right)^2 = \frac{5}{9} \approx 0.5556.$$

Three percentages, three denominators

Retained: $5/9 = 55.6\%$ of the solid-model mass.

Removed: $4/9 = 44.4\%$ of the solid-model mass.

Solid prediction relative to hollow mass:

$$\frac{m_s - m_h}{m_h} = \frac{4/9}{5/9} = 0.80,$$

so it is 80% too large on that denominator.

Checks

$0 < 5/9 < 1$; as $R_i \rightarrow 0$, the ratio tends to 1; as $R_i \rightarrow R_o^-$, it tends to 0.

Interpretation

A disagreement with weighing does not uniquely identify the bore. Incorrect density, a blind bore, zero offset or attached material could also matter. Inspection plus weighing is more informative than either alone.

Measurement quality and reporting

Distinguish the ideas

- **Resolution:** smallest displayed/meaningful increment.
- **Repeatability/precision:** closeness of repeated indications under stated conditions.
- **Accuracy:** closeness to the measurand's true quantity value; it is qualitative, not a number to attach automatically.
- **Calibration:** relation between indications and reference values under stated conditions.

Chapter support: Chapter 1 §1.5.

A useful indication model

$$x_j = x^* + b + \epsilon_j.$$

A common offset b shifts every indication but leaves pairwise differences and sample spread unchanged.

Reporting discipline

Error is not the same as uncertainty. Significant figures are a reporting convention, not a probability model. A Type A/Type B classification describes how uncertainty information is evaluated, not whether it is “random” or “systematic” by itself.

Worked Example 1.6 - complete statement

Two instruments measuring the same reference

A constructed reference has $x_{\text{ref}} = 20.000 \text{ mm}$ and standard uncertainty $u_{\text{ref}} = 0.010 \text{ mm}$. Two synthetic instruments produce:

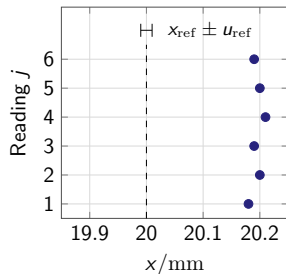
j	1	2	3	4	5	6
A/mm	20.18	20.20	20.19	20.21	20.20	20.19
B/mm	19.9	20.1	20.0	20.0	19.9	20.1

Instrument A has display increment 0.01 mm and was constructed with offset $+0.20 \text{ mm}$; its additional deviations are $(-0.02, 0, -0.01, +0.01, 0, -0.01) \text{ mm}$. Instrument B has increment 0.10 mm and zero stipulated offset, with the displayed deviations about 20.0 mm . These are controlled teaching data, not random draws or laboratory observations.

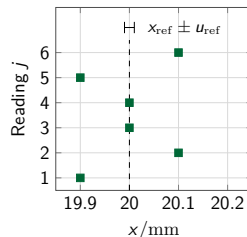
Tasks. Calculate means and sample spreads; compare resolution, clustering and reference residuals; subtract the stipulated A offset and interpret what changes and what does not.

Worked Example 1.6 - solution and aligned plots

A: fine display, common positive offset



B: coarser display, centred synthetic readings



$$\bar{x}_A = 20.195 \text{ mm}, \quad s_A \simeq 0.01049 \text{ mm},$$

$$\bar{x}_B = 20.000 \text{ mm}, \quad s_B \simeq 0.08944 \text{ mm}.$$

After subtracting 0.20 mm from every A reading,

$$\bar{x}_{A,\text{corr}} = 19.995 \text{ mm}, \quad s_{A,\text{corr}} = s_A.$$

Interpretation. A is more finely resolved and tightly clustered, yet its uncorrected indications lie farther from the reference. B's coincident mean does not prove exactness. The capped reference marker is a stated *standard uncertainty*, not a tolerance or guaranteed coverage band.

Worked Example 1.8 - calibration over two ranges

Complete setup

An idealised displacement instrument has input x in millimetres and output y in volts. Constructed data are generated from

$$y_C = 0.10 \text{ V} + (0.20 \text{ V mm}^{-1})x + (0.020 \text{ V mm}^{-2})x^2.$$

Initially only the low-range pairs

$$(x/\text{mm}, y/\text{V}) = (0, 0.10), (1, 0.32), (2, 0.58)$$

are supplied for calibration. **Test a linear endpoint calibration and a quadratic description, then predict held-out outputs at $x = 4$ and 6 mm .**

Endpoint line (defined explicitly; not least-squares).

$$a_L = \frac{0.58 - 0.10}{2 - 0} = 0.24 \text{ V mm}^{-1}, \quad b_L = 0.10 \text{ V},$$

$$y_L = 0.10 \text{ V} + (0.24 \text{ V mm}^{-1})x.$$

Worked Example 1.8 - solution and held-out prediction

For $y_C = b + ax + c_2x^2$, the three training points give

$$\begin{aligned}b &= 0.10 \text{ V}, & a + c_2 &= 0.22, \\2a + 4c_2 &= 0.48.\end{aligned}$$

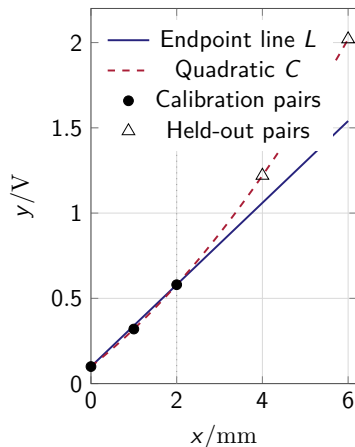
so

$$\begin{aligned}a &= 0.20 \text{ V mm}^{-1}, \\c_2 &= 0.020 \text{ V mm}^{-2}.\end{aligned}$$

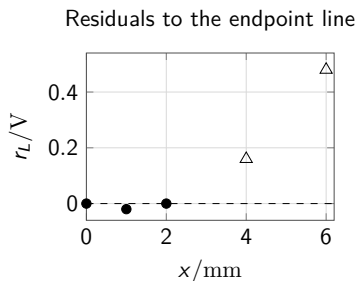
x/mm	y_L/V	y_C/V	held-out y/V
4	1.06	1.22	1.22
6	1.54	2.02	2.02

Do not reuse calibration as validation

Three parameters can reproduce three training points exactly; zero residuals there describe interpolation, not independent predictive success.



Model testing with residuals

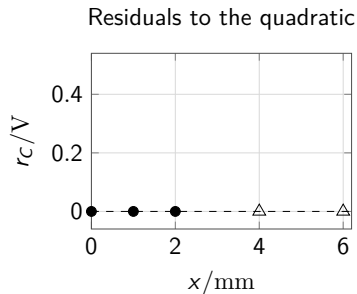


With $r = y_{\text{data}} - y_{\text{model}}$, the endpoint-line held-out residuals are

$$+0.16 \text{ V}, \quad +0.48 \text{ V}.$$

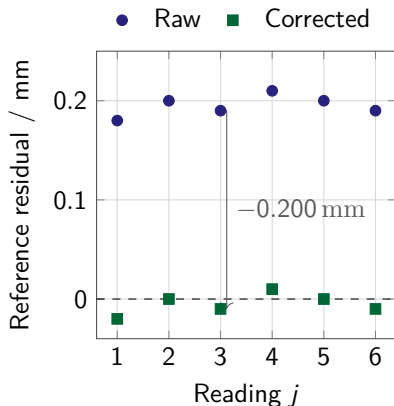
Unit invariance

Changing millimetres to metres changes numerical coefficients but must leave physical predictions unchanged.



The quadratic held-out residuals are zero **by construction of this teaching dataset**. One held-out agreement does not prove a universal sensor law.

A correction is not new data



Common correction

Subtracting the same 0.200 mm from every A reading translates all reference residuals by the same amount. Their spread is unchanged.

Preserve raw observations

A correction produces derived values; it does not create new measurements. Keep raw readings and acquisition order so a correction can be revised later.

Core modelling checklist. Define measurand/system; state assumptions; carry units; check dimensions, signs, limits and scale; distinguish direct, inferred and corrected values; distinguish bounds from uncertainty; seek genuinely new evidence.

Problem Set 1 - Problem 5: find a cubic conversion error



Complete problem statement

A script obtains $V = 1250 \text{ mm}^3$ and converts it to 1.250 m^3 by dividing by 1000. With $\rho = 2.40 \text{ g cm}^{-3}$, find the correct mass using two coherent unit routes. By what factor is the script's converted volume wrong? Connect the diagnosis to Figure F01-02.

Before calculating

Choose one coherent unit route and carry the *entire quantity*, including the cubic unit factor. Then repeat by a genuinely different unit route as an independent check.

Problem Set 1 - Problem 5 solution



Convert the complete volume

$$1250 \text{ mm}^3 = 1250(10^{-3} \text{ m})^3 = 1.250 \times 10^{-6} \text{ m}^3 = 1.250 \text{ cm}^3.$$

Route 2: SI

Route 1: g and cm^3

$$m = (2.40)(1.250) \text{ g} = 3.00 \text{ g}.$$

$$\rho = 2400 \text{ kg m}^{-3},$$

$$\begin{aligned} m &= (2400)(1.250 \times 10^{-6}) \text{ kg} \\ &= 0.00300 \text{ kg}. \end{aligned}$$

Diagnosis

The script's 1.250 m^3 is 10^6 times too large. Dividing by only 10^3 applies the length conversion once; a volume requires the cube. The two independent routes agree on the same physical mass.

Problem Set 2 - Problem 22: do not reuse a test

Complete problem statement

A quadratic has been chosen to fit the three training points in Example 1.8 exactly. A student calls those zero residuals “three independent validations”. Repair the statement. Using the held-out point at 4 mm, compare the line and quadratic predictions. Rewrite their coefficients for a length input in metres and verify the same predictions.

Known models from Example 1.8

$$y_L = 0.10 \text{ V} + (0.24 \text{ V mm}^{-1})x,$$

$$y_C = 0.10 \text{ V} + (0.20 \text{ V mm}^{-1})x + (0.020 \text{ V mm}^{-2})x^2.$$

The held-out constructed response at $x = 4 \text{ mm}$ is 1.22 V.

Problem Set 2 - Problem 22 solution

Predict before comparing

At $x = 4 \text{ mm}$,

$$y_L = 1.06 \text{ V}, \quad y_C = 1.22 \text{ V}.$$

With $r = y_{\text{data}} - y_{\text{model}}$,

$$r_L = +0.16 \text{ V}, \quad r_C = 0.$$

The training-point zeros are interpolation, not independent validation.

Rewrite for metre-valued input

$$y_L = 0.10 \text{ V} + (240 \text{ V m}^{-1})x,$$

$$y_C = 0.10 \text{ V} + (200 \text{ V m}^{-1})x \\ + (20000 \text{ V m}^{-2})x^2.$$

At $x = 0.004 \text{ m}$ these give the same 1.06 V and 1.22 V predictions.

Interpretation

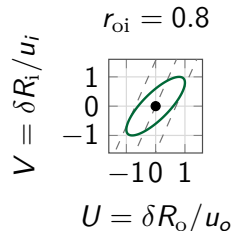
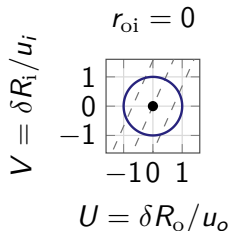
One constructed held-out agreement supports the quadratic over the line at that point; it does not prove the quadratic universally correct. Real calibration also requires input/calibration uncertainties and further validation points.

Advanced Topics for Technical and Engineering Students - sensitivity and covariance

For a small change in $y = f(x_1, \dots, x_n)$,

$$\delta y \simeq \sum_i c_i \delta x_i, \quad c_i = \frac{\partial f}{\partial x_i},$$

$$u_y^2 \simeq \sum_{ij} c_i c_j \text{Cov}(x_i, x_j).$$



For the spacer,

$$\frac{\delta m}{m} \simeq \frac{\delta \rho}{\rho} + \frac{\delta L}{L} + \frac{2R_o \delta R_o - 2R_i \delta R_i}{R_o^2 - R_i^2}.$$

Interpretation. For $W = R_o^2 - R_i^2$ the derivatives have opposite signs, so positive correlation can reduce u_W here. This is not universal; covariance contours are not automatically confidence regions.

Chapter support: Chapter 1 advanced section; Figure F01-07.

Advanced Topics for Technical and Engineering Students — dimensionless modelling



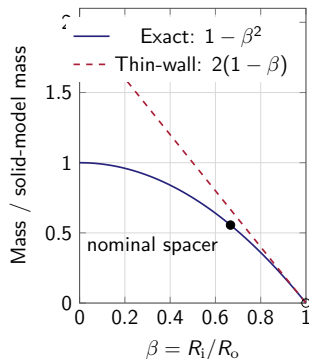
Define

$$\Pi_m = \frac{m}{\rho R_o^3}, \quad \lambda = \frac{L}{R_o},$$
$$\beta = \frac{R_i}{R_o}.$$

Uniform-cylinder geometry gives

$$\Pi_m = \pi \lambda (1 - \beta^2), \quad \frac{m}{\rho \pi L R_o^2} = 1 - \beta^2.$$

Meaning. Nondimensionalisation separates overall size from shape. Geometrically similar parts share β and λ ; at equal density their masses scale with the cube of length.



Chapter support: Chapter 1 advanced section; Figure F01-08.

Advanced Topics for Technical and Engineering Students - when is a thin wall thin enough?



Write $t = R_o - R_i$. The exact annular area is

$$A = \pi t(2R_o - t).$$

Replacing the mean-radius factor by the outer radius gives

$$A_{\text{thin}} = 2\pi R_o t,$$

with relative overestimate

$$\epsilon_{\text{thin}} = \frac{A_{\text{thin}} - A}{A} = \frac{t}{2R_o - t} = \frac{1 - \beta}{1 + \beta}.$$

Model-validity lesson. Approximation quality is controlled by a dimensionless shape parameter, not by millimetre size alone.

Chapter support: Chapter 1 advanced thin-wall section and Example 1.13.

A quantitative 1% criterion

$$\frac{1 - \beta}{1 + \beta} < 0.01 \implies \beta > \frac{0.99}{1.01} \simeq 0.980198,$$

so $t/R_o < 1.98\%$.

Nominal spacer

For $\beta = 2/3$, $t/R_o = 1/3$ and

$$\epsilon_{\text{thin}} = 0.20.$$

It is **not** thin-walled under the 1% criterion.

Chapter synthesis and bridge to Chapter 2

Seven questions to ask before trusting a number

- 1 What physical quantity/measurand is requested?
- 2 What system boundary and model connect it to the available information?
- 3 Are the units and dimensions consistent?
- 4 Does the result have the correct sign, scale and limiting behaviour?
- 5 What was directly measured, what was inferred, and what was stipulated?
- 6 What does the uncertainty or bound statement actually mean?
- 7 Did the comparison use independent information, or data already used to construct the model?

Next: Chapter 2 - Motion Along a Straight Line

Position, displacement, velocity and acceleration require the same discipline in coordinates, units, graphs, model assumptions and interpretation. Chapter 1 supplies the quantitative habits; Chapter 2 applies them to motion.

