

Applications of Newton's Laws

Physics I - Chapter 6

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Can the proposed motion actually occur?

A conveyor can accelerate a package through friction. A guide can bend a trajectory through its normal force. A fluid can limit a falling body's speed through drag.

Newton's law calculates the force that a proposed motion requires. The interaction model decides whether that force is available.

The chapter's organizing question

Calculate the demand, test the capacity, and change the motion model when a contact constraint fails.

The route begins with sticking and sliding, extends to curved motion, then develops velocity-dependent forces and their time histories.

Chapter support: Revised Chapter 6

Learning route and prerequisites

- 1 Resolve forces and test each frictional contact.
- 2 Separate speed change from direction change on a curved path.
- 3 Use relative velocity to construct sign-correct drag laws.
- 4 Solve and check the resulting initial-value problem.
- 5 Extend the method to translating, nonrotating frames.

Preparation

Signed velocity, vector components, trigonometry, Newton's laws and free-body diagrams from Chapters 2–5. Inequalities, curvature and separable differential equations receive local explanations here.

Chapter support: Revised Chapter 6

Learning goals

By the end of the core lecture, you should be able to:

- Calculate static-friction demand and decide whether sticking survives.
- Resolve inclined and connected systems and test every contact.
- Calculate radial and tangential resultants, banked-turn intervals and shared traction limits.
- Derive drag transients and check their signs, dimensions and limiting behaviour.
- Infer force-law parameters and demonstrate numerical accuracy through an independent benchmark.

Advanced work compares laboratory and platform equations and tests sticking, sliding and detachment on an accelerating incline.

Chapter support: Revised Chapter 6

A model includes both equations and admissibility

Choose the body, draw its real external forces and define signed axes. Then propose the motion and solve $\sum \mathbf{F} = m\mathbf{a}$.

$$N \geq 0, \quad T \geq 0, \quad |f_{s,\text{req}}| \leq \mu_s N.$$

A one-sided surface pushes, a cable pulls, and static friction supplies a bounded signed response. Each condition belongs to its own interaction.

Conventions throughout this lecture

Use $g = 9.81 \text{ m s}^{-2}$, SI units and particle models unless stated otherwise. Bold symbols are vectors. Magnitudes are nonnegative; components can have either sign. Angular derivatives use radians.

If a constraint fails, release it and solve the new regime. Clipping a force while retaining the old acceleration violates Newton's law.

Chapter support: Revised Chapter 6

Static friction: demand inside a force band

For sticking on a level belt, $f_{s,\text{req}} = ma_b$ and $N = mg$. Thus $|a_b| \leq \mu_s g$. Equality describes impending slip.

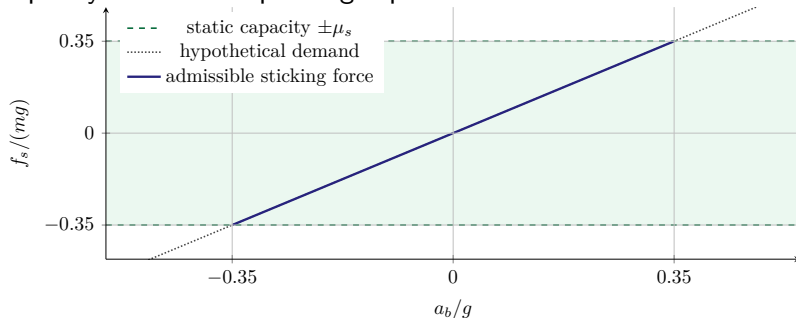


Figure 6.1: Static-friction demand for the horizontally accelerating package, with $\mu_s = 0.35$. The shaded band is a permitted force range. Only the solid segment describes admissible sticking. The dotted continuation diagnoses a failed assumption; it does not predict the sliding force.

Chapter support: Revised Chapter 6

The package-only force equation

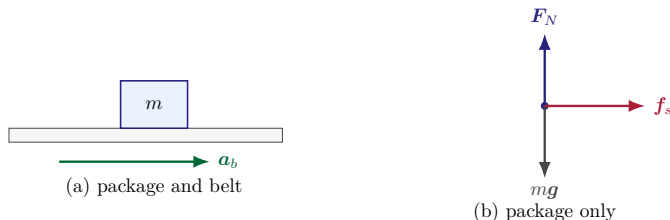


Figure 6.2: The physical setup and its separate package-only free-body diagram for rightward sticking acceleration. Belt acceleration is a kinematic quantity, so it appears in the setup rather than as an extra force in the free-body diagram. Arrow lengths are schematic.

$$N - mg = 0, \quad f_s = ma_b \quad \text{if sticking holds.}$$

Static contact means no relative slip, even during acceleration.

Chapter support: Revised Chapter 6

Kinetic friction follows relative sliding

$$v_{\text{rel}} = v_{\text{body}} - v_{\text{surface}}, \quad f_k = -\mu_k N \operatorname{sgn}(v_{\text{rel}}) \quad (v_{\text{rel}} \neq 0).$$

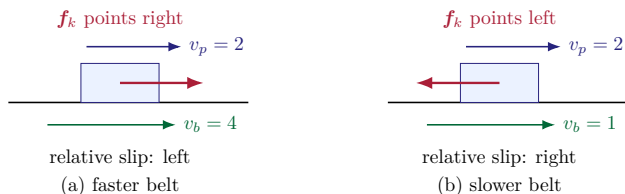


Figure 6.3: Velocities are prescribed in m s^{-1} , with rightward positive. In (a), $v_{\text{rel}} = 2 - 4 = -2$; in (b), $v_{\text{rel}} = 2 - 1 = +1$. Thus the friction force reverses even though the package moves rightward in both laboratory descriptions. Drawing lengths are schematic.

At $v_{\text{rel}} = 0$, test the required static force before choosing the next regime. Equal instantaneous velocities alone do not ensure continued sticking.

Chapter support: Revised Chapter 6

Example 6.1: conveyor, first sticking test

A 12.0 kg package initially rests relative to a level belt. The stipulated coefficients are $\mu_s = 0.350$ and $\mu_k = 0.250$. Determine its friction force and acceleration when the belt begins accelerating rightward at 2.40 m s^{-2} . Repeat for 4.20 m s^{-2} .

The package-only vertical equation gives $N = 12.0(9.81) = 117.72 \text{ N}$. Propose $a_p = a_b$ and compare:

$$\mu_s N = 41.202 \text{ N}, \quad f_{s,\text{req}} = 12.0(2.40) = 28.8 \text{ N}.$$

The test passes. The friction is 28.8 N rightward and the package acceleration is 2.40 m s^{-2} .

The 4.20 m s^{-2} case is completed on the next slide.

Chapter support: Revised Chapter 6: Example 6.1

Example 6.1: failure of sticking

At $a_b = 4.20 \text{ m s}^{-2}$, sticking would require

$$f_{s,\text{req}} = 12.0(4.20) = 50.4 \text{ N} > 41.202 \text{ N}.$$

The belt moves ahead. The package slips left relative to it, so the actual kinetic force is rightward:

$$f_k = \mu_k mg = 29.43 \text{ N}, \quad a_p = f_k/m = 2.4525 \text{ m s}^{-2}.$$

Report $a_p = 2.45 \text{ m s}^{-2}$ rightward. Because $a_p < a_b$, the emerging slip direction is consistent.

Independent threshold check

The static acceleration limit is $\mu_s g = 3.4335 \text{ m s}^{-2}$. It lies between the two tested belt accelerations. Mass cancels because both demand and capacity scale with mass.

Chapter support: Revised Chapter 6: Example 6.1

Example 6.2: re-sticking is an event

Use the sliding conveyor of the preceding example. From $t = 0$ to 1.00 s the belt accelerates at 4.20 m s^{-2} . It then keeps the velocity it has reached. The package starts from rest. Find the time at which sliding ends, the friction afterward, and the total lag of the package behind the belt.

Restored context from Example 6.1

$m = 12.0\text{ kg}$, $\mu_s = 0.350$, $\mu_k = 0.250$, level belt, rightward positive. Both package and belt begin at rest at the same reference position. Sliding gives $a_k = \mu_k g = 2.4525\text{ m s}^{-2}$.

Integrate the sliding law only until the first zero of v_{rel} . Then calculate the force needed to keep the velocities equal and test the static bound. Complete time and lag calculations follow on slide 13.

Chapter support: Revised Chapter 6: Example 6.2

Example 6.2: catch-up time and positional lag

For $0 \leq t \leq 1.00 \text{ s}$, $v_b = 4.20t$ and $v_p = 2.4525t$. At $t_1 = 1.00 \text{ s}$:

$$v_b - v_p = 1.7475 \text{ m s}^{-1}, \quad \ell_1 = \frac{1}{2}(4.20 - 2.4525)t_1^2 = 0.87375 \text{ m}.$$

Write $s = t - t_1$. During the second interval, $v_{\text{rel}} = -1.7475 + a_k s$, whose first zero gives

$$s_c = \frac{1.7475}{2.4525} = 0.712538 \text{ s}, \quad t_c = 1.71254 \text{ s}.$$

The now constant-speed belt requires $f_{s,\text{req}} = 0$. Sticking is admissible and friction becomes zero.

$$\ell_c = \ell_1 + 1.7475s_c - \frac{1}{2}a_k s_c^2 = 1.49633 \text{ m} \simeq 1.50 \text{ m}.$$

The relative-velocity area gives the same lag. Continuing $+\mu_k mg$ after t_c would violate the assumed slip direction.

Chapter support: Revised Chapter 6: Example 6.2

An angled force changes the normal load

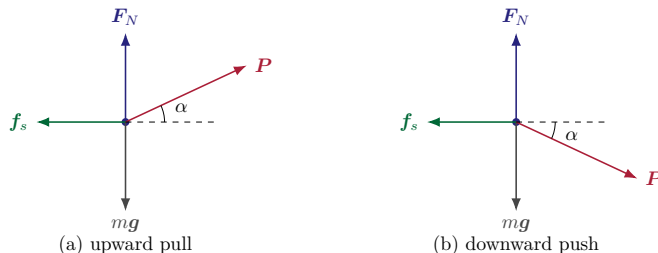


Figure 6.5: Separate body-only force diagrams for a crate that is just about to slide right. The applied force has the same magnitude and horizontal projection in the two cases, but its vertical projection changes sign. Normal-force magnitudes must be calculated; the arrows are not to scale.

For zero vertical acceleration on a level floor:

$$N_{\text{pull}} = mg - P \sin \alpha, \quad N_{\text{push}} = mg + P \sin \alpha.$$

Impending rightward slip requires $P \cos \alpha = \mu_s N$.

Chapter support: Revised Chapter 6

Example 6.3: pulling and pushing the same crate

A 15.0 kg crate rests on a level floor with $\mu_s = 0.400$. Find the force needed to initiate rightward sliding when the force is directed 25.0° above horizontal, and when it is directed 25.0° below horizontal. Ignore tipping by treating the crate as a particle.

With $\alpha = 25.0^\circ$, the threshold equations give

$$P_{\text{pull}} = \frac{\mu_s mg}{\cos \alpha + \mu_s \sin \alpha} = 54.7 \text{ N},$$

$$P_{\text{push}} = \frac{\mu_s mg}{\cos \alpha - \mu_s \sin \alpha} = 79.8 \text{ N}.$$

For the pull, $N = 124.0 \text{ N} > 0$. The push denominator is positive for these data, so both solutions are admissible.

At $\alpha = 0$, both reduce to $\mu_s mg = 58.9 \text{ N}$. A nonpositive push denominator would remove a finite sliding threshold in this ideal particle model.

Chapter support: Revised Chapter 6: Example 6.3

Core practice 1: P06-01, conveyor threshold

A 6.0 kg case rests on a horizontal conveyor with $\mu_s = 0.30$ and $\mu_k = 0.22$. (a) Find the largest belt acceleration for which the case can remain fixed relative to the belt. (b) If the belt accelerates right at 4.0 m s^{-2} , find the case acceleration immediately after sliding is established. Describe the relative slip direction.

Solution (a). Sticking requires $N = mg$ and $f_s = ma_b$, hence

$$|a_b| \leq \mu_s g, \quad a_{b,\max} = 2.943 \simeq 2.94 \text{ m s}^{-2}.$$

Solution (b). The required 24.0 N exceeds the 17.658 N capacity. The belt outruns the case, so it slips left relative to the belt and receives rightward friction:

$$f_k = 12.9492 \text{ N}, \quad a_p = \mu_k g = 2.1582 \simeq 2.16 \text{ m s}^{-2}.$$

The relative acceleration $a_p - a_b = -1.8418 \text{ m s}^{-2}$ confirms the slip direction.

Chapter support: Revised Chapter 6: P06-01

Example 6.4: stopping ends the sliding branch

A right-sliding body on a stationary level floor has $a = -\mu_k g$ only while $v > 0$. At $v = 0$, test the rest state rather than extending the same force direction.

A case initially slides at 6.00 m s^{-1} on a stationary level floor. With $\mu_k = 0.250$, determine its stopping time and distance. Use Newton's law and kinematics, without assuming an energy method from the next chapter.

$$t_s = \frac{v_0}{\mu_k g} = 2.44648 \text{ s} \simeq 2.45 \text{ s},$$

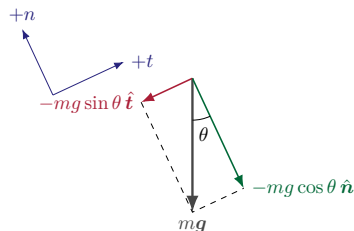
$$d = \int_0^{t_s} (v_0 - \mu_k g t) dt = \frac{v_0^2}{2\mu_k g} = 7.34 \text{ m}.$$

Afterward $v = 0$ and $f_s = 0$. The triangular velocity area $v_0 t_s / 2$ independently reproduces the distance.

Doubling initial speed doubles stopping time and quadruples stopping distance under this constant-coefficient model.

Chapter support: Revised Chapter 6: Example 6.4

Incline geometry before contact forces



Projection construction:
components
sum to gravity.
Not three physical forces.

Figure 6.7: Gravity resolved in incline-aligned axes, drawn at $\theta = 25^\circ$. The dashed parallelogram verifies the vector sum. This is a component construction rather than a free-body diagram. The rotated-angle construction method is adapted from Izaak Neutelings, *Friction*, TikZ.net, CC BY-SA 4.0.

Choose $+t$ uphill and $+n$ outward. Gravity's projections are $-mg \sin \theta$ and $-mg \cos \theta$: two components of one interaction.

Chapter support: Revised Chapter 6

A bare incline: equilibrium demand and threshold

Rest requires $f_s = mg \sin \theta$ uphill and $N = mg \cos \theta$. Therefore $\tan \theta \leq \mu_s$ and $\theta_c = \arctan \mu_s$.

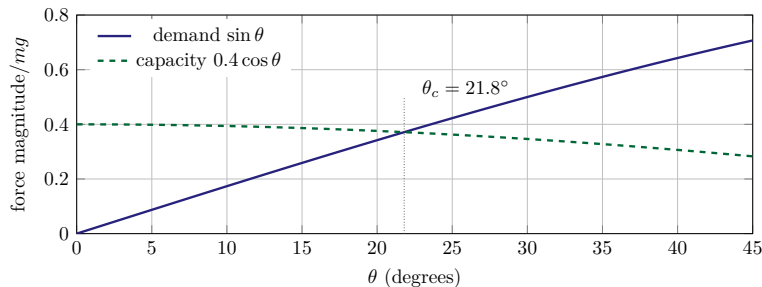


Figure 6.8: Competing force magnitudes on a bare incline with $\mu_s = 0.400$. The demand and capacity cross at $\arctan(0.400) = 21.8^\circ$. To the right, the plotted demand is a failed equilibrium requirement, rather than the actual kinetic friction. Above the threshold, use the sliding law. For established downhill slip, $a_{\text{down}} = g(\sin \theta - \mu_k \cos \theta)$.

Chapter support: Revised Chapter 6

Example 6.5: rest below the critical angle

A 20.0 kg case rests on a 18.0° incline, with $\mu_s = 0.400$. Determine whether equilibrium is possible, and find the actual friction. Explain why using the maximum friction would give the wrong acceleration.

Resolve weight using the axes established on slide 18:

$$N = 20.0(9.81) \cos 18.0^\circ = 186.597 \text{ N},$$

$$f_{s,\text{req}} = 20.0(9.81) \sin 18.0^\circ = 60.6291 \text{ N} < \mu_s N = 74.6389 \text{ N}.$$

Equilibrium is possible. The actual force is 60.6 N uphill.

Using the maximum would invent a 14.0 N uphill resultant, contradicting rest. The equivalent test $\tan 18.0^\circ = 0.324920 < 0.400$ checks the conclusion independently.

Chapter support: Revised Chapter 6: Example 6.5

A cable can reverse friction on an incline

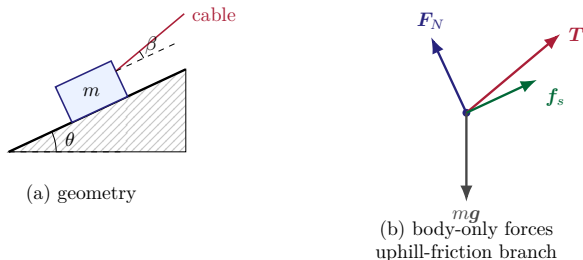


Figure 6.9: A cable on an incline with geometry generated from $\theta = 25^\circ$ and $\beta = 15^\circ$. The separate free-body diagram displays the uphill-friction branch; a sufficiently large tension requires the opposite friction direction. The normal is perpendicular to the plane. Rotated-angle construction adapted from Izaak Neutelings, *Friction*, TikZ.net, CC BY-SA 4.0.

For rest, with $f_s > 0$ uphill and $T \geq 0$:

$$f_{s,\text{req}} = mg \sin \theta - T \cos \beta, \quad N = mg \cos \theta - T \sin \beta.$$

Tension reduces the normal load and can reverse the friction demand.

Chapter support: Revised Chapter 6

Both sides of the static tension interval

Substitute the signed demand and normal load into

$$-\mu_s N \leq mg \sin \theta - T \cos \beta \leq \mu_s N, \quad N \geq 0, \quad T \geq 0.$$

The positive-friction endpoint has $f_s = +\mu_s N$ and gives

$$T_{\min} = mg \frac{\sin \theta - \mu_s \cos \theta}{\cos \beta - \mu_s \sin \beta}.$$

The negative-friction endpoint has $f_s = -\mu_s N$ and gives

$$T_{\max} = mg \frac{\sin \theta + \mu_s \cos \theta}{\cos \beta + \mu_s \sin \beta}.$$

These interval expressions require the displayed denominators to be positive and the resulting range to satisfy $T \geq 0$ and $N \geq 0$. Otherwise solve the original inequalities with their actual signs.

Chapter support: Revised Chapter 6

Example 6.6: the complete equilibrium range

An 18.0 kg equipment case rests on a 25.0° incline. A cable pulls uphill at 15.0° above the plane, and $\mu_s = 0.350$. Find the entire tension range consistent with static contact, the tension at which friction vanishes, and the friction at $T = 70.0 \text{ N}$.

The positive denominators permit the formulas on slide 22:

$$T_{\min} = 21.2642 \text{ N}, \quad T_{\max} = 123.651 \text{ N}.$$

Thus the static interval is $21.3 \leq T \leq 123.7 \text{ N}$ to the displayed rounding. At $T_{\max}$, $N = 128.0 \text{ N} > 0$.

Contact alone would permit $T \leq mg \cos \theta / \sin \beta = 618.3 \text{ N}$. Here the friction bound fails first. The zero-friction and 70.0 N cases are completed on slide 24.

Chapter support: Revised Chapter 6: Example 6.6

Example 6.6: reversal inside the static band

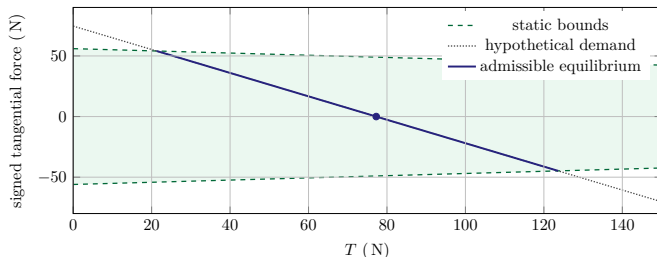


Figure 6.10: The cable example's decreasing friction demand and narrowing static-force band. The solid segment lies inside the band from $T_{\min}$ to $T_{\max}$. Its zero marks a friction reversal within equilibrium, rather than the onset of slip. Positive friction is uphill.

Zero friction occurs at $T_0 = mg \sin \theta / \cos \beta = 77.2585$ N.

At $T = 70.0$ N, $N = 141.918$ N and $f_s = +7.01112$ N: 7.01 N uphill, below the 49.6715 N capacity.

Endpoint substitution gives $f_s = \pm \mu_s N$; the interior zero is a direction reversal within equilibrium.

Chapter support: Revised Chapter 6: Example 6.6

Connected bodies: each contact needs its own test

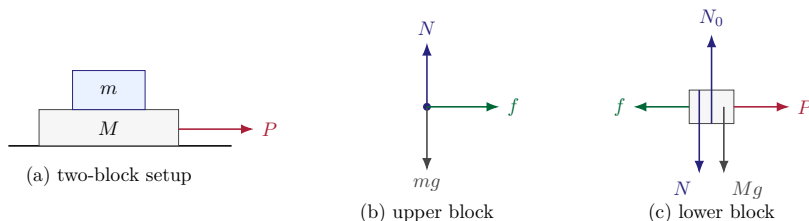


Figure 6.11: The upper and lower bodies require separate contact equations. Their interface forces are opposite third-law partners; the floor force N_0 is a different interaction. The isolated lower-block diagram offsets the origins of its parallel vertical forces to distinguish their interactions; their directions remain vertical. Force lengths are schematic.

On a smooth floor, sticking gives $a = P/(M + m)$. The upper body alone requires $f_s = ma$.

$$N_{\text{interface}} = mg, \quad |P| \leq (M + m)\mu_s g.$$

The distinct floor contact supports $(M + m)g$.

Chapter support: Revised Chapter 6

Example 6.7: a common acceleration that survives

A 4.00 kg upper block rests on a 10.0 kg lower block. The floor is frictionless, and the block–block coefficients are $\mu_s = 0.300$ and $\mu_k = 0.220$. Initially both are at rest. A force pulls the lower block rightward. Find both accelerations for $P = 40.0$ N and $P = 50.0$ N.

At $P = 40.0$ N, propose a common acceleration:

$$a = \frac{40.0}{14.0} = 2.85714 \text{ m s}^{-2},$$

$$f_{s,\text{req}} = 4.00a = 11.4286 \text{ N}.$$

Capacity is $\mu_s mg = 11.772$ N. Both accelerations are 2.86 m s^{-2} rightward. The largest compatible force is $(M + m)\mu_s g = 41.202$ N. Slide 27 completes the larger-force case.

Chapter support: Revised Chapter 6: Example 6.7

Example 6.7: two accelerations after interface slip

At $P = 50.0 \text{ N}$, the hypothetical common motion requires 14.2857 N at the interface, above its 11.772 N capacity.

The lower block moves ahead. Kinetic friction on the upper block is rightward, and its third-law partner on the lower block is leftward.

$$f_k = 0.220(4.00)(9.81) = 8.6328 \text{ N},$$

$$a_m = f_k/m = 2.1582 \text{ m s}^{-2}, \quad a_M = (50.0 - f_k)/10.0 = 4.13672 \text{ m s}^{-2}.$$

Report 2.16 and 4.14 m s^{-2} rightward, respectively. Because $a_M > a_m$, the assumed relative slip is consistent.

Independent force-accounting check

$ma_m + Ma_M = 50.0 \text{ N}$. The internal friction cancels even when a common acceleration no longer exists.

Chapter support: Revised Chapter 6: Example 6.7

A friction coefficient is an inference from a model

Slowly tilt a fixed bare incline and record the onset of sustained sliding. If sliding occurs before tipping and the surface remains unchanged,

$$\mu_s = \tan \theta_c, \quad u_{\mu_s} \simeq \sec^2 \theta_c u_\theta.$$

The derivative uses radians. Convert an angular uncertainty in degrees by multiplying it by $\pi/180$.

Repeated observations

Independent trial scatter contributes $s_\theta/\sqrt{n}$ to the uncertainty of the mean. A common calibration offset remains common to all trials. Combine independent standard uncertainty contributions in quadrature.

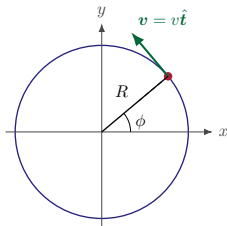
For kinetic tests, fit force against independently determined normal load. An intercept and structured residuals can expose a sensor offset or an inadequate constant-coefficient model.

Chapter support: Revised Chapter 6

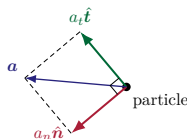
Curved motion changes the velocity direction

The product rule for $\mathbf{v} = v\hat{\mathbf{t}}$, using $d\hat{\mathbf{t}}/dt = (v/R)\hat{\mathbf{n}}$, gives

$$\mathbf{a} = \dot{v}\hat{\mathbf{t}} + (v^2/R)\hat{\mathbf{n}}.$$



(a) geometry and velocity



$$a_t = \dot{v} \text{ may have either sign;}$$
$$a_n = v^2/R \geq 0$$

(b) acceleration components

Figure 6.13: Circular geometry and the acceleration decomposition. Both panels use equal spatial units on the two axes. The acceleration sketch represents increasing speed; for decreasing speed the tangential arrow reverses while the normal component remains inward. Polar point placement and angle offsets follow the construction method studied in Neutelings' *Uniform circular motion* example; the present diagram and physical model are independently constructed.

Chapter support: Revised Chapter 6

A flat turn uses inward static friction

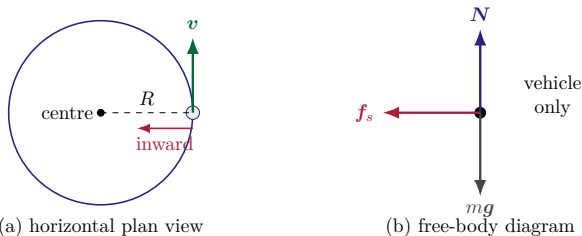


Figure 6.14: A flat circular turn requires inward static friction. The velocity appears in the geometry panel, and only real external forces appear in the free-body diagram. Arrow lengths are schematic.

For level motion with no other horizontal force:

$$N = mg, \quad f_s = mv^2/R \leq \mu_s N, \quad v_{\max} = \sqrt{\mu_s g R}.$$

“Centripetal” names the real-force inward resultant.

Chapter support: Revised Chapter 6

Example 6.9: speed limit and actual friction

A vehicle of model mass 900 kg follows a level turn of radius 40.0 m. The stipulated coefficient is $\mu_s = 0.650$. Find the largest constant speed admitted by the model, then test a speed of 12.0 m s^{-1} .

Vertical balance gives $N = 900(9.81) = 8829 \text{ N}$. Hence

$$v_{\max} = \sqrt{(0.650)(9.81)(40.0)} = 16.0 \text{ m s}^{-1}.$$

At 12.0 m s^{-1} :

$$f_s = \frac{900(12.0)^2}{40.0} = 3240 \text{ N} < \mu_s N = 5739 \text{ N}.$$

The actual friction is 3240 N inward, using 0.565 of capacity. No slip is compatible with the model.

Mass cancels from the limiting speed; $\mu_s \rightarrow 0$ removes every nonzero-speed turn in this two-force vertical model.

Chapter support: Revised Chapter 6: Example 6.9

Banking: the normal supplies inward force

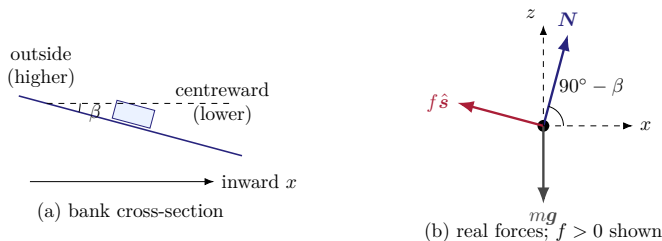


Figure 6.15: Bank geometry and actual forces, drawn at $\beta = 15^\circ$ with equal spatial scales. Positive signed friction f points up the bank, outward and upward. If the calculation gives $f < 0$, its actual direction is down the bank. The axis arrows are auxiliary directions and do not represent interactions.

Use inward x , upward z and uphill-positive f . Put $a = v^2/R$:

$$N \sin \beta - f \cos \beta = ma, \quad N \cos \beta + f \sin \beta = mg.$$

With $f = 0$, division gives the single ideal speed $v_0 = \sqrt{Rg \tan \beta}$.

Chapter support: Revised Chapter 6

The full no-slip speed window on a bank

Solving the component equations gives

$$N = m(g \cos \beta + a \sin \beta), \quad f = m(g \sin \beta - a \cos \beta).$$

For $0 \leq \beta < 90^\circ$ and $a \geq 0$, $N > 0$. Apply $-\mu_s N \leq f \leq \mu_s N$:

$$v_{\min}^2 = Rg \max \left[0, \frac{\sin \beta - \mu_s \cos \beta}{\cos \beta + \mu_s \sin \beta} \right],$$

$$v_{\max}^2 = Rg \frac{\sin \beta + \mu_s \cos \beta}{\cos \beta - \mu_s \sin \beta}, \quad \cos \beta > \mu_s \sin \beta.$$

If the last denominator is nonpositive, this ideal inequality gives no finite upper speed. If $\mu_s \geq \tan \beta$, the lower limit is zero.

Checks: $\mu_s = 0$ collapses the interval to v_0 ; $\beta = 0$ recovers the flat-turn bound.

Chapter support: Revised Chapter 6

Example 6.10: the banked-turn statement and geometry

For the bank in Figure 6.16, find the ideal speed, full no-slip speed interval and the signed friction and normal force at $v = 10.0 \text{ m s}^{-1}$ for a 1000 kg model vehicle.

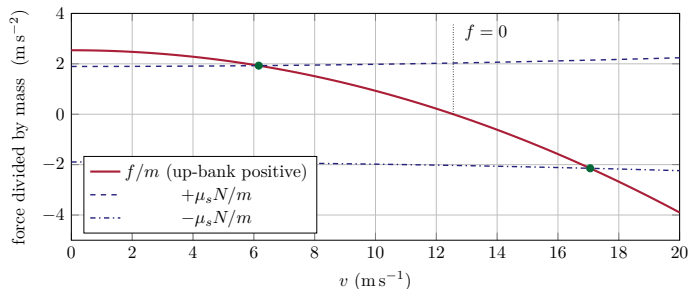


Figure 6.16: Calculated signed friction demand for $R = 60.0 \text{ m}$, $\beta = 15.0^\circ$ and $\mu_s = 0.200$. Static contact is possible where the red solid curve lies between the two blue limits. The lower-speed endpoint requires uphill friction; the higher-speed endpoint requires downhill friction. Values are model calculations, not road measurements.

Chapter support: Revised Chapter 6: Example 6.10

Example 6.10: both limits and a trial speed

Using $R = 60.0 \text{ m}$, $\beta = 15.0^\circ$ and $\mu_s = 0.200$:

$$v_0 = 12.6 \text{ m s}^{-1}, \quad 6.16 \leq v \leq 17.1 \text{ m s}^{-1}.$$

At $v = 10.0 \text{ m s}^{-1}$, $a = v^2/R = 1.6667 \text{ m s}^{-2}$. For $m = 1000 \text{ kg}$:

$$\begin{aligned} N &= 1000(9.81 \cos 15^\circ + 1.6667 \sin 15^\circ) \\ &= 9.91 \times 10^3 \text{ N}. \end{aligned}$$

$$f = 1000(9.81 \sin 15^\circ - 1.6667 \cos 15^\circ) = 929 \text{ N}.$$

Friction is uphill and $|f| < \mu_s N = 1.98 \times 10^3 \text{ N}$, so the trial is admissible. Component checks give 9810 N vertically and 1666.7 N inward. At the lower limit $f = +\mu_s N$; at the upper limit $f = -\mu_s N$.

Chapter support: Revised Chapter 6: Example 6.10

Conical pendulum: geometry sets the circle radius

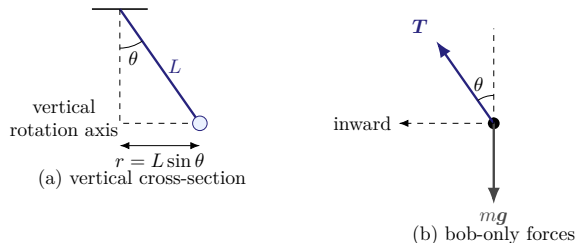


Figure 6.17: A conical pendulum at $\theta = 35^\circ$. The cross-section is drawn with equal scales and uses a string angle measured from the vertical. The circular path is perpendicular to this page; its radius is the horizontal distance to the axis. The body-only force diagram shows tension and weight, with no additional centripetal interaction.

A light taut string, fixed pivot and negligible air resistance give

$$r = L \sin \theta, \quad T \cos \theta = mg, \quad T \sin \theta = mv^2/r.$$

Thus $v^2 = rg \tan \theta$ and $\mathcal{T} = 2\pi \sqrt{L \cos \theta / g}$ for $0 < \theta < 90^\circ$.

Chapter support: Revised Chapter 6

Example 6.11: infer the conical motion

A 0.200 kg bob on a 1.20 m string maintains $\theta = 35.0^\circ$. Determine radius, tension, speed and period under the model above.

The stated model is a light taut string, fixed pivot, negligible air resistance and a particle bob. Apply the geometry and two force components:

$$r = 1.20 \sin 35.0^\circ = 0.688 \text{ m}, \quad T = \frac{0.200(9.81)}{\cos 35.0^\circ} = 2.40 \text{ N},$$

$$v = \sqrt{rg \tan 35.0^\circ} = 2.17 \text{ m s}^{-1}, \quad \mathcal{T} = 2\pi r/v = 1.99 \text{ s}.$$

Checks: $T \cos \theta = 1.962 \text{ N} = mg$ and $T \sin \theta = 1.37 \text{ N} = mv^2/r$ before rounding.

Mass affects tension but cancels from speed and period. The small-angle period limit describes small conical motion; an exactly resting bob does not complete revolutions.

Chapter support: Revised Chapter 6: Example 6.11

Inside and outside contacts have opposite signs

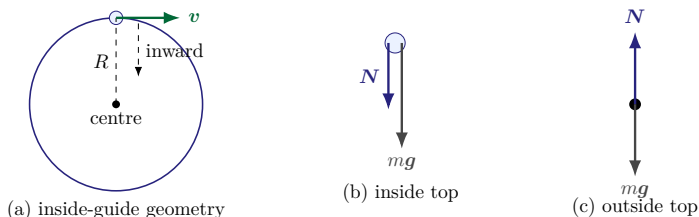


Figure 6.18: Inside-guide and outside-hill contact at the top of a vertical circle. The geometry panel contains velocity and an auxiliary inward marker; the two body-only panels contain the actual forces for different contact arrangements. Force lengths are schematic and a negative calculated N signals failure of the respective assumed contact.

At the top, choose inward/downward positive:

$$\begin{aligned}\text{Inside:} \quad & N = m(v^2/R - g), \quad v \geq \sqrt{gR}, \\ \text{Outside:} \quad & N = m(g - v^2/R), \quad v \leq \sqrt{gR}.\end{aligned}$$

Both require $N \geq 0$. Negative N rejects the assumed contact.

Chapter support: Revised Chapter 6

Example 6.12: local contact at an inside-loop top

A 40.0 kg carriage is intended to follow the inside of a circular guide of radius 3.00 m. Find the minimum local top speed compatible with the stated contact, then test 5.00 and 6.20 m s⁻¹.

Weight and the one-sided guide normal both point inward at the top:

$$v_{\min} = \sqrt{(9.81)(3.00)} = 5.42 \text{ m s}^{-1}.$$

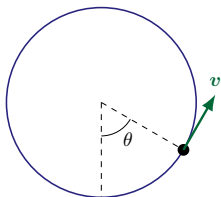
For 5.00 m s⁻¹, $N = 40.0(25.0/3.00 - 9.81) = -59.1 \text{ N}$. Contact is inadmissible.

For 6.20 m s⁻¹, $N = 40.0(38.44/3.00 - 9.81) = 120 \text{ N} > 0$. Contact is locally possible.

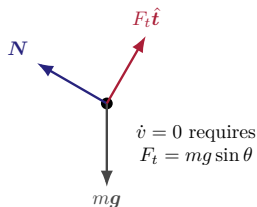
At the threshold, $N = 0$ and gravity alone supplies the required inward resultant. The given local speeds do not establish the entire earlier motion. Free flight after release uses the actual release position and tangent velocity as initial conditions.

Chapter support: Revised Chapter 6: Example 6.12

A driven vertical circle needs tangential force



(a) position from the bottom



(b) actual forces at $\theta = 60^\circ$

Figure 6.19: A prescribed-speed carriage on an inside vertical circle. The position angle is measured from the bottom and increases along the rising right side. The displayed tangential drive is an actual interaction needed to cancel the tangential gravity component; it is not a consequence of circular kinematics alone. Geometry uses equal scales; force lengths are schematic.

With θ measured from the bottom along the rising right side:

$$F_t - mg \sin \theta = m\dot{v}, \quad N - mg \cos \theta = mv^2/R.$$

For prescribed constant speed, $F_t = mg \sin \theta$ and $N = m(v^2/R + g \cos \theta)$. The actual drive cancels tangential gravity.

Chapter support: Revised Chapter 6

Example 6.13: loads around a driven circle

A 2.00 kg carriage moves at a prescribed constant 6.00 m s^{-1} in an inside circle of radius 2.00 m. A tangential actuator maintains the speed. Determine the normal load at bottom, side and top, and the drive force at $\theta = 60.0^\circ$.

Here $v^2/R = 18.0 \text{ m s}^{-2}$ and $mg = 19.62 \text{ N}$.

$$N_{\text{bottom}} = m(18.0 + g) = 55.6 \text{ N},$$

$$N_{\text{side}} = m(18.0) = 36.0 \text{ N}, \quad N_{\text{top}} = m(18.0 - g) = 16.4 \text{ N}.$$

All are positive. The minimum occurs at the top, so one-sided contact is compatible with the whole prescribed-speed circle.

$$F_t(60.0^\circ) = mg \sin 60.0^\circ = 17.0 \text{ N}$$

along the rising tangent. It balances tangential gravity, giving $\dot{v} = 0$. The independent speed threshold $\sqrt{gR} = 4.43 \text{ m s}^{-1} < 6.00 \text{ m s}^{-1}$ agrees.

Chapter support: Revised Chapter 6: Example 6.13

Local curvature extends circular dynamics

A smooth path has $\mathbf{a} = \dot{v}\hat{\mathbf{t}} + (v^2/R_c)\hat{\mathbf{n}}$. For a graph, $\tan\psi = y'$ and $ds/dx = \sqrt{1 + y'^2}$ give

$$R_c = (1 + y'^2)^{3/2}/|y''|.$$

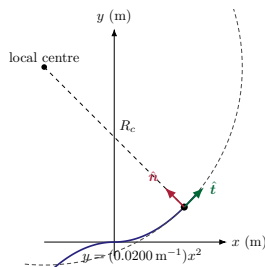


Figure 6.21: Local curvature of an analytic parabolic path at $x = 25.0$ m. The tangent makes 45° with the horizontal, the inward normal is perpendicular to it, and the dashed osculating circle has radius 70.7 m. Equal axis scales and analytic derivatives determine every geometric direction. Only the relevant portion of the circle is shown.

Chapter support: Revised Chapter 6

Example 6.15: force demand on a parabolic path

A 3.00 kg mobile platform is modelled as a particle following $y = (0.0200 \text{ m}^{-1})x^2$ at $x = 25.0 \text{ m}$. At that instant its speed is 10.0 m s^{-1} and it is speeding up at 2.00 m s^{-2} . Find the local radius, the resultant force components and the Cartesian acceleration.

At $x = 25.0 \text{ m}$, $y' = 1.00$ and $y'' = 0.0400 \text{ m}^{-1}$, so $R_c = 70.7107 \text{ m}$. For increasing x :

$$\hat{\mathbf{t}} = (1, 1)/\sqrt{2}, \quad \hat{\mathbf{n}} = (-1, 1)/\sqrt{2}, \quad a_n = 1.4142 \text{ m s}^{-2}.$$

Thus $\sum F_t = ma_t = 6.00 \text{ N}$ and $\sum F_n = ma_n = 4.24 \text{ N}$.

$$\mathbf{a} = 2\hat{\mathbf{t}} + \sqrt{2}\hat{\mathbf{n}} = (0.414, 2.41) \text{ m s}^{-2}.$$

Magnitude check: $|\mathbf{a}| = \sqrt{a_t^2 + a_n^2} = \sqrt{6} = 2.45 \text{ m s}^{-2}$.

These are net requirements. Individual forces need a support/propulsion model.

Chapter support: Revised Chapter 6: Example 6.15

Turning and acceleration share one traction budget

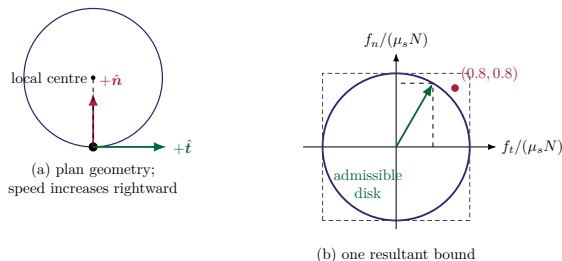


Figure 6.22: Local path geometry and the resultant friction constraint. The tangent and inward normal follow the actual path geometry. In component space the isotropic friction circle bounds the resultant, while separate component tests allow the dashed square and wrongly admit its corners. The marked $(0.8, 0.8)$ demand violates the circular bound even though each component is individually below one. For level motion with no other horizontal force, $N = mg$, $f_t = ma_t$ and $f_n = mv^2/R_c$. The vector bound is

$$a_t^2 + (v^2/R_c)^2 \leq (\mu_s g)^2.$$

Chapter support: Revised Chapter 6

Example 6.16: shared traction while speeding up

A vehicle is modelled as a particle on a level road with $\mu_s = 0.700$. It follows a radius $R = 50.0\text{ m}$ while increasing its speed at $a_t = 3.00\text{ m s}^{-2}$. Find the maximum instantaneous speed compatible with no slip in the isotropic friction-circle model, and compare it with the constant-speed limit.

The total acceleration capacity is $\mu_s g = 6.867\text{ m s}^{-2}$. Subtract the squared tangential demand before converting to speed:

$$a_{n,\max} = \sqrt{6.867^2 - 3.00^2} = 6.17703\text{ m s}^{-2},$$

$$v_{\max} = \sqrt{50.0 a_{n,\max}} = 17.6\text{ m s}^{-1} = 63.3\text{ km h}^{-1}.$$

The constant-speed limit is $\sqrt{\mu_s g R} = 18.5\text{ m s}^{-1}$.

At the combined limit, $f_t/(\mu_s N) = 0.437$ and $f_n/(\mu_s N) = 0.900$; their squared sum is one before rounding. Braking at -3.00 m s^{-2} uses the same magnitude budget.

Chapter support: Revised Chapter 6: Example 6.16

Core practice 2: P06-16, remaining acceleration

In the simplified friction-circle model, a vehicle with $\mu_s = 0.75$ negotiates a 60 m radius turn at 15 m s^{-1} . Find the largest simultaneous tangential acceleration magnitude without exceeding the force budget. Explain why the mass cancels.

Solution. The assumptions give $N = mg$, $f_t = ma_t$ and $f_n = ma_n$. Dividing the single force bound by mass gives

$$a_n = \frac{15^2}{60} = 3.75 \text{ m s}^{-2}, \quad |a_t|_{\max} = \sqrt{(0.75g)^2 - 3.75^2} = 6.33 \text{ m s}^{-2}.$$

Mass cancels because demand and capacity both scale with it. The normalized boundary components are 0.860362 and 0.509684, whose squared sum is one.

The magnitude bound admits either speeding up or braking in this ideal symmetric model. It does not establish equal real-vehicle performance in both directions.

Chapter support: Revised Chapter 6: P06-16

Drag depends on velocity relative to the medium

With $v_{\text{rel}} = v - u$ and prescribed medium velocity u :

$$F_d = -bv_{\text{rel}}, \quad [b] = \text{kg s}^{-1},$$

$$F_d = -c|v_{\text{rel}}|v_{\text{rel}}, \quad [c] = \text{kg m}^{-1}.$$

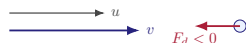
body slower than medium



body matches medium



body faster than medium



Positive direction: right. Arrow lengths are schematic.

Figure 6.24: The medium-relative velocity determines the sign of drag. All three bodies move right in the laboratory, but the drag can point right, vanish, or point left. The arrows compare velocities within a row; they are not measured data.

Both forces oppose relative motion, including for $v_{\text{rel}} < 0$.

Chapter support: Revised Chapter 6

Terminal motion is a stable force balance

In a stationary medium without buoyancy, terminal balance is $mg = |F_d|$:

$$v_{t,L} = mg/b, \quad v_{t,Q} = \sqrt{mg/c}.$$

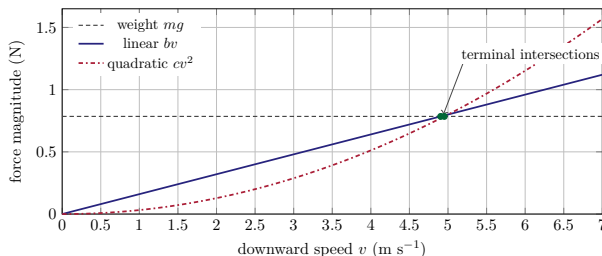


Figure 6.25: The stipulated baseline models have $m = 0.0800 \text{ kg}$, $b = 0.160 \text{ kg s}^{-1}$, and $c = 0.0320 \text{ kg m}^{-1}$. Their terminal intersections nearly coincide, but their force predictions away from the intersections differ. These are calculated force magnitudes; the direction of drag is upward for downward motion.

Below equilibrium weight wins; above it drag wins. Both restore speed toward the positive terminal value.

Chapter support: Revised Chapter 6

Example 6.17: similar terminals, different transients

A body has $m = 0.0800 \text{ kg}$. Compare the linear model $b = 0.160 \text{ kg s}^{-1}$ with the quadratic model $c = 0.0320 \text{ kg m}^{-1}$. Find both terminal speeds and the acceleration at downward speed 2.50 m s^{-1} .

For the stationary medium and downward-positive velocity, $mg = 0.7848 \text{ N}$:

$$v_{t,L} = 0.7848/0.160 = 4.905 \text{ m s}^{-1},$$
$$v_{t,Q} = \sqrt{0.7848/0.0320} = 4.95227 \text{ m s}^{-1}.$$

At $v = 2.50 \text{ m s}^{-1}$, linear drag is 0.400 N and quadratic drag is 0.200 N :

$$a_L = 9.81 - 0.400/0.0800 = 4.81 \text{ m s}^{-2},$$
$$a_Q = 9.81 - 0.200/0.0800 = 7.31 \text{ m s}^{-2}.$$

The terminal difference is only 0.0473 m s^{-1} , but the transient accelerations differ by 2.50 m s^{-2} . Both lie between zero and g , as their subterminal speeds require.

Chapter support: Revised Chapter 6: Example 6.17

Linear drag: solve the initial-value problem

Define $\tau = m/b$ and $v_t = mg/b = g\tau$. Newton's equation becomes $\dot{v} = (v_t - v)/\tau$. Separate variables for $v_0 \neq v_t$:

$$\int_{v_0}^{v(t)} \frac{dw}{v_t - w} = \frac{t}{\tau}, \quad -\ln \left| \frac{v_t - v(t)}{v_t - v_0} \right| = \frac{t}{\tau}.$$

Continuity preserves the sign of $v_t - v$, giving

$$v(t) = v_t + (v_0 - v_t)e^{-t/\tau}.$$

The constant solution $v_0 = v_t$ is included. Integrate $\dot{y} = v$ with downward-positive y :

$$y - y_0 = v_t t + \tau(v_0 - v_t)(1 - e^{-t/\tau}).$$

Differentiation recovers the ODE and $\dot{y} = v$; $t = 0$ recovers both initial conditions. From rest at small t , $v \simeq gt$ and $y - y_0 \simeq gt^2/2$.

Chapter support: Revised Chapter 6

Example 6.19: release from rest, complete statement

For the baseline linear model, find the speed and fall distance after 0.500, 1.50, and 2.50 s. Determine when the speed first reaches 90.0% of terminal speed.

Baseline: stationary medium, downward positive, $v_0 = y_0 = 0$.

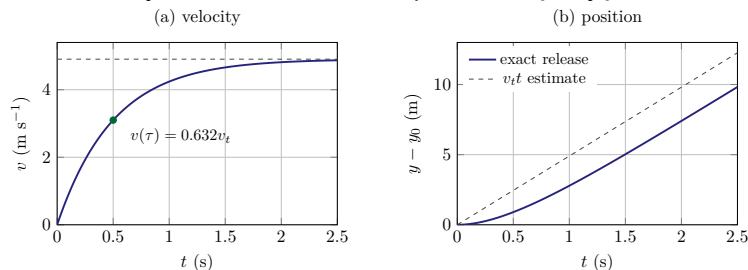


Figure 6.27: A linear-drag release from rest with $m = 0.0800 \text{ kg}$ and $b = 0.160 \text{ kg s}^{-1}$ has $\tau = 0.500 \text{ s}$ and $v_t = 4.905 \text{ m s}^{-1}$. The initial position curve has zero slope; replacing the full transient by terminal motion overestimates the fall distance. Both panels are analytical calculations.

Chapter support: Revised Chapter 6: Example 6.19

Example 6.19: speeds, distances and ninety percent

The baseline has $\tau = 0.500 \text{ s}$ and $v_t = 4.905 \text{ m s}^{-1}$:

$$v = 4.905(1 - e^{-t/0.500}), \quad y = 4.905[t - 0.500(1 - e^{-t/0.500})].$$

Here numerical t is entered in seconds, giving v in m s^{-1} and y in metres.

$t \text{ (s)}$	$v \text{ (m s}^{-1}\text{)}$	$y \text{ (m)}$
0.500	3.10	0.902
1.50	4.66	5.03
2.50	4.87	9.83

Solving $p = 1 - e^{-t_p/\tau}$ gives $t_p = -\tau \ln(1 - p)$, hence $t_{90} = 0.500 \ln 10 = 1.15 \text{ s}$.

At one, three and five time constants, the speed fractions are 63.2%, 95.0% and 99.3%. Every distance is below $v_t t$, and $v_t/\tau = g$ checks the initial acceleration.

Chapter support: Revised Chapter 6: Example 6.19

Quadratic drag: derive the release branch

For release from rest, $v \geq 0$. Set $q = v/v_t$ with $v_t = \sqrt{mg/c}$. Separation gives

$$\int_0^q \frac{dw}{1-w^2} = \frac{1}{2} \ln \frac{1+q}{1-q} = \frac{gt}{v_t}.$$

Solving, with $z = gt/v_t$:

$$q = \frac{e^{2z} - 1}{e^{2z} + 1} \equiv \tanh z, \quad v = v_t \tanh(gt/v_t).$$

Since $\cosh z = (e^z + e^{-z})/2$ and $d \ln \cosh z / dz = \tanh z$, integration gives

$$y - y_0 = \frac{v_t^2}{g} \ln \cosh(gt/v_t).$$

The derivative of $\tanh z$ is $1 - \tanh^2 z$, which verifies the ODE. At small t , $y - y_0 \simeq gt^2/2$; at large t , the slope approaches v_t . This branch assumes $v_0 = 0$.

Chapter support: Revised Chapter 6

Example 6.21: quadratic speed, distance and approach

For $m = 0.0800 \text{ kg}$ and $c = 0.0320 \text{ kg m}^{-1}$, find the speed and fall distance after 1.00 s and the time to reach $0.900 v_t$.

Release is from rest at $y_0 = 0$ in a stationary medium, downward positive, with no buoyancy. Here $v_t = 4.95227 \text{ m s}^{-1}$ and $gt/v_t = 1.98091$ at one second:

$$v(1.00) = 4.95227 \tanh(1.98091) = 4.77 \text{ m s}^{-1},$$

$$y(1.00) = \frac{4.95227^2}{9.81} \ln \cosh(1.98091) = 3.27 \text{ m}.$$

The exponential definition yields

$$t_p = \frac{v_t}{2g} \ln \frac{1+p}{1-p}, \quad t_{90} = 0.743 \text{ s}.$$

The speed is below v_t and the distance below the no-drag 4.905 m . Its shorter 90% time reflects weaker subterminal resistance than the baseline linear model.

Chapter support: Revised Chapter 6: Example 6.21

Identical terminal speeds do not identify the law

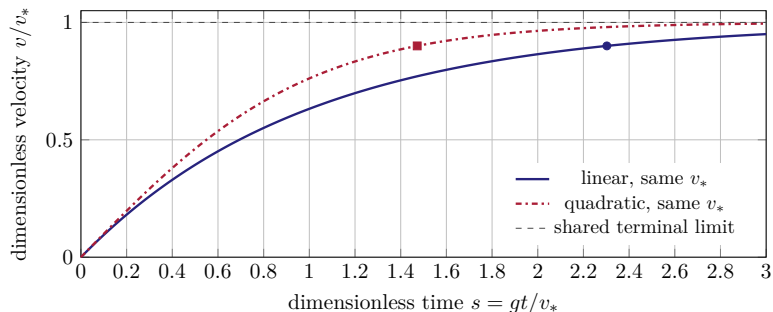


Figure 6.28: Two models calibrated to the same terminal speed v_* still predict distinct transients from rest. For this comparison, $b = mg/v_*$ and $c = mg/v_*^2$; it is a normalized analytical comparison, separate from the slightly different baseline terminal speeds. The 90% times are $s = \ln 10$ and $s = \frac{1}{2} \ln 19$.

At $v = \alpha v_*$, $a_L = g(1 - \alpha)$ while $a_Q = g(1 - \alpha^2)$. They differ for $0 < \alpha < 1$. Both models begin at $a = g$ and approach $a = 0$. Intermediate observations identify differences that either endpoint misses.

Chapter support: Revised Chapter 6

Numerical integration: slopes and step size

For $\dot{v} = f(v) = g - (c/m)v|v|$, a time step h advances velocity. The explicit midpoint method is

$$k_1 = f(v_n), \quad v_{n+1/2} = v_n + \frac{1}{2}hk_1, \quad v_{n+1} = v_n + hf(v_{n+1/2}).$$

Fourth-order Runge–Kutta (RK4) uses

$$\begin{aligned} k_1 &= f(v_n), & k_2 &= f(v_n + hk_1/2), \\ k_3 &= f(v_n + hk_2/2), & k_4 &= f(v_n + hk_3), \\ v_{n+1} &= v_n + \frac{h}{6}(k_1 + 2k_2 + 2k_3 + k_4). \end{aligned}$$

Every k_i has acceleration units and hk_i has velocity units. The intermediate states are estimates used to sample slopes.

Over a fixed smooth interval, small-step global errors approach order h^2 for midpoint and h^4 for RK4. Refinement must demonstrate that regime.

Chapter support: Revised Chapter 6

Example 6.23: independent transient benchmark

Integrate the baseline quadratic release on $0 \leq t \leq 2.00\text{ s}$ using midpoint and RK4. Compare each run with the analytical solution at all its time nodes and interpret the refinement pattern.

Baseline restored: $m = 0.0800\text{ kg}$, $c = 0.0320\text{ kg m}^{-1}$, stationary medium, $v(0) = 0$.

$$E_h = \max_n |v_n - v_t \tanh(gt_n/v_t)|.$$

$h\text{ (s)}$	midpoint $E_h\text{ (m s}^{-1}\text{)}$	RK4 $E_h\text{ (m s}^{-1}\text{)}$
0.200	7.0133×10^{-2}	2.6170×10^{-3}
0.100	1.3860×10^{-2}	1.3619×10^{-4}
0.050	3.1439×10^{-3}	7.7038×10^{-6}
0.025	7.4613×10^{-4}	4.5782×10^{-7}
0.0125	1.8174×10^{-4}	2.7910×10^{-8}

Exact endpoint: $v(2.00) = 4.948687\text{ m s}^{-1}$. Interpretation follows on slide 58.

Chapter support: Revised Chapter 6: Example 6.23

Example 6.23: refinement validates the computation

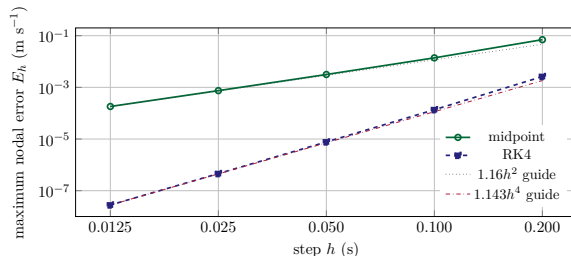


Figure 6.31: Independent benchmark errors on $0 \leq t \leq 2.00$ s reveal second- and fourth-order refinement. The two guide lines show slopes, not exact error laws. For the displayed formulas h is entered as its numerical value in seconds and error is shown in m s^{-1} .

Final halving ratios are 4.11 (midpoint) and 16.4 (RK4), approaching 2^2 and 2^4 .

Midpoint at $h = 0.025$ s meets $E_h < 0.001 \text{ m s}^{-1}$. Terminal attraction can mask larger early errors in an endpoint-only check.

Refinement tests the ODE solution; measurements test the model.

Chapter support: Revised Chapter 6: Example 6.23

Testing a force law: fitting and residuals

A force-speed test should measure speed relative to the body, subtract sensor zero offset and control geometry and medium conditions. Terminal speed supplies only one balance equation.

For an origin-constrained model $F_i = \alpha x_i$ with equal independent force uncertainties, minimize

$$S(\alpha) = \sum_i (F_i - \alpha x_i)^2, \quad \frac{dS}{d\alpha} = -2 \sum_i x_i (F_i - \alpha x_i) = 0.$$

Therefore

$$\hat{\alpha} = \frac{\sum_i x_i F_i}{\sum_i x_i^2}, \quad r_i = F_i - \hat{\alpha} x_i, \quad \chi^2 = \sum_i (r_i / \sigma_F)^2.$$

Choose $x_i = v_i$ for linear drag and $x_i = v_i^2$ for quadratic drag. Inspect residual structure as well as its magnitude. Synthetic teaching data are distinct from performed measurements.

Chapter support: Revised Chapter 6

Core synthesis and bridge to the next chapter

- Contact: find the required force, then test $N \geq 0$ and $|f_s| \leq \mu_s N$. A failed regime needs new kinematics.
- Curved motion: real interactions supply $m\dot{v}$ tangentially and mv^2/R_c inward. Shared traction is one resultant bound.
- Resistance: medium-relative velocity fixes drag direction. Terminal balance and complete transients answer different questions.
- Evidence: residuals test a force model; independent benchmarks and refinement test its numerical solution.

Chapter 7 develops work, kinetic energy and power from Newton's law. Those methods can compare states efficiently, while the contact admissibility tests remain necessary.

Reading: the supplied chapter's omitted examples and exercises, especially hill release (6.14), friction calibration (6.8), upward launch (6.20), and full force-speed fitting (6.22).

Chapter support: Revised Chapter 6

Advanced: translating frames from vector geometry

For parallel, nonrotating axes, differentiate $\mathbf{r} = \mathbf{R} + \mathbf{r}'$ twice:

$$\mathbf{a} = \mathbf{A} + \mathbf{a}', \quad \sum \mathbf{F}_{\text{real}} - m\mathbf{A} = m\mathbf{a}'.$$

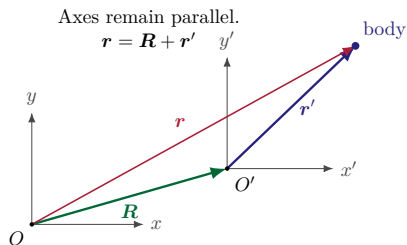


Figure 6.32: Vector geometry for a nonrotating translating frame. The transformation is valid for a prescribed origin motion $\mathbf{R}(t)$, including time-dependent translation. For the steady examples below the platform acceleration is constant. Lengths in this coordinate sketch are schematic.

The inertial term is a coordinate correction with no third-law partner. $\mathbf{A} = 0$ recovers inertial equations; rotating axes need additional derivative terms.

Chapter support: Revised Chapter 6

Advanced worked comparison: P06-32

A platform accelerates right at 4.0 m s^{-2} . A block rests on its level floor with $\mu_s = 0.35$. Decide whether it can remain fixed relative to the platform using (a) the laboratory frame and (b) the platform frame. Explain what information is needed to predict the ensuing sliding acceleration.

Laboratory (a): sticking demands $a_x = A = 4.0 \text{ m s}^{-2}$. Since $N = mg$, required $f_s = mA$ exceeds $0.35mg$:

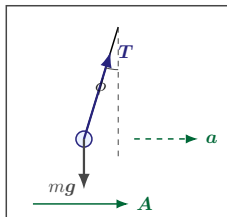
$$\mu_s g = 3.4335 \text{ m s}^{-2} < 4.0 \text{ m s}^{-2}.$$

Platform (b): hypothetical rest has $a'_x = 0$, so $f_s - mA = 0$. The same inequality fails. The conclusions agree.

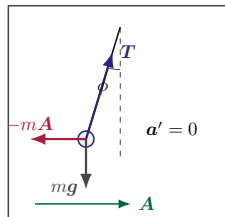
The block lags left relative to the platform. With a supplied kinetic coefficient, $a_x = \mu_k g$ and $a'_x = \mu_k g - A$. No numerical sliding acceleration is determined here because μ_k is not given.

Chapter support: Revised Chapter 6: P06-32

Advanced accelerometer: geometry before calibration



(a) laboratory: real forces



(b) platform: inertial term added

Figure 6.33: Correct string geometry for rightward platform acceleration: the bob hangs left of its support and tension pulls upward and right. Both panels show the same physical configuration, with tilt $\phi = \tan^{-1}(3.00/9.81)$. The laboratory acceleration arrow is dashed to distinguish a kinematic quantity from a force; the platform panel alone includes the inertial term. Force lengths are schematic. For a settled bob, $T \sin \phi = mA$ and $T \cos \phi = mg$, so $\tan \phi = A/g$. Platform balance uses the same real forces plus $-m\mathbf{A}$, with $\mathbf{a}' = 0$.

Chapter support: Revised Chapter 6

Example 6.24: tilt and tension in two frames

A 0.250 kg bob hangs from a light string in a nonrotating platform accelerating right at 3.00 m s^{-2} . Find the steady tilt from vertical and the string tension. Then infer the acceleration corresponding to a measured steady tilt of $(12.0 \pm 0.5)^\circ$.

For a settled bob behind the support, real force components give

$$\phi = \arctan(A/g) = \arctan(3.00/9.81) = 17.0^\circ,$$

$$T = m\sqrt{g^2 + A^2} = 0.250\sqrt{9.81^2 + 3.00^2} = 2.56 \text{ N}.$$

In the platform frame, $\mathbf{T} + m\mathbf{g} - m\mathbf{A} = 0$ gives the same components. The inverse measured-angle part and its uncertainty are completed on slide 65.

Chapter support: Revised Chapter 6: Example 6.24

Example 6.24: inverse calibration and uncertainty

For the measured steady tilt $(12.0 \pm 0.5)^\circ$, use the exact calibration:

$$A = g \tan 12.0^\circ = 2.08518 \text{ m s}^{-2}.$$

Interpret the stated angular uncertainty as a small standard uncertainty and hold g fixed. Convert to radians:

$$\sigma_\phi = 0.5\pi/180 = 0.00872665 \text{ rad}, \quad \sigma_A = g \sec^2 \phi \sigma_\phi = 0.0895 \text{ m s}^{-2}.$$

Report $(2.09 \pm 0.09) \text{ m s}^{-2}$ under that uncertainty model. A leftward bob displacement indicates rightward acceleration.

Mass cancels from the angle calibration but affects tension. As $A \rightarrow 0$, $\phi \rightarrow 0$ and $T \rightarrow mg$.

The inference assumes settled motion, no rotation and no vertical platform acceleration. More generally, $\tan \phi = |A_x|/(g + A_y)$ for $g + A_y > 0$; angle alone then cannot isolate A_x .

Chapter support: Revised Chapter 6: Example 6.24

Advanced incline: project the effective field

Let the plane rise rightward at θ , with signed platform acceleration A positive rightward. Choose

$$\hat{\mathbf{t}} = (\cos \theta, \sin \theta), \quad \hat{\mathbf{n}} = (-\sin \theta, \cos \theta).$$

The effective force per mass is $(-A, -g)$. Its projections are $-D$ and $-Q$, where

$$D = g \sin \theta + A \cos \theta, \quad Q = g \cos \theta - A \sin \theta.$$

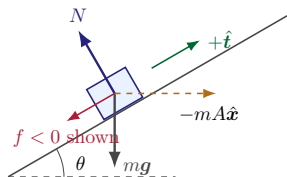
For maintained contact with zero relative normal acceleration:

$$N = mQ, \quad f_{s,\text{req}} = mD, \quad Q \geq 0, \quad |D| \leq \mu_s Q.$$

During established slip, $a'_t = -D - \mu_k Q \operatorname{sgn}(v'_t)$. If $Q < 0$, set $N = f = 0$ after detachment and use $a'_n = -Q > 0$.

Chapter support: Revised Chapter 6

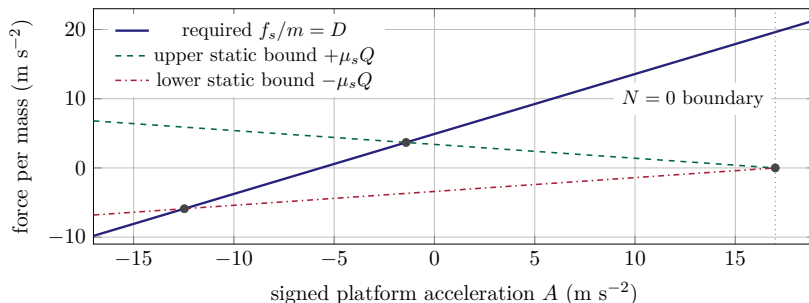
Advanced incline: the contact and static envelopes



$$D = g \sin \theta + A \cos \theta$$

$$Q = g \cos \theta - A \sin \theta$$

Contact requires $Q \geq 0$.



Full caption and challenge statement follow on slide 68.

Chapter support: Revised Chapter 6

Advanced challenge: Example 6.25, full statement

Figure 6.34: A plane rising rightward on a translating platform. The upper panel shows the chosen directions and real forces plus the explicitly identified platform-frame inertial term for a leftward acceleration; its arrow lengths are schematic. The lower panel is quantitative for $\theta = 30.0^\circ$ and $\mu_s = 0.400$. Sticking occurs where the required signed friction lies between the two static bounds and $N \geq 0$. Beyond the contact boundary the supported-contact formulas are inapplicable.

A 2.00 kg block rests on an incline rising rightward at 30.0° inside a horizontally translating platform. The stipulated coefficients are $\mu_s = 0.400$ and $\mu_k = 0.300$. Find the signed acceleration range permitting rest relative to the incline. Test $A = -6.00, 0, -15.0$, and $+20.0\text{ m s}^{-2}$.

The complete interval derivation and the four case tests follow on slides 69–70.

Chapter support: Revised Chapter 6: Example 6.25

Example 6.25: interval, sticking and downhill slip

For $\theta = 30.0^\circ$, $D = 4.905 + 0.866025A$ and $Q = 8.49571 - 0.500A$ in m s^{-2} . Contact requires $A \leq 16.9914 \text{ m s}^{-2}$.

Solve $-\mu_s Q \leq D \leq \mu_s Q$, with positive denominators:

$$-\frac{g(\mu_s \cos \theta + \sin \theta)}{\cos \theta - \mu_s \sin \theta} \leq A \leq \frac{g(\mu_s \cos \theta - \sin \theta)}{\cos \theta + \mu_s \sin \theta}.$$

The range $-12.47 \leq A \leq -1.413 \text{ m s}^{-2}$ also satisfies $N \geq 0$.

$A = -6.00$: $N = 22.99 \text{ N}$, $f_s = -0.5823 \text{ N}$ downhill, and $\mu_s N = 9.20 \text{ N}$.

Sticking survives.

$A = 0$: $Q = 8.49571 > 0$, but $D = 4.905 > 0.400Q$. Downhill slip gives $a'_t = -D + \mu_k Q = -2.36 \text{ m s}^{-2}$, consistent with its emerging direction.

Chapter support: Revised Chapter 6: Example 6.25

Example 6.25: uphill slip, detachment and synthesis

$A = -15.0 \text{ m s}^{-2}$: $Q = 15.9957 > 0$ and $D = -8.08538 < -0.400Q$.
Contact survives, but uphill slip emerges:

$$a'_t = -D - \mu_k Q = +3.29 \text{ m s}^{-2}.$$

$A = +20.0 \text{ m s}^{-2}$: $Q = -1.50429 \text{ m s}^{-2}$ would require $N = mQ = -3.01 \text{ N}$.
The plane cannot pull, so the block detaches:

$$N = f = 0, \quad a'_n = -Q = +1.50 \text{ m s}^{-2}.$$

Its detached relative acceleration is $\mathbf{a}' = (-20.0, -9.81) \text{ m s}^{-2}$. Adding $\mathbf{A} = (20.0, 0)$ returns laboratory free fall, $\mathbf{a} = (0, -9.81)$.

Synthesis. Transform accelerations consistently, then test each real interaction. Frictionless relative rest needs $D = 0$, so $A = -g \tan \theta$ and $Q = g / \cos \theta > 0$: acceleration is toward the lower end.

Chapter support: Revised Chapter 6: Example 6.25

A top-down view of a wooden desk with a spiral-bound notebook in the center. The notebook's pages are white, and the words "THANK YOU!" are printed in a large, bold, black sans-serif font. The exclamation mark is red. To the top-left of the notebook, a pair of gold-rimmed glasses with black temples lies on the desk. To the right, a silver and black ballpoint pen is positioned vertically. In the bottom-left corner, a small white pot holds a green succulent plant with thick, pointed leaves. The desk surface is made of light-colored wood with a visible grain.