

Kinetic Energy and Work

Physics I - Chapter 7

Hamzeh Khanpour

hamzeh.khanpour@agh.edu.pl

Faculty of Physics and Applied Computer Science
AGH University of Krakow

Lecturer information and contact details



Lecturer

Hamzeh Khanpour

High Energy Particle Physicist, AGH University of Krakow || CERN-TH

How to contact me

E-mail: hamzeh.khanpour@fis.agh.edu.pl; hamzeh.khanpour@agh.edu.pl

CERN e-mail: hamzeh.khanpour@cern.ch

Where to find me

Office & Room: D10 & 218

You are welcome to contact me or stop by for discussion at any time.

Website and course materials

<https://view.fis.agh.edu.pl/staff/hamzeh.khanpour/>

<https://home.agh.edu.pl/~hamzeh.khanpour/>

<https://hamzeh-khanpour.web.cern.ch/>

How do forces change speed?

Newton's laws give acceleration along a trajectory. Work connects the initial and final motion through a scalar energy transfer: how much does a force transfer as its application point moves?

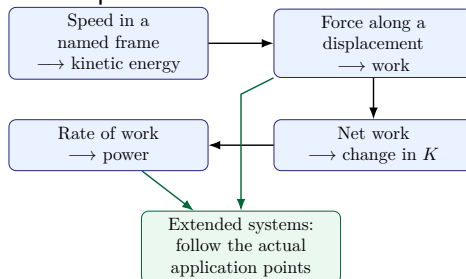


Figure 7.1: Conceptual route through Chapter 7. The core links particle speed, work and kinetic energy; later sections use the same accounting ideas for power, measurements, computation and deformable systems.

Chapter support: Revised Chapter 7, opening roadmap and Figure 7.1.

Learning route and prerequisites



- 1 **Describe motion:** name the inertial frame and calculate kinetic energy from speed.
- 2 **Calculate transfers:** project constant forces; integrate signed variable-force work.
- 3 **Predict and test:** derive $W_{\text{net}} = \Delta K$, find speeds and check every accessible position.
- 4 **Extend the model:** resolve work rates, numerical data and actual force-application points.

Tools: vectors, dot products, trigonometry, Newton's second law, derivatives and definite integrals. The advanced block introduces parametrized curves and relative motion.

Chapter support: Revised Chapter 7: Conventions and notation.

What you will be able to do



- Calculate K in a named frame and distinguish speed from signed velocity.
- Compute work by each force, preserving projections, signs and path orientation.
- Derive the particle work theorem; predict speed, maxima and turning points with a whole-path check.
- Integrate gravity and spring forces and explain power, efficiency and finite-speed drive laws.
- Estimate work from data and distinguish numerical error from missing physical information.
- Use actual application-point motion; extend the calculation to line integrals and deformable systems.

Chapter support: Revised Chapter 7: Chapter learning goals.

Kinetic energy retains speed, not direction



For a fixed-mass particle in a named inertial frame, at nonrelativistic speed,

$$K = \frac{1}{2} m \mathbf{v} \cdot \mathbf{v} = \frac{1}{2} m (v_x^2 + v_y^2 + v_z^2) \geq 0.$$

One joule is $1 \text{ kg m}^2 \text{ s}^{-2}$. In one dimension, equal $+v_x$ and $-v_x$ give equal K .

Scaling and an answered check

Doubling speed multiplies K by four; doubling mass at fixed speed multiplies it by two. Does $K = 0$ imply zero force? No: a particle can be instantaneously at rest with nonzero acceleration.

Chapter support: Revised Chapter 7, Section 7.1.

Changing the observer: Example 7.1



A frame moving at constant velocity $\mathbf{u}$ measures $\mathbf{v}' = \mathbf{v} - \mathbf{u}$, so $K' = \frac{1}{2}m|\mathbf{v} - \mathbf{u}|^2$. Mass stays fixed; the velocity and energy assignments change.

Full statement: moving inspection station

A 1200 kg inspection carriage moves at 8.0 m s^{-1} relative to the laboratory. A second inspection station moves in the same direction at 5.0 m s^{-1} . Find the carriage kinetic energy in the laboratory and in the moving station frame.

Chapter support: Chapter 7, Example 7.1.

Example 7.1: two consistent energies

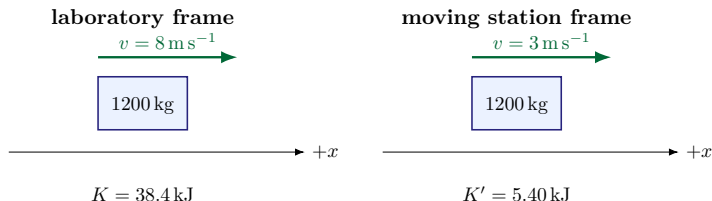


Figure 7.2: The carriage speed is 8.0 m s^{-1} in the laboratory and 3.0 m s^{-1} in a station moving at $u = 5.0 \text{ m s}^{-1}$ in the same direction. The arrows specify direction and are schematic in length. Each kinetic energy uses the speed measured in its labelled inertial frame.

In the station frame, $v' = 8.0 - 5.0 = 3.0 \text{ m s}^{-1}$. Direct substitution gives $\frac{1}{2}(1200)(8.0)^2 = 38400 \text{ J}$ and $\frac{1}{2}(1200)(3.0)^2 = 5400 \text{ J}$. A carriage rider measures $K'' = 0$; each result belongs to its frame.

Chapter support: Chapter 7, Example 7.1.

Example 7.2: equal energy, unequal mass



Inverting the definition gives $v = \sqrt{2K/m}$ for speed. It supplies no independent direction information.

Full statement

Two translating instrument packages move on horizontal low-friction guides in the same laboratory. Package A has mass 0.080 kg; package B has mass 0.320 kg. Each has kinetic energy 4.00 J. Find their speeds. If A moves toward $+x$, does that information determine B's velocity direction? Finally, how much kinetic energy would A have at twice its calculated speed?

Chapter support: Chapter 7, Example 7.2.

Example 7.2: speeds and direction



Treat each translating package as a particle, with negligible rotation. At common K ,

$$\frac{v_A}{v_B} = \sqrt{\frac{m_B}{m_A}} = \sqrt{4} = 2.$$

$$v_A = \sqrt{\frac{2(4.00)}{0.080}} = 10.0 \text{ m s}^{-1}, \quad v_B = \sqrt{\frac{2(4.00)}{0.320}} = 5.00 \text{ m s}^{-1}.$$

B can move either way along its guide. A's direction does not constrain B. Doubling A's speed gives $K'_A = 4K_A = 16.0 \text{ J}$.

Check

Substitution into $\frac{1}{2}mv^2$ gives 4.00 J for both. Equal energy establishes neither equal speed nor equal velocity.

Chapter support: Chapter 7, Example 7.2.

Work is a signed projection



A specified constant force transfers work through the displacement of its application point:

$$W_{\mathbf{F}} = \mathbf{F} \cdot \Delta \mathbf{r} = F \Delta r \cos \theta = F_x \Delta x + F_y \Delta y.$$

θ is the angle between these two vectors. Work is positive along the force, zero for perpendicular displacement and negative against the force.

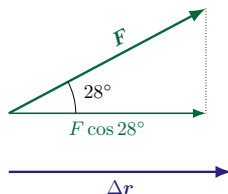
Keep the force identity

If several forces act on the same particle, $W_{\text{net}} = \sum_i W_i$. Work by a cable alone is a different quantity from net work. Their SI unit is $\text{N m} = \text{J}$.

Chapter support: Revised Chapter 7, Section 7.2.

Geometry before arithmetic: an angled cable

(a) Projection geometry



(b) Particle free-body diagram

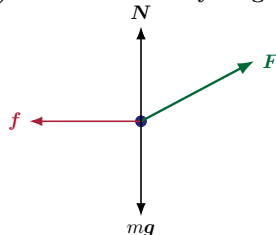


Figure 7.4: An angled cable pulls a cart horizontally. Panel (a) resolves the cable force; the projection is an auxiliary construction. Panel (b) contains each actual force once, with schematic arrow lengths. Gravity and the normal force do zero work in this horizontal particle model. Projection-coordinate construction follows the method studied in Izaak Neutelings, *Work & energy*, TikZ.net; the physics and layout are reconstructed for this chapter.

Chapter support: Revised Chapter 7, Figure 7.4.

Example 7.3: an angled service cable



Full statement

A service cart is pulled 6.0 m horizontally by a constant cable force of 85 N directed 28° above the horizontal. A constant rolling-resistance force of 22 N opposes the motion. Find the work done by the cable, the work done by resistance and the net work. Assume no vertical displacement.

Model: translating particle; fixed angle and force magnitudes. Write the actual displacement first: $\Delta \mathbf{r} = (6.0 \text{ m})\hat{i}$. Vertical force components therefore contribute no work.

Chapter support: Chapter 7, Example 7.3.

Example 7.3: selected and net work



Cable projection:

$$W_c = (85 \text{ N})(6.0 \text{ m}) \cos 28^\circ = 450.3 \text{ J}.$$

Resistance: opposite the motion,

$$W_r = -(22)(6.0) = -132 \text{ J}, \quad W_g = W_N = 0.$$

$$W_{\text{net}} = 450.3 - 132 = 318.3 \text{ J}.$$

Check: $(85 \cos 28^\circ - 22)(6.0) = 318.3 \text{ J}$. The net force's projection gives the same sum. The cable's positive transfer exceeds the net transfer because resistance removes energy.

Chapter support: Chapter 7, Example 7.3.

Return journeys: the force law matters

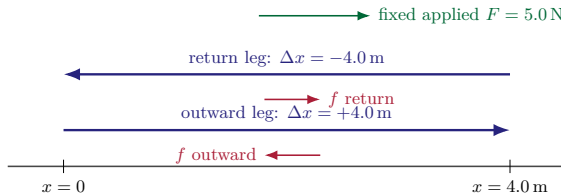


Figure 7.6: A round trip has zero total displacement but nonzero total distance. A fixed rightward applied force has opposite work signs on the two legs. Resistance reverses with the motion and does negative work on both. The separated arrows represent successive legs on the same straight guide.

A fixed vector uses total displacement and cancels on a return. Resistance reverses with velocity: $dW_r = -f d\ell$, where $d\ell > 0$ is distance. Its negative works add.

Chapter support: Revised Chapter 7, Section 7.2.

Practice 1: two forces on a round trip



Full statement: Example 7.4

A guided 2.0 kg trolley travels from $x = 0$ to $x = 4.0$ m and later returns to $x = 0$. Throughout both legs a device exerts a fixed force $F_x = +5.0$ N. A resistance of magnitude 2.0 N opposes the motion on each leg. Find the work of the device and of resistance, both separately by leg and for the whole trip. May one conclude that the trolley has no change in kinetic energy simply because its total displacement is zero?

Model: the path is specified; another interaction may be needed to arrange the turn and the return. Calculate only the two named forces.

Chapter support: Chapter 7, Example 7.4.

Practice 1: fixed force cancels; resistance does not



Choose rightward positive. The device stays rightward on both legs; resistance opposes each leg.

$$W_{F,\text{out}} = 5.0(4.0) = 20.0 \text{ J}, \quad W_{r,\text{out}} = -2.0(4.0) = -8.00 \text{ J}.$$

$$W_{F,\text{back}} = 5.0(-4.0) = -20.0 \text{ J}, \quad W_{r,\text{back}} = 2.0(-4.0) = -8.00 \text{ J}.$$

Thus $W_F = 0$ but $W_r = -16.0 \text{ J}$. Zero total displacement does not establish zero kinetic-energy change: the work of every interaction is needed. **Check:** total resistance work is $-f$ times total distance, $-(2.0)(8.0) = -16.0 \text{ J}$. If another actuator arranges equal endpoint speeds, it must compensate that 16.0 J loss.

Chapter support: Chapter 7, Example 7.4.

Variable work: add signed small transfers



On a monotonic one-dimensional leg, $\Delta W_j \simeq F_x(x_j^*)\Delta x_j$. Taking the limit gives

$$W = \int_{x_i}^{x_f} F_x(x) dx.$$

For increasing x , the geometric areas above and below the axis carry positive and negative signs. Reversing the same position-only force law reverses the integral limits.

The model used next

We stipulate $F_x = F_0(1 - x/L)$, with $F_0, L > 0$.

$$F_x(x) = F_0 \left(1 - \frac{x}{L}\right). \quad (7.8)$$

Zero force at $x = L$ does not establish zero speed or zero accumulated work.

Chapter support: Revised Chapter 7, Section 7.3.

Signed area: positive and negative regions

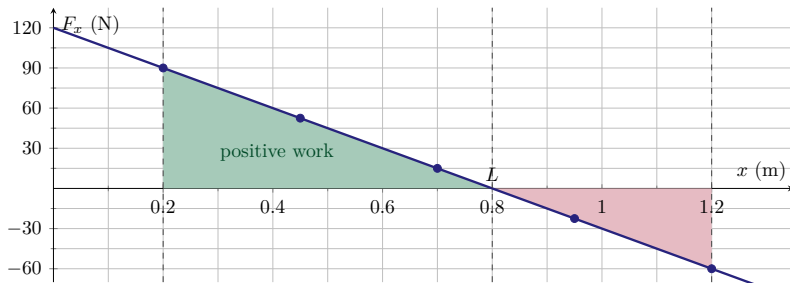


Figure 7.7: Analytical force–position curve for $F_x = F_0(1 - x/L)$ with $F_0 = 120$ N and $L = 0.80$ m. The shaded work from $x_i = 0.20$ m to $x_f = 1.20$ m is the positive green area plus the negative red area.

Chapter support: Revised Chapter 7, Figure 7.7.

Example 7.5: integrate a sign-changing force



For the force in Eq. (7.8), let $F_0 = 120 \text{ N}$ and $L = 0.80 \text{ m}$. Find the work as the particle moves from $x_i = 0.20 \text{ m}$ to $x_f = 1.20 \text{ m}$.

$$W = F_0 \left[(x_f - x_i) - \frac{x_f^2 - x_i^2}{2L} \right] = 120 \left[1.00 - \frac{1.40}{1.60} \right] = 15.0 \text{ J}.$$

The same result follows from the two triangles:

$$W_+ = \frac{1}{2}(0.60)(90) = 27.0 \text{ J}, \quad W_- = -\frac{1}{2}(0.40)(60) = -12.0 \text{ J}.$$

Thus $W_+ + W_- = 15.0 \text{ J}$. Reversal under the same field gives -15.0 J ; replacing the areas by magnitudes would give the wrong sign accounting.

Chapter support: Chapter 7, Example 7.5.

Cumulative work is not local force



Define running work from a fixed starting point:

$$W(x) = \int_{x_i}^x F_x(\xi) d\xi, \quad \frac{dW}{dx} = F_x(x).$$

Force sets the slope, rather than the value, of cumulative work. A maximum of smooth $W(x)$ has $F_x = 0$ even if a large work has accumulated.

Choose the appropriate average

For distinct endpoints, $\overline{F_x}^{(x)} = W/(x_f - x_i)$ is a displacement average. A time average weights slow regions longer and generally differs. For a force specified in time, use the trajectory: $W = \int F_x(t) v_x(t) dt$.

Chapter support: Revised Chapter 7, Section 7.3.

Deriving the work–kinetic-energy theorem



In an inertial frame for constant mass, start with Newton's vector law and use $d\mathbf{r} = \mathbf{v} dt$:

$$\begin{aligned}\mathbf{F}_{\text{net}} \cdot d\mathbf{r} &= m \frac{d\mathbf{v}}{dt} \cdot \mathbf{v} dt = m\mathbf{v} \cdot d\mathbf{v} \\ &= d\left(\frac{1}{2}m\mathbf{v} \cdot \mathbf{v}\right) = dK.\end{aligned}$$

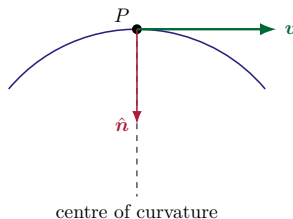
$$W_{\text{net}} = \sum_i \int \mathbf{F}_i \cdot d\mathbf{r} = K_f - K_i.$$

The differential identity $d(\mathbf{v} \cdot \mathbf{v}) = 2\mathbf{v} \cdot d\mathbf{v}$ supplies the central step. All forces refer to the same particle, path and frame; the path may be curved.

Chapter support: Revised Chapter 7, Section 7.4.

A normal force can redirect motion

(a) Exact local geometry



(b) Force components

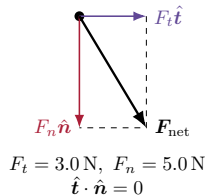


Figure 7.8: At the top of a clockwise circular path, the tangent is horizontal and the inward normal vertical. Panel (b) draws force components on a common force scale: their ratio is 3 : 5, and their vector sum is the diagonal. The normal component redirects motion; only $F_t v$ contributes to the instantaneous change of kinetic energy. Velocity and force are shown in separate panels and use different display scales.

The derivation used no division by speed, so it remains valid at $v = 0$. A turning particle can have $K = 0$ with a nonzero restoring force.

Chapter support: Revised Chapter 7, Section 7.4.

Example 7.6: tangent versus normal



A 0.75 kg guided slider moves at 4.0 m s^{-1} along a curved track. At one instant the net force has tangential component $F_t = 3.0\text{ N}$ in the direction of motion and normal component $F_n = 5.0\text{ N}$. Find the instantaneous rate of change of kinetic energy. What role does the normal component play?

$$\frac{dK}{dt} = \mathbf{F}_{\text{net}} \cdot \mathbf{v} = F_t v = (3.0)(4.0) = 12\text{ W}.$$

F_n is perpendicular to $\mathbf{v}$ and contributes zero rate of work. It changes direction through the normal acceleration $a_n = F_n/m$; the tangent changes speed through $a_t = F_t/m$. **Check:** $dK/dt = mv(dv/dt) = mv(F_t/m) = F_t v$, giving the same rate. The force and velocity arrows have different display scales.

Chapter support: Chapter 7, Example 7.6.

Reachability is a whole-path test



For a specified net force on a one-dimensional leg,

$$K(x) = K_i + \int_{x_i}^x F_{\text{net}}(\xi) d\xi, \quad v(x) = \sqrt{\frac{2K(x)}{m}} \quad (K \geq 0).$$

Velocity direction still comes from the trajectory. Check $K \geq 0$ throughout the continuous path, rather than only at the endpoint.

Interpreting a zero

At a first zero, inspect the force. If it points back toward the accessible region, the particle turns. A later positive formal K behind a negative interval is inaccessible. A zero at equilibrium or a model boundary needs its own continuation analysis.

Chapter support: Revised Chapter 7, Section 7.4.

A force graph becomes a kinetic-energy graph

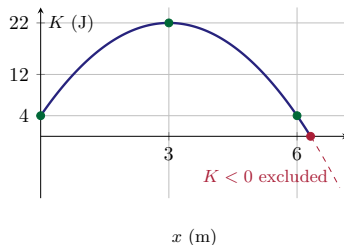
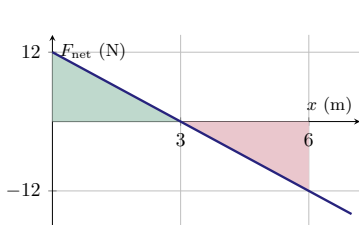


Figure 7.9: The net force $F_{\text{net}} = (12 \text{ N}) - (4 \text{ N m}^{-1})x$ and the corresponding $K(x)$ for $m = 2.00 \text{ kg}$, $x_i = 0$, and $v_i = 2.00 \text{ m s}^{-1}$. Equal positive and negative force areas between 0 and 6 m restore the initial kinetic energy. The maximum K is at the force zero. The dashed negative continuation is a mathematical prediction that the forward trajectory cannot realize.

Chapter support: Revised Chapter 7, Figure 7.9.

Example 7.7: acceleration, braking and reversal

Full statement

A 2.00 kg slider enters a horizontal guide at $x = 0$ moving toward $+x$ with speed 2.00 m s^{-1} . The net force is

$$F_{\text{net}}(x) = (12.0 \text{ N}) - (4.00 \text{ N m}^{-1})x.$$

Find the speed at $x = 3.00 \text{ m}$ and at $x = 6.00 \text{ m}$, the maximum speed on the forward leg, and the forward turning point. Decide whether the slider reaches $x = 7.00 \text{ m}$ under this model.

The statement already specifies the *net* force. Add no extra resistance term; integrate it once to obtain the entire kinetic-energy profile.

Chapter support: Chapter 7, Example 7.7.

Example 7.7: running energy and peak speed



For $F = A - Bx$, with $A = 12.0 \text{ N}$ and $B = 4.00 \text{ N m}^{-1}$,

$$K_i = \frac{1}{2}(2.00)(2.00)^2 = 4.00 \text{ J}, \quad K(x) = K_i + Ax - \frac{1}{2}Bx^2.$$

Entering x in metres, $K(x) = 4 + 12x - 2x^2$ joules.

$$K(3) = 22.0 \text{ J}, \quad v(3) = \sqrt{22} = 4.69 \text{ m s}^{-1};$$

$$K(6) = 4.00 \text{ J}, \quad v(6) = 2.00 \text{ m s}^{-1}.$$

The maximum satisfies $dK/dx = A - Bx = 0$, so $x_{\text{max}} = 3.00 \text{ m}$ and $v_{\text{max}} = 4.69 \text{ m s}^{-1}$. Equal $+18.0$ and -18.0 J work areas to 6 m restore the initial speed.

Chapter support: Chapter 7, Example 7.7.

Example 7.7: the first forward turning point



Set $K(x) = 0$ and select the root on the forward leg:

$$x_{\text{turn}} = \frac{A + \sqrt{A^2 + 2BK_i}}{B} = 3 + \sqrt{11} = 6.32 \text{ m.}$$

The second root $3 - \sqrt{11} < 0$ lies outside that leg. At the positive root,

$$F_{\text{net}} = -13.3 \text{ N,}$$

so the stopped particle accelerates left and reverses. At $x = 7.00 \text{ m}$, formal $K = 4 + 84 - 98 = -10.0 \text{ J}$: the particle cannot reach it. **Checks:** differentiating K restores F ; Bx^2 has energy units; the turning point exceeds 6 m , where speed is still positive.

Chapter support: Chapter 7, Example 7.7.

Braking removes a kinetic-energy budget

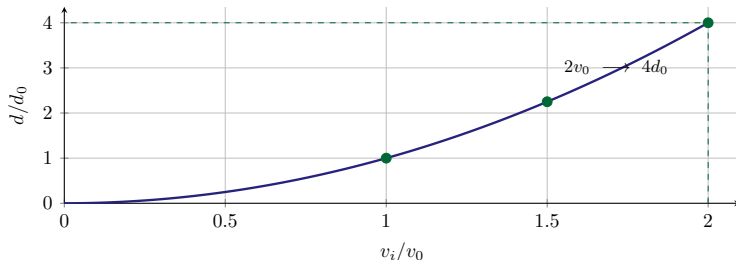


Figure 7.10: For the same mass and constant opposing net force, stopping distance scales with the square of initial speed. Here $d_0 = mv_0^2/(2B)$. This comparison excludes any delay before the braking force acts and any change in force with speed. A constant opposing net force $B > 0$ gives $-Bd = -\frac{1}{2}mv_i^2$, hence $d = mv_i^2/(2B)$. If $B = \mu_k mg$ the mass cancels; if B is a fixed actuator limit, it does not.

Chapter support: Revised Chapter 7, Section 7.4.

Example 7.8: a fixed braking-force limit



Full statement

A 600 kg guided vehicle enters a braking region at 12.0 m s^{-1} . Its approximately constant opposing net force is 1.80 kN . Find the stopping distance. Compare the distance at an entry speed of 18.0 m s^{-1} . If the second vehicle must stop within the first distance, what opposing net force is required?

The stated force is the complete opposing longitudinal resultant. The mechanical stopping distance begins when braking begins.

$$1.80\text{ kN} = 1800\text{ N}.$$

Chapter support: Chapter 7, Example 7.8.

Example 7.8: stopping distance and force



Use $d = K_i/B$ and keep the model fixed when comparing speeds.

Entry speed	K_i	d at $B = 1800 \text{ N}$
12.0 m s^{-1}	43.2 kJ	24.0 m
18.0 m s^{-1}	97.2 kJ	54.0 m

To stop the second vehicle within 24.0 m,

$$B' = 97200/24.0 = 4050 \text{ N} = 4.05 \text{ kN}.$$

Independent check: $a = B/m = 3.00 \text{ m s}^{-2}$ and $0 = v_i^2 - 2ad$ give $d = 144/6 = 24.0 \text{ m}$. A speed ratio 1.50 gives an energy and distance ratio 2.25.

Chapter support: Chapter 7, Example 7.8.

Inverse dynamics: force is the slope of K

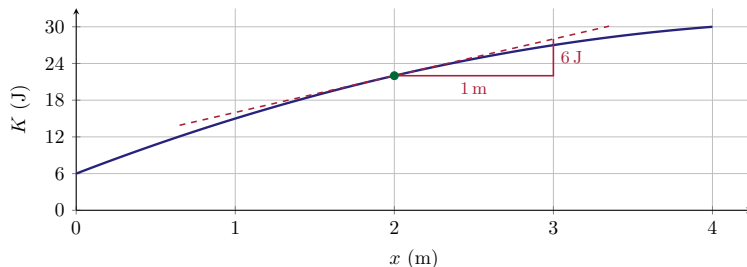


Figure 7.11: The tangent slope of $K(x) = 6 + 10x - x^2$ (SI numerical values) at $x = 2$ m is 6 J m^{-1} , so the net longitudinal force is 6 N there. The dashed tangent is local information, not an alternative kinetic-energy law over the entire guide. On a smooth one-dimensional leg, $F_{\text{net}} = dK/dx$. The units are $\text{J/m} = \text{N}$. This identifies the resultant; separate actuator and resistance contributions need additional information.

Chapter support: Revised Chapter 7, Section 7.4.

Example 7.9: infer an actuator contribution

Full statement

On the outward leg of a guided 3.00 kg slider, a controller produces the theoretical profile

$$K(x) = (6.00 \text{ J}) + (10.0 \text{ N})x - (1.00 \text{ N m}^{-1})x^2, \quad 0 \leq x \leq 4.00 \text{ m}.$$

A separate constant resistance of magnitude 1.50 N opposes the motion. Find the net force and required actuator force at $x = 2.00 \text{ m}$, the speed there, and the net work from 0 to 4.00 m.

Chapter support: Chapter 7, Example 7.9.

Example 7.9: slope, speed and work



Differentiate the profile: $F_{\text{net}} = 10.0 - 2.00x$ N for x in metres. On the outward leg $F_r = -1.50$ N.

$$F_{\text{net}}(2) = 6.00 \text{ N}, \quad F_A(2) = 6.00 - (-1.50) = 7.50 \text{ N}.$$

$$K(2) = 22.0 \text{ J}, \quad v(2) = \sqrt{44/3.00} = 3.83 \text{ m s}^{-1}.$$

$$W_{\text{net}} = K(4) - K(0) = 30.0 - 6.00 = 24.0 \text{ J}.$$

Checks: $\int_0^4 (10 - 2x) dx = 24.0 \text{ J}$. Resistance does -6.00 J ; the actuator does 30.0 J . $K > 0$ throughout $[0, 4]$, so this outward interval is accessible.

Chapter support: Chapter 7, Example 7.9.

Uniform gravity: follow the height change



Choose $+y$ upward and take g constant over the height range:

$$\mathbf{F}_g = -mg\hat{\mathbf{j}}, \quad W_g = \int_{y_i}^{y_f} (-mg)dy = mg(y_i - y_f).$$

Horizontal displacement contributes zero. Gravity's work depends on end-point heights in this model: positive down, negative up.

Full statement: Example 7.11

A 4.0 kg inspection tool is lowered from $y_i = 12.0$ m to $y_f = 7.5$ m. What work does gravity do?

$$W_g = (4.0)(9.81)(12.0 - 7.5) = 176.58 \text{ J} \simeq 177 \text{ J}.$$

The positive sign follows independently from aligned downward force and displacement.

Chapter support: Chapter 7, Example 7.11.

An ideal spring: define the signed coordinate

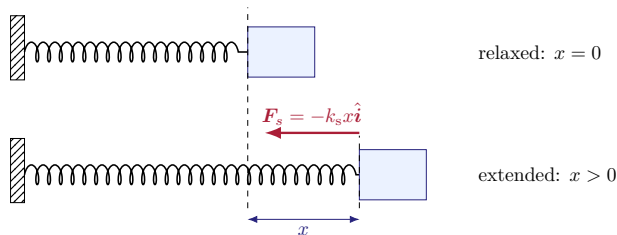


Figure 7.14: Two configurations of an ideal spring with the same fixed left support. The signed extension x is the displacement of the moving spring attachment from its relaxed position; it is positive for extension and negative for compression. Object sizes and spring coils are schematic. Coordinate-based coils and reference guides use the construction method studied in Neutelings, *Springs & masses*, TikZ.net.

Chapter support: Revised Chapter 7, Figure 7.14.

Spring work: transform coordinate and differential



For signed extension x from the relaxed length, $F_s = -k_s x$, with k_s in N/m. Integration gives

$$W_s = \int_{x_i}^{x_f} -k_s x dx = \frac{1}{2} k_s (x_i^2 - x_f^2).$$

On the compressed side, let $q = -x \geq 0$. Then $F_s = +k_s q$ but $dx = -dq$, so

$$dW_s = (k_s q)(-dq) = -k_s q dq.$$

Further compression gives negative work; relaxation gives positive work. Do not combine a Cartesian force component with an oppositely oriented coordinate differential. **Scope:** the linear law applies only within its stated ideal range.

Chapter support: Revised Chapter 7, Section 7.5.

Example 7.12: further compression from motion

Full statement

A 1.50 kg carriage moves on a horizontal frictionless guide with speed 2.50 m s^{-1} when the spring is already compressed by $q_i = 0.040\text{ m}$. Here $q = -x \geq 0$ is the compression magnitude; the signed extension is $x_i = -0.040\text{ m}$. It continues in the compression direction until $q_f = 0.120\text{ m}$. The spring constant is $k_s = 320\text{ N m}^{-1}$. Find the spring work and the carriage speed at q_f . Also find the maximum compression predicted by the ideal model.

Chapter support: Chapter 7, Example 7.12.

Example 7.12: speed and maximum compression



Only the ideal spring does work; the guide is frictionless and horizontal.

$$W_s = \frac{1}{2}(320)(0.040^2 - 0.120^2) = -2.048 \text{ J.}$$

$$K_i = \frac{1}{2}(1.50)(2.50)^2 = 4.6875 \text{ J}, \quad K_f = 2.6395 \text{ J.}$$

$$v_f = \sqrt{2(2.6395)/1.50} = 1.88 \text{ m s}^{-1}.$$

At the first zero of $K(q)$,

$$q_{\text{turn}} = \sqrt{q_i^2 + \frac{mv_i^2}{k_s}} = 0.176 \text{ m.}$$

Check: $K(q)$ decreases during compression. The requested 0.120 m is reached before the first zero; larger q beyond q_{turn} is inaccessible on this leg.

Chapter support: Chapter 7, Example 7.12.

Practice 2: identify spring and resistance

Full statement: Problem 46

Identify a spring and friction from stopping data. An unknown ideal spring and constant friction stop a block during positive compression q . A first trial has incident kinetic energy 2.0 J and stops at $q = 0.100$ m; a second has 6.0 J and stops at $q = 0.200$ m. Infer stiffness and friction magnitude. Predict stopping compression for $K_i = 8.0$ J. Explain why two identical compression values would not provide independent equations for these two parameters.

Start here: for each stopping trial,

$$E_i = \frac{1}{2}k_s q_i^2 + f q_i.$$

The spring and resistance contributions are distinct. Use the two trials as independent equations for the two model parameters.

Chapter support: Chapter 7, Problem 46.

Practice 2: solve the inverse model



In SI units, the two stopping equations are

$$0.005k_s + 0.100f = 2.0, \quad 0.020k_s + 0.200f = 6.0.$$

Subtract twice the first from the second: $0.010k_s = 2.0$, giving $k_s = 200 \text{ N m}^{-1}$ and then $f = 10.0 \text{ N}$.

$$100q^2 + 10q - 8 = 0, \quad q = \frac{-10 + \sqrt{3300}}{200} = 0.237 \text{ m}.$$

The negative root -0.337 m violates $q \geq 0$. Since $dK/dq = -200q - 10 < 0$, the accepted root is the first reachable stop. **Identifiability:** identical q values produce identical coefficient rows. They cannot determine both parameters; different energies at the same q would contradict this model.

Chapter support: Chapter 7, Problem 46.

Power: rate of a specified transfer



Average power uses a specified work and time interval. Instantaneous power uses the velocity of the force's application point:

$$\mathcal{P}_{\text{av}} = \frac{W}{\Delta t}, \quad \mathcal{P} = \frac{dW}{dt} = \mathbf{F} \cdot \mathbf{v}.$$

One watt is one joule per second. Parallel force and velocity give positive power; opposite vectors give negative power; perpendicular vectors give zero.

Particle versus mechanical port

Net particle power is dK/dt . A selected actuator power need not equal it when other forces also act. A stationary load receives zero mechanical power from a holding force, even if the apparatus consumes energy internally.

Chapter support: Revised Chapter 7, Section 7.6.

Constant force and constant power differ

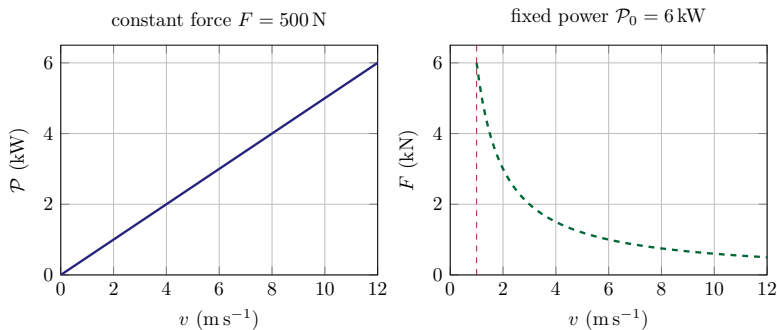


Figure 7.18: analytical operating-law comparison. With constant force, power grows linearly with speed. With fixed positive power, the ideal force varies as $1/v$. The displayed fixed-power branch begins at $v = 1 \text{ m s}^{-1}$ and cannot be extrapolated to $v = 0$.

Chapter support: Revised Chapter 7, Figure 7.18.

Example 7.16: an ideal lifting actuator



Full statement

An ideal lifting mechanism raises a 650 kg load vertically at constant speed 0.75 m s^{-1} . Find the useful mechanical power delivered to the load. If an explicitly specified overall energy-transfer efficiency is $\eta = 0.82$, what input power is required in this simplified model?

Model: steady upward translation with gravity as the only opposing load force. First find the useful mechanical transfer to the load; then apply the stipulated input/output definition.

Chapter support: Chapter 7, Example 7.16.

Example 7.16: useful and input power



Constant speed gives zero acceleration: $F_A - mg = 0$.

$$\mathcal{P}_{\text{useful}} = F_A v = mgv = (650)(9.81)(0.75) = 4.7824 \text{ kW} \simeq 4.78 \text{ kW}.$$

Define the stated efficiency for these two transfers:

$$\eta = \frac{\mathcal{P}_{\text{useful}}}{\mathcal{P}_{\text{in}}}, \quad \mathcal{P}_{\text{in}} = \frac{4.7824}{0.82} = 5.83 \text{ kW}.$$

Checks: input exceeds useful output for $0 < \eta < 1$; mgv has units $\text{N m/s} = \text{W}$. Over any steady interval Δt , multiplying both powers by Δt reproduces the same energy ratio. The given 0.82 is a teaching parameter; no additional device performance is inferred.

Chapter support: Chapter 7, Example 7.16.

Average the product, not the factors



During T , the work is $W = \int_0^T \mathbf{F}(t) \cdot \mathbf{v}(t) dt$. Thus

$$\mathcal{P}_{\text{av}} = \langle \mathbf{F} \cdot \mathbf{v} \rangle \neq \langle \mathbf{F} \rangle \cdot \langle \mathbf{v} \rangle \quad \text{in general.}$$

In one dimension, write each quantity as its mean plus its deviation:

$$\langle Fv \rangle = \langle F \rangle \langle v \rangle + \langle (F - \langle F \rangle)(v - \langle v \rangle) \rangle.$$

The last term is the time covariance. With larger force at larger speed it can be positive. For constant F , $\langle Fv \rangle = F \langle v \rangle$ exactly. Force and velocity records must refer to the same contact point and time.

Chapter support: Revised Chapter 7, Section 7.6.

Example 7.15: a dynamically consistent history



A 2.00 kg particle moves on a horizontal frictionless guide. Its initial velocity is 1.00 m s^{-1} , and its only horizontal force is $F(t) = (4.00 \text{ N s}^{-1})t$ for $0 \leq t \leq 2.00 \text{ s}$. Find the work and average power; compare the result with the product of the time-averaged force and velocity. **Derive the trajectory before integrating power.** Newton's law gives

$$a(t) = F(t)/m = (2.00 \text{ m s}^{-3})t.$$

$$v(t) = 1.00 \text{ m s}^{-1} + (1.00 \text{ m s}^{-3})t^2.$$

This velocity follows from the supplied net force and initial condition. The force law is a time law; it cannot be inserted into a position integral without this trajectory.

Chapter support: Chapter 7, Example 7.15.

Example 7.15: integrate and compare averages



With numerical t in seconds, $\mathcal{P}(t) = 4t + 4t^3$ watts:

$$W = \int_0^2 (4t + 4t^3) dt = [2t^2 + t^4]_0^2 = 24.0 \text{ J}, \quad \mathcal{P}_{\text{av}} = 12.0 \text{ W}.$$

$$\langle F \rangle = 4.00 \text{ N}, \quad \langle v \rangle = \frac{1}{2} \int_0^2 (1 + t^2) dt = \frac{7}{3} \text{ m s}^{-1}.$$

Their product is $28/3 = 9.33 \text{ W}$. The missing covariance is $8/3 = 2.67 \text{ W}$, not a rounding error. **Independent check:** $v_f = 5.00 \text{ m s}^{-1}$ and $\Delta K = \frac{1}{2}(2.00)(5.00^2 - 1.00^2) = 24.0 \text{ J}$. Multiplying separate means would incorrectly give only 18.7 J .

Chapter support: Chapter 7, Example 7.15.

Finite startup joined to a power cap

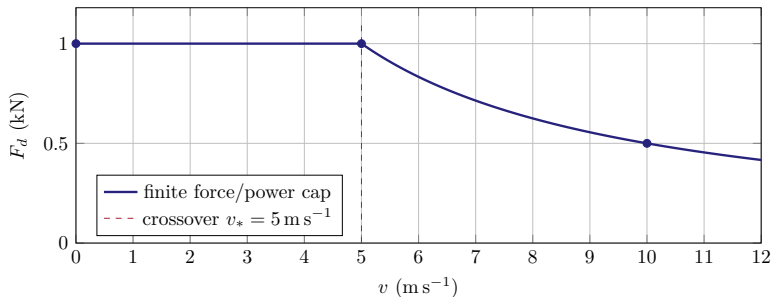


Figure 7.21: A bounded forward drive law. The horizontal branch includes startup at $v = 0$, with finite force and zero mechanical power. The decreasing branch begins at the finite crossover speed. Its construction avoids extrapolating $\mathcal{P}_{\max}/v$ to rest. For forward motion the bounded law is $F_d = F_{\max}$ below $v_* = \mathcal{P}_{\max}/F_{\max}$, then $F_d = \mathcal{P}_{\max}/v$ above v_* . The two branches meet; finite startup force gives zero power at rest.

Chapter support: Revised Chapter 7, Section 7.6.

Example 7.18: startup under two caps



A 100 kg carriage starts from rest on a frictionless horizontal guide. Its drive has $F_{\max} = 1000 \text{ N}$ and $\mathcal{P}_{\max} = 5000 \text{ W}$, following the bounded law above. Find the time and distance to reach 10.0 m s^{-1} , the total work, and compare with an uncapped constant 1000 N drive. **Force-limited phase:**

$$v_* = 5000/1000 = 5.00 \text{ m s}^{-1}, \quad a = 1000/100 = 10.0 \text{ m s}^{-2}.$$

$$t_* = v_*/a = 0.500 \text{ s}, \quad x_* = \frac{1}{2}at_*^2 = 1.250 \text{ m}.$$

Its work is $F_{\max}x_* = 1250 \text{ J}$. At the crossover both branches give 1000 N and 5000 W.

Chapter support: Chapter 7, Example 7.18.

Example 7.18: integrate the finite-speed phase



For $t \geq t_*$, $mv \, dv/dt = \mathcal{P}_{\max}$. Hence

$$\Delta t = \frac{m(v_f^2 - v_*^2)}{2\mathcal{P}_{\max}} = 0.750 \text{ s}, \quad t_f = 1.250 \text{ s}.$$

For distance, $dt = (m/\mathcal{P}_{\max})v \, dv$, so

$$\Delta x = \frac{m(v_f^3 - v_*^3)}{3\mathcal{P}_{\max}} = 5.833 \text{ m}, \quad x_f = 7.083 \text{ m}.$$

Work is $1250 + 5000(0.750) = 5000 \text{ J} = \frac{1}{2}(100)(10.0)^2$. **Comparison:** uncapped 1000 N reaches the same speed in 1.00 s over 5.00 m, but its final power is 10000 W. The same work need not imply the same time, distance or peak power.

Chapter support: Chapter 7, Example 7.18.

Extended bodies: use actual application points



For force $\mathbf{F}_i$ applied at material point i , use that point's path:

$$W_{\text{ext}} = \sum_i \int \mathbf{F}_i \cdot d\mathbf{r}_i.$$

The centre-of-mass relation instead tracks translation only:

$$\Delta K_{\text{CM}} = \int \mathbf{F}_{\text{ext,net}} \cdot d\mathbf{R}_{\text{CM}}, \quad K_{\text{CM}} = \frac{1}{2} M V_{\text{CM}}^2.$$

For pure rigid translation, all $d\mathbf{r}_i = d\mathbf{R}_{\text{CM}}$ and the expressions agree. Rotation or deformation can make them differ. **Answered check:** zero net external force does not ensure zero external work; equal opposite forces can act through different displacements.

Chapter support: Revised Chapter 7, Section 7.7.

A connector can stretch with fixed centre of mass

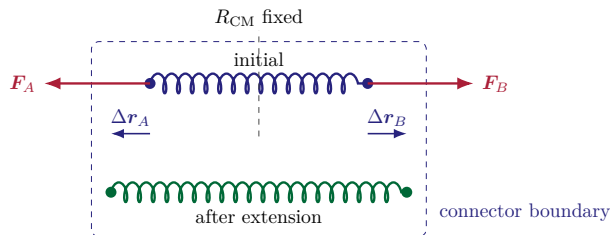


Figure 7.23: Equal and opposite endpoint forces can have zero net force and zero centre-of-mass displacement while both application points move along their respective forces. The dashed boundary selects the connector; endpoint forces transfer energy inward through positive work. The centre-of-mass path is a fixed point, so its pseudowork is zero. Force arrows denote forces, not energy-flow rates.

Chapter support: Revised Chapter 7, Figure 7.23.

Example 7.20: endpoint work survives force cancellation



A deformable connector has two ends A and B . A constant force $\mathbf{F}_A = -400\hat{\mathbf{i}}\text{ N}$ acts at the left end and $\mathbf{F}_B = +400\hat{\mathbf{i}}\text{ N}$ acts at the right end. During a slow symmetric extension, A moves 0.030 m left and B moves 0.030 m right. The centre of mass remains fixed. Find the actual external work and compare it with the centre-of-mass work.

$$W_A = (-400)(-0.030) = 12.0\text{ J}, \quad W_B = (400)(0.030) = 12.0\text{ J}.$$

Thus $W_{\text{ext}} = 24.0\text{ J}$. But $\mathbf{F}_{\text{ext,net}} = 0$ and $\Delta\mathbf{R}_{\text{CM}} = 0$, so the centre-of-mass contribution is zero. **Interpretation:** energy enters through the relative endpoint motions and changes the connector's internal state. Factoring out one common displacement would contradict the deformation geometry. Units and both positive force–displacement alignments independently check the sign.

Chapter support: Chapter 7, Example 7.20.

Work from data: the interpolation is a model



For oriented samples (x_j, F_j) , segmentwise trapezoids give

$$W_T = \sum_{j=0}^{N-1} \frac{1}{2} (F_j + F_{j+1}) (x_{j+1} - x_j).$$

This exactly integrates the piecewise linear interpolant. With equal spacing h and even N , Simpson integrates paired quadratic interpolants:

$$W_S = \frac{h}{3} \left[F_0 + F_N + 4 \sum_{j \text{ modd}} F_j + 2 \sum_{\substack{j \text{ meven} \\ 0 < j < N}} F_j \right].$$

Keep chronological branch order, signs and SI units; do not sort a loading cycle into one increasing- x curve. A constant sensor offset b biases a monotonic work by $b\Delta x$; finer integration does not remove it.

Chapter support: Revised Chapter 7, Section 7.8.

Practice 3: sparse force samples



Force samples are $F(0) = 20 \text{ N}$, $F(0.50 \text{ m}) = 15 \text{ N}$ and $F(1.00 \text{ m}) = 4.0 \text{ N}$. Estimate work with trapezoidal and Simpson rules. Explain what their disagreement says about the data, and what further evidence would be needed to declare either estimate accurate. There are two equal intervals, $h = 0.50 \text{ m}$:

$$W_T = 0.50 \left[\frac{20 + 4}{2} + 15 \right] = 13.5 \text{ J},$$

$$W_S = \frac{0.50}{3} (20 + 4 \cdot 15 + 4) = 14.0 \text{ J}.$$

The 0.5 J disagreement reflects different interpolation curvature. It does not identify either exact integral. Additional samples, an independent benchmark or a defensible force model are needed; if a quadratic law were independently established, Simpson would be exact.

Chapter support: Chapter 7, Problem 22.

Convergence tests the numerical calculation

For synthetic $F_x = 90e^{-x/(0.50\text{ m})} - 15\text{ N}$ on $[0, 1.00]\text{ m}$, direct integration gives $W_* = 45(1 - e^{-2}) - 15 = 23.909912\text{ J}$.

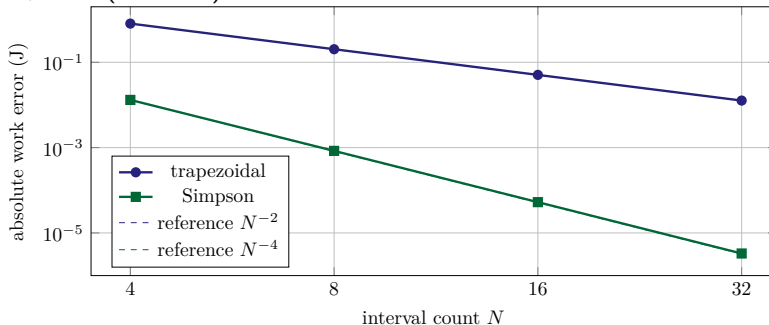


Figure 7.26: Quadrature convergence using full-precision samples of the chapter's exponential force law. Dashed reference lines are anchored at $N = 4$ and display the expected asymptotic slopes. The near-parallel trends verify numerical behaviour for this analytic test; they do not calibrate a force sensor or validate the force law for a real device.

Chapter support: Revised Chapter 7, Figure 7.26.

From numerical work to a checked speed



Original teaching application, based on the chapter benchmark. A 4.00 kg particle starts at $x = 0$ moving toward $+x$ with $K_i = 16.0$ J. Its net force is the exponential law on slide 58. Find its speed at 1.00 m and justify accessibility.

$$K_f = 16.0 + 23.909912 = 39.909912 \text{ J}, \quad v_f = \sqrt{2K_f/4.00} = 4.47 \text{ m s}^{-1}.$$

The force decreases and crosses zero once at $x = 0.50 \ln 6 = 0.896$ m. Thus $K(x)$ rises, then falls; its minimum on the interval is one of the endpoints, both positive. The endpoint is accessible. **Numerical check:** the $N = 8$ Simpson work is 23.910750 J, an energy error 0.000838 J. Accuracy of this synthetic calculation does not calibrate a sensor.

Chapter support: Chapter 7 numerical benchmark; original teaching application.

Core synthesis and the bridge to Chapter 8



- **Name the frame and system.** $K = \frac{1}{2}mv^2$ gives speed information; Newton's vector law supplies direction and constraint tests.
- **Name each force and path.** $W = \int \mathbf{F} \cdot d\mathbf{r}$ is a signed transfer; only net particle work equals ΔK .
- **Name the interval and port.** $\mathcal{P} = \mathbf{F} \cdot \mathbf{v}$ is a rate at an actual application point; peak and average powers differ.
- **Test the model.** Check units, the whole reachable path and numerical resolution. Positive energy alone does not guarantee surface contact.

Gravity and spring work can be written in terms of endpoint configurations. Chapter 8 develops potential energy, suitable force fields and mechanical-energy conservation from that observation.

Chapter support: Revised Chapter 7, Section 7.10.

Advanced: work on a parametrized path



Let $\mathbf{r}(s)$ describe an oriented prescribed curve C , with $s_a \leq s \leq s_b$. The parameter need not be time:

$$d\mathbf{r} = \mathbf{r}'(s)ds, \quad W_C = \int_{s_a}^{s_b} \mathbf{F}(\mathbf{r}(s)) \cdot \mathbf{r}'(s)ds.$$

If $d\ell > 0$ is arc length and $\hat{\mathbf{t}}$ its oriented tangent, $d\mathbf{r} = \hat{\mathbf{t}}d\ell$, giving $W_C = \int_C \mathbf{F}_t d\ell$.

Force and path used next

We stipulate $\mathbf{F} = ay\hat{\mathbf{i}}$, $[a] = \text{N/m}$, and $y = H(x/L)^p$ for $0 \leq x \leq L$, $p > 0$. These are prescribed paths; a guide or controller may be needed. The specified field work is not automatically net work.

Chapter support: Revised Chapter 7, Section 7.9.

Path shape controls the sampled force

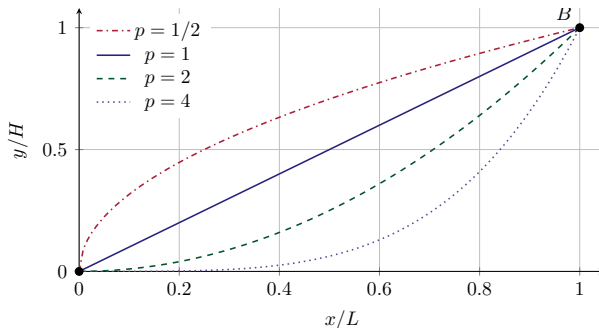


Figure 7.29: Analytical prescribed paths in the field $\mathbf{F} = ay\hat{\mathbf{i}}$. The endpoints coincide, but the height sampled during horizontal motion changes with p . Axes are normalized by the physical lengths L, H ; the four curves are distinguished by line style as well as colour.

Since $F_y = 0$, $W_p = \int_0^L aH(x/L)^p dx = aHL/(p+1)$. Use the physical scales L, H even when the plot uses $X = x/L$, $Y = y/H$.

Chapter support: Revised Chapter 7, Figure 7.29.

Advanced challenge: parameter and reversal



Full statement: Example 7.27

For $a = 30 \text{ N m}^{-1}$, $L = 0.40 \text{ m}$ and $H = 0.30 \text{ m}$, find the work along the parabola $y = H(x/L)^2$. Verify it using $x = Lq^3$, then find the work when the same path is traversed backwards.

Restored field: $\mathbf{F} = ay\hat{\mathbf{i}}$; traverse from $O = (0, 0)$ to $B = (L, H)$. **Chain rule:** if $s = g(q)$ increases along the same curve, $d\mathbf{r}/dq = (d\mathbf{r}/ds)g'(q)$. The displacement conversion is essential. Reversing orientation is a different operation from changing parameter.

Chapter support: Chapter 7, Example 7.27.

Advanced challenge: three consistent integrals



For $p = 2$, direct integration gives

$$W = \int_0^L aH(x/L)^2 dx = \frac{aHL}{3} = 1.20 \text{ J.}$$

With $0 \leq q \leq 1$, $x = Lq^3$, $y = Hq^6$ and $dx = 3Lq^2 dq$:

$$W = \int_0^1 (aHq^6)(3Lq^2) dq = 3aHL \left[\frac{q^9}{9} \right]_0^1 = 1.20 \text{ J.}$$

Reversing the same position-only field changes limits and gives $W_{B \rightarrow O} = -1.20 \text{ J}$. **Checks:** $(\text{N/m})(\text{m})(\text{m})$ is J. The monotonic mapping remains valid with zero derivative at $q = 0$. A friction force that reverses with slip would require a different reversal argument.

Chapter support: Chapter 7, Example 7.27.

Advanced: relative motion adds external work



For application point i , write $\mathbf{r}_i = \mathbf{R}_{\text{CM}} + \boldsymbol{\rho}_i$. Then

$$dW_{\text{ext}} = \mathbf{F}_{\text{ext,net}} \cdot d\mathbf{R}_{\text{CM}} + \sum_i \mathbf{F}_i \cdot d\boldsymbol{\rho}_i.$$

The second term vanishes in pure rigid translation; deformation need not make it vanish.

Full statement: Example 7.26

A symmetric connector is pulled quasistatically outward at both ends with equal and opposite force magnitude

$$F(\delta) = k_s \delta + b\delta^3,$$

where $\delta = \ell - \ell_0$ is the increase in total end-to-end length, $k_s = 180 \text{ N m}^{-1}$, and $b = 2.0 \times 10^5 \text{ N m}^{-3}$. The centre of mass remains fixed. Find the actual external work required to increase the length by $\delta_f = 0.040 \text{ m}$.

Chapter support: Chapter 7, Example 7.26.

Example 7.26: the deformation coordinate fixes the factor



The coordinate δ is the *total* separation increase. Each end moves outward by $d\delta/2$, along its own force:

$$dW_{\text{ext}} = F \frac{d\delta}{2} + F \frac{d\delta}{2} = F(\delta) d\delta.$$

$$W_{\text{ext}} = \int_0^{\delta_f} (k_s \delta + b\delta^3) d\delta = \frac{1}{2} k_s \delta_f^2 + \frac{1}{4} b \delta_f^4.$$

$$W_{\text{ext}} = 0.144 + 0.128 = 0.272 \text{ J}, \quad W_{\text{CM}} = 0.$$

Checks: both terms have joule units. Integrating each endpoint contribution separately gives the same total. The coordinate already includes both ends; another factor two would double-count them. The stipulated law supplies no measurement of rate dependence or hysteresis.

Chapter support: Chapter 7, Example 7.26.

Advanced: friction follows relative slip

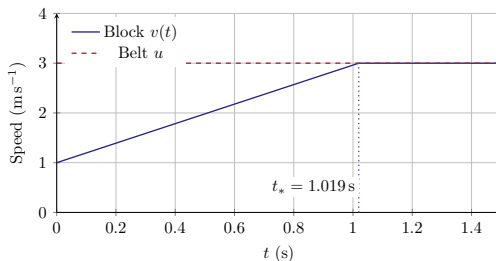


Figure 7.32: A block speeds up on a faster belt in the laboratory frame. The relative slip speed is the vertical difference between the curves. The ideal friction law changes at matching speed: with no other horizontal force, the subsequent contact force can be zero.

For $0 < v < u$, friction on the block points forward: $f = \mu_k mg$. Block power is $fv > 0$; the opposite belt contact power is $-fu$. Their sum $-f(u - v) < 0$ is the mechanical loss at sliding contact. A driver maintains the prescribed belt speed.

Chapter support: Revised Chapter 7, Section 7.9.

Example 7.28: a driven belt in two frames

Full statement

A 2.00 kg block starts at $v_0 = 1.00 \text{ m s}^{-1}$ on a belt moving at $u = 3.00 \text{ m s}^{-1}$. Take $\mu_k = 0.200$ and $g = 9.81 \text{ m s}^{-2}$. Find the matching time, block work, work on the belt and interface dissipation. Recalculate block work in the inertial frame moving with the belt.

Normal balance gives $N = mg$. While the block slips,

$$f = \mu_k mg = (0.200)(2.00)(9.81) = 3.924 \text{ N}, \quad a = f/m = 1.962 \text{ m s}^{-2}.$$

Friction points toward $+x$ because the belt is faster, not because the block is moving forward. Analyze only the slipping interval up to matching speed.

Chapter support: Chapter 7, Example 7.28.

Example 7.28: laboratory transfer and dissipation



Uniform acceleration until matching gives

$$t_* = (u - v_0)/a = 1.01937 \text{ s}, \quad s_b = \frac{1}{2}(v_0 + u)t_* = 2.03874 \text{ m}.$$

$$W_b = fs_b = 8.00 \text{ J}, \quad s_{\text{belt}} = ut_* = 3.05810 \text{ m}, \quad W_{\text{belt}} = -12.00 \text{ J}.$$

A driver supplies +12.00 J to keep the belt speed fixed.

$$E_{\text{diss}} = f(s_{\text{belt}} - s_b) = 4.00 \text{ J}.$$

Checks: $\Delta K_b = \frac{1}{2}(2.00)(3.00^2 - 1.00^2) = 8.00 \text{ J}$; driver work is $8.00 + 4.00 = 12.00 \text{ J}$. After matching, zero horizontal contact force is possible when other resistance is absent.

Chapter support: Chapter 7, Example 7.28.

Example 7.28: the belt frame closes the check



In the inertial belt frame, $v'_0 = 1.00 - 3.00 = -2.00 \text{ m s}^{-1}$ and $v'_f = 0$. The same positive friction acts through

$$s'_b = s_b - ut_* = -1.01937 \text{ m}, \quad W'_b = fs'_b = -4.00 \text{ J} = \Delta K'_b.$$

The belt contact is stationary, so its contact work is zero. Relative slip and $E_{\text{diss}} = 4.00 \text{ J}$ are unchanged.

$$\mathcal{P}'_i = \mathcal{P}_i - \mathbf{F}_i \cdot \mathbf{u}.$$

For the equal opposite contact pair the frame corrections cancel. Individual work and kinetic-energy changes are frame dependent; the slip loss is invariant. For positive friction, eliminating t_* gives $E_{\text{diss}} = \frac{1}{2}m(u - v_0)^2$. At zero friction no matching occurs, so that eventual-matching result cannot be extrapolated to $\mu_k = 0$.

Chapter support: Chapter 7, Example 7.28.

